MATHEMATICAL METHODS FORMULA SHEET

Properties of derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x}\Big(f(x)\,g(x)\Big)=f'(x)g(x)+f(x)g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right)=\frac{f'(x)g(x)-f(x)g'(x)}{\left[g(x)\right]^2}$$

$$\frac{\mathrm{d}}{\mathrm{d}x}f\Big(g(x)\Big)=f'\Big(g(x)\Big)g'(x)$$

Quadratic equations

If $ax^2+bx+c=0$ then $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Discrete random variables

The mean or expected value of a discrete random variable is:

$$\mu_X=\sum xp(x),$$

where $p(x)$ is the probability function for achieving result x.

The standard deviation of a discrete random variable is:

$$\sigma_X=\sqrt{\sum\left[x-\mu_X\right]^2 p(x)},$$

where μ_X is the expected value and $p(x)$ is the probability function for achieving result x.

Bernoulli distribution

The mean of the Bernoulli distribution is p, and the standard deviation is:

$$\sqrt{p(1-p)}.$$

Binomial distribution

The mean of the binomial distribution is np, and the standard deviation is:

$$\sqrt{np(1-p)},$$

where p is the probability of success in a single Bernoulli trial and n is the number of trials.

The probability of k successes from n trials is:

$$\Pr(X=k)=C_k^n p^k(1-p)^{n-k},$$

where p is the probability of success in the single Bernoulli trial.

Population proportions

The sample proportion is $\hat{p}=\dfrac{X}{n}$,

where a sample of size n is chosen, and X is the number of elements with a given characteristic.

The distribution of a sample proportion has a mean of p and a standard deviation of

$$\sqrt{\frac{p(1-p)}{n}}.$$

The upper and lower limits of a confidence interval for the population proportion are:

$$\hat{p}-z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\le p\le \hat{p}+z\sqrt{\frac{\hat{p}(1-\hat{p})}{n}},$$

where the value of z is determined by the confidence level required.

Continuous random variables

The mean or expected value of a continuous random variable is:

$$\mu_X=\int_{-\infty}^{\infty}x f(x)\,\mathrm{d}x,$$

where $f(x)$ is the probability density function.

The standard deviation of a continuous random variable is:

$$\sigma_X=\sqrt{\int_{-\infty}^{\infty}\left[x-\mu_X\right]^2 f(x)\,\mathrm{d}x},$$

where $f(x)$ is the probability density function.

Normal distributions

The probability density function for the normal distribution with mean μ and standard deviation σ is:

$$f(x)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}.$$

All normal distributions can be transformed to the standard normal distribution with $\mu=0$ and $\sigma=1$ by:

$$Z=\frac{X-\mu}{\sigma}.$$

Sampling and confidence intervals

If $\bar{x}$ is the sample mean of a sufficiently large sample, and σ is the population standard deviation, then the upper and lower limits of the confidence interval for the population mean are:

$$\bar{x}-z\frac{\sigma}{\sqrt{n}}\le\mu\le\bar{x}+z\frac{\sigma}{\sqrt{n}},$$

where the value of z is determined by the confidence level required.