LIST OF MATHEMATICAL FORMULAE FOR USE IN STAGE 2 MATHEMATICAL METHODS

Standardised Normal Distribution

A measurement scale X is transformed into a standard scale Z using the formula

$$Z = \frac{X - \mu}{\sigma}$$

where μ is the population mean and σ is the standard deviation for the population distribution.

Confidence Interval — Mean

A 95% confidence interval for the mean μ of a normal population with standard deviation σ, based on a simple random sample of size n with sample mean $\bar{x}$, is

$$\bar{x} - 1.96\frac{\sigma}{\sqrt{n}} \leq \mu \leq \bar{x} + 1.96\frac{\sigma}{\sqrt{n}}.$$

For suitably large samples, an approximate 95% confidence interval can be obtained by using the sample standard deviation s in place of σ.

Sample Size — Mean

The sample size n required to obtain a 95% confidence interval of width w for the mean of a normal population with standard deviation σ is

$$n = \left(\frac{2 \times 1.96\sigma}{w}\right)^2.$$

Confidence Interval — Population Proportion

An approximate 95% confidence interval for the population proportion p, based on a large simple random sample of size n with sample proportion

$\hat{p} = \frac{X}{n}$, is

$$\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \leq p \leq \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}.$$

Sample Size — Proportion

The sample size n required to obtain an approximate 95% confidence interval of approximate width w for a proportion is

$$n = \left(\frac{2 \times 1.96}{w}\right)^2 p^*\left(1 - p^*\right)$$

where p^* is a given preliminary value for the proportion.

Binomial Probability

$$P(X = k) = C_k^n p^k (1-p)^{n-k}$$

where p is the probability of a success in one trial and the possible values of X are $k = 0, 1, \ldots n$ and

$$C_k^n = \frac{n!}{(n-k)!k!} = \frac{n(n-1)\ldots(n-k+1)}{k!}.$$

Binomial Mean and Standard Deviation

The mean and standard deviation of a binomial count X and a proportion of successes $\hat{p} = \frac{X}{n}$ are

$$\mu_X = np \qquad \mu(\hat{p}) = p$$

$$\sigma_X = \sqrt{np(1-p)} \qquad \sigma(\hat{p}) = \sqrt{\frac{p(1-p)}{n}}$$

where p is the probability of a success in one trial.

Derivatives

$f(x) = y$	$f'(x) = \frac{\mathrm{d}y}{\mathrm{d}x}$
x^n	nx^{n-1}
e^{kx}	ke^{kx}
$\ln x = \log_e x$	$\frac{1}{x}$

Properties of Derivatives

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(x) + g(x)\} = f'(x) + g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(x) - g(x)\} = f'(x) - g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{k\,f(x)\} = k\,f'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\{f(x)g(x)\} = f'(x)g(x) + f(x)g'(x)$$

$$\frac{\mathrm{d}}{\mathrm{d}x}f(g(x)) = f'(g(x))g'(x)$$

Laws of Logarithms

$$\log A + \log B = \log AB$$

$$\log A - \log B = \log\frac{A}{B}$$

$$\log A^n = n\log A$$