Self-directed Clarifying Activity

Assessment Type 2: Folio - Functions and Reciprocals

Purpose

The purpose of this activity is to support teachers to interpret and apply performance standards consistently to students' work in Stage 1 Mathematics. To get the most value from this activity it is recommended that teachers spend time to complete step one before accessing the annotated student work sample in step two.

Steps

1. **Determine a grade for this student work sample.**

   Please note: grades are determined by using the Stage 1 Mathematics performance standards and considering whether evidence of learning demonstrates the specific features predominantly within a particular grade e.g. a B grade level work sample should demonstrate specific features predominantly at the B grade band.

2. **Access the annotated performance standards and student work sample.**

3. **Use the annotated performance standards and student work sample to compare your interpretation of the performance standards and recalibrate your assessment decision (if necessary).**

Once you have made an assessment decision
Access the annotated performance standards and student work sample by holding the 'Ctrl' key and clicking here.

Performance Standards for Stage 1 Mathematics

	Mathematical Knowledge and Skills and Their Application	Mathematical Modelling and Problem-solving	Communication of Mathematical Information
A	Comprehensive knowledge of content and understanding of concepts and relationships. Appropriate selection and use of mathematical algorithms and techniques (implemented electronically where appropriate) to find efficient solutions to complex questions. Highly effective and accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Development and effective application of mathematical models. Complete, concise, and accurate solutions to mathematical problems set in applied and theoretical contexts. Concise interpretation of the mathematical results in the context of the problem. In-depth understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made.	Highly effective communication of mathematical ideas and reasoning to develop logical arguments. Proficient and accurate use of appropriate notation, representations, and terminology.
B	Some depth of knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to complex questions. Accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Attempted development and appropriate application of mathematical models. Mostly accurate and complete solutions to mathematical problems set in applied and theoretical contexts. Complete interpretation of the mathematical results in the context of the problem. Some depth of understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made.	Effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly accurate use of appropriate notation, representations, and terminology.
C	Generally competent knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find mostly correct solutions to routine questions. Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Appropriate application of mathematical models. Some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts. Generally appropriate interpretation of the mathematical results in the context of the problem. Some understanding of the reasonableness and possible limitations of the interpreted results and some recognition of assumptions made.	Appropriate communication of mathematical ideas and reasoning to develop some logical arguments. Use of generally appropriate notation, representations, and terminology, with some inaccuracies.
D	Basic knowledge of content and some understanding of concepts and relationships. Some use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to routine questions. Sometimes accurate application of knowledge and skills to answer questions set in applied or theoretical contexts.	Application of a mathematical model, with partial effectiveness. Partly accurate and generally incomplete solutions to mathematical problems set in applied or theoretical contexts. Attempted interpretation of the mathematical results in the context of the problem. Some awareness of the reasonableness and possible limitations of the interpreted results.	Some appropriate communication of mathematical ideas and reasoning. Some attempt to use appropriate notation, representations, and terminology, with occasional accuracy.
E	Limited knowledge of content. Attempted use of mathematical algorithms and techniques (implemented electronically where appropriate) to find limited correct solutions to routine questions. Attempted application of knowledge and skills to answer questions set in applied or theoretical contexts, with limited effectiveness.	Attempted application of a basic mathematical model. Limited accuracy in solutions to one or more mathematical problems set in applied or theoretical contexts. Limited attempt at interpretation of the mathematical results in the context of the problem. Limited awareness of the reasonableness and possible limitations of the results.	Attempted communication of emerging mathematical ideas and reasoning. Limited attempt to use appropriate notation, representations, or terminology, and with limited accuracy.

STAGE 1 MATHEMATICS
FOLIO
FUNCTIONS AND RECIPROCALS

Purpose
To demonstrate your ability to:
- use mathematical modelling and problem-solving strategies as well as your knowledge, skills, and understanding of mathematical ideas and processes
- effectively and appropriately communicate relevant mathematical information within your solutions

Description of assessment
You will be investigating the patterns that emerge when comparisons are made between the graphs of straight lines and quadratic functions and their reciprocal graphs. Graphs can be created using a graphics calculator or graphing software.
This assessment allows you to show your skills in understanding and appropriate use of the mathematical concepts, processes, and strategies in the following subtopics:

Subtopic 12.1: Interpreting Points
Subtopic 12.5: Linking the Algebraic and Graphical Representations of a Relationship.

Assessment conditions
You have two weeks to complete this assessment task. Technology (graphics calculators or graphing software) may be used. You can present your investigation in written or multimodal mode e.g. a Power Point presentation. Your investigation should include:
- an introduction that outlines the problem to be explored.
- the method required to find a solution, in terms of the mathematical model or strategy to be used
- the appropriate application of the mathematical model or strategy, including
  - the generation or collection of relevant data and/or information, with details of the process of collection
  - mathematical calculations and results, and appropriate representations
  - the analysis and interpretation of results
  - reference to the limitations of the original problem as well as appropriate refinements and/or extensions
- a statement of the results and conclusions in the context of the original problem
- appendices and a bibliography as appropriate

Learning Requirements	*Assessment Design Criteria*	*Capabilities*
1. understand mathematical concepts and relationships, making use of electronic technology where appropriate to aid and enhance understanding 2. identify, collect, and organise mathematical information relevant to investigating and finding solutions to questions/problems taken from social, scientific, economic, or historical contexts 3. recognise and apply the mathematical techniques needed when analysing and finding a solution to a question/problem in context 4. interpret results, draw conclusions, and reflect on the reasonableness of these in the context of the question/problem 5. communicate mathematical reasoning and ideas , using appropriate language and representations 6. work both independently and cooperatively in planning, organising, and carrying out mathematical activities.	Mathematical Knowledge and Skills and Their Application The specific features are as follows: ▪MKSA1 Knowledge of content and understanding of mathematical concepts and relationships. ▪MKSA2 Use of mathematical algorithms and techniques (implemented electronically as appropriate) to find solutions to routine and complex questions. ▪MKSA3 Application of knowledge and skills to answer questions set in applied and theoretical contexts. Mathematical Modelling and Problem-solving The specific features are as follows: ▪MMP1 Application of mathematical models. ▪MMP2 Development of solutions to mathematical problems set in applied and theoretical contexts. ▪MMP3 Interpretation of the mathematical results in the context of the problem. ▪MMP4 Understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. Communication of Mathematical Information The specific features are as follows: ▪CMI1 Communication of mathematical ideas and reasoning to develop logical arguments. ▪CMI2 Use of appropriate mathematical notation, representations, and terminology.	Communication Citizenship Personal Development Work Learning

STAGE 1 MATHEMATICS
FOLIO
FUNCTIONS AND RECIPROCALS

Introduction

During this investigation you will see patterns emerging when comparisons are made between graphs of straight lines and quadratic functions, and their reciprocal graphs. You may use your graphics calculator or graphing software (www.graphsketch.com), but sketches should contain asymptotes (when appropriate) and labelled axes.

Mathematical Investigations, Analysis and Discussions

Part One

Sketch $y_1 = x - 3$ and $y_2 = \frac{1}{x-3}$ ***on the same set of axes.***

(a) What is the relationship between the zero of y_1 and the vertical asymptote of y_2?

(b) Present an argument to support your observation in part (a), i.e. consider *x* values close to 3, below and above 3, in the rational function.

(c) When y_1 has a large positive value, *explain* what happens to the value of y_2.
Similarly, when y_1 has a large negative value what happens to y_2? Explain why this happens.

(d) Discuss y_1 and y_2 as $x \to \pm\infty$.

Part Two

Sketch $y_1 = x^2 - 5x + 6$ ***and its reciprocal on the same set of axes.***

(a) What is the relationship between the zeros of y_1 and the vertical asymptotes of the reciprocal function?

(b) Present an argument to support your observation in part (a).

(c) Find the vertex of y_1 and state whether it is a maximum or minimum.
(i) Consider the *x* coordinate of the vertex. Find the point in the reciprocal function at this *x* value. What is the feature of the graph at this point?
(ii) Explain this observation by considering the value of y_1, and the corresponding effect on $1/y_1$ at this point.
(d) Discuss y_1 and the consequences to $1/y_1$ as $x \to \pm\infty$.

Part Three

Give an example of a quadratic whose reciprocal has no vertical asymptotes.

(a) Explain why you chose this quadratic.

(b) Sketch the quadratic and its reciprocal on the same set of axes.

(c) Compare the vertex of the quadratic and the corresponding feature of the reciprocal. Record your observations.
(d) Will your observation hold for all quadratics of the form $y = ax^2 + bx + c$ whose reciprocal has no vertical asymptotes? Give a detailed argument to support your answer.

(e) Discuss both graphs for $x \to \pm\infty$.

Part Four

Investigate a quadratic, which touches the x-axis and its reciprocal.

Your investigation should include:

- A clear graph and details of the two functions on the same set of axes.
- A discussion about the vertical asymptotes of the reciprocal and why they occur.
- A consideration of the vertex (or turning point) of the quadratic and the corresponding feature of the reciprocal.
- A discussion for both graphs for $x \to \pm \infty$.

Conclusion

Write a summary of the relationship between the characteristics of the graphs of

- a linear function and its reciprocal function; &
- a quadratic function and its reciprocal function.

Describe the similarities and differences between each graph and it's reciprocal [this could be presented in table form].

Student Work Sample

Introduction?

Maths investigation – Relations and Functions

Task one:

Sketch Y1 = x-3 and Y2 = 1/(x-3) on the same set of axes.

a) What is the relationship between the zero of Y1 and the vertical asymptote of Y2?
Y1 Intersects with x axis at x=3 Y_2 = 1/Y1, and anything over zero has an undefined value. Y2 does not exist when Y1 is zero asymptote at x = 3.

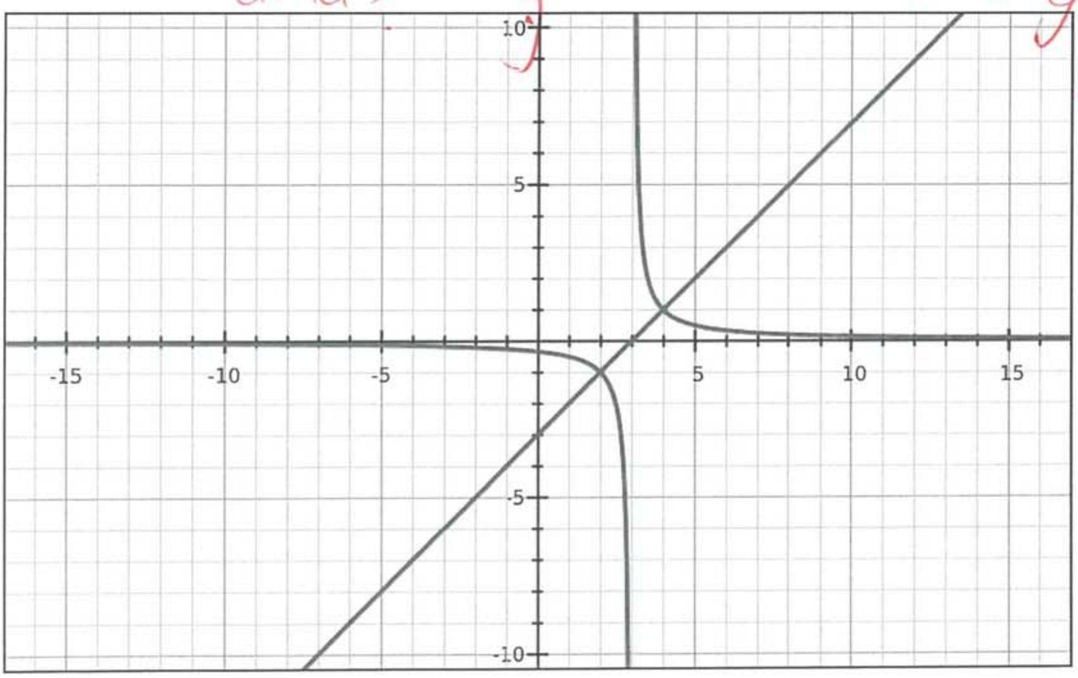
b) Present an argument to support your observation in part a) i.e consider x values close to 3, below 3 and above 3, in the rational function. Values close to 3?
Considering 1/(x-3) the value of x is 3, making the denominator 0. Dividing anything by 0 will give us undefined, which then becomes an asymptote.

c) When Y1 has a large positive value, explain what happens to the value of Y2. Similarly, when Y1 has a large negative value, what happens to Y2? Explain why this happens.
As Y1 increases, Y2 decreases. Y2 = 1/Y2, therefore it will end up following a pattern like this 1/1, 1/2, 1/3, 1/4. As Y1 the denominator gets larger, Y2 becomes smaller as a result.
and if y2 has large neg value

Y1 is x=3 which is on the denominator of Y2. Therefore, Y2=1/(x-3) and it is 1 divided by Y1.
If Y22 has a large positive value, Y2 approaches 0.

Notation

d) Discuss y_1 and y_2 as x approaches $\pm\infty$.
As Y1 x goes to infinity, Y2 approaches zero. which is y_1 and y_2
and?



Task Two:

Sketch $y_1 = x^2$-5x+6 and its reciprocal $y_2 = (1/x^2$ -5x+6)

In factorised form: Y1 = (x-2)(x-3) and 1/((x-2)(x-3))

a) What is the relationship between the zeros of y_1 and the vertical asymptotes of the reciprocal function?

When Y1 = 0, Y2 has 2 vertical asymptotes. 1 at x=2 and x=3.

What are the zero's of y_1?

b) Present an argument to support the observation in part (a) [Recall the approach in Take one part (b).

Y2 = it 1/Y1 and 1 Y1 = 0 at x=2 and x=3, therefore Y has an asymptote, and when a value is divided by 0, it is an undefined value. Values around 2, 3 ?

c) Find the vertex (or turning point) of y_1 and state whether it is a maximum or minimum.

incorrect

The turning point of Y1 is at (2.5, -0.49). The process of finding the x coordinate: The two x values were added and afterwards divided by 2, which gives us the x coordinate. In order to find the discriminant, the formula x=-b/2a was used. The discriminant found was x = 0.1. Afterwards, the discriminant was substituted into the original equation of x-^{2}-5x+6. The result was -0.49, which is the minimum turning point.

i) Consider the x coordinate of the vertex. Find the point in the reciprocal function at this x value. What is the feature of the graph at this point?

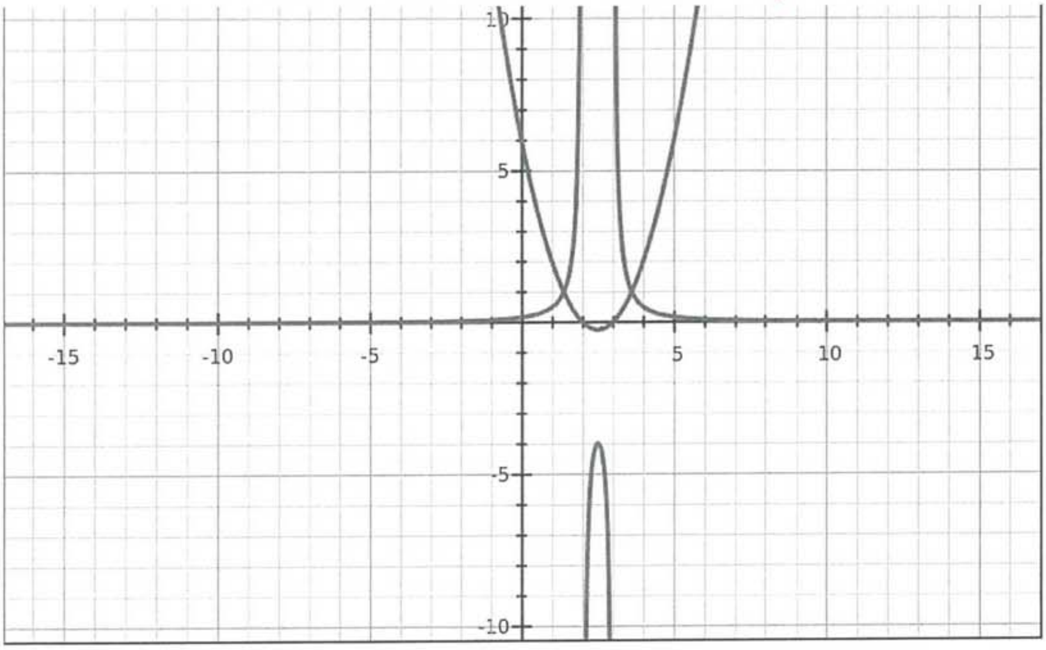
The feature of the x value on the reciprocal function is that it is the maximum turning point

ii) Explain this observation by considering the value of Y1 and the corresponding effect on 1/Y1 at this point.

This occurs because, as Y1 becomes smaller, just as this in this graph. The reciprocal becomes greater: 1/ (0.5), 1/ (0.25), and so on/etc. ?

d) Discuss y_1 and the consequences to $1/y_1$ as x goes to infinity.

As x goes to infinity, the asymptote of 1/Y1 approaches zero. ?



Task three:

Give an example of a quadratic whose reciprocal has no vertical asymptotes.
An example of this is x^2+5 and its reciprocal $1/x^2+5x$.

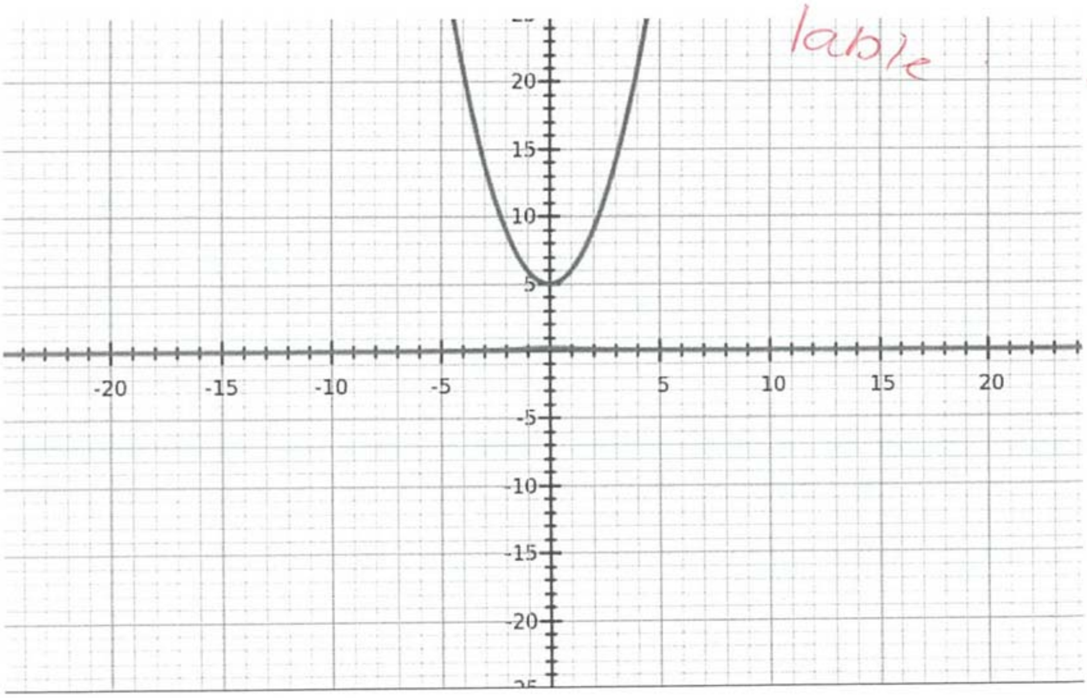
a) This quadratic was chosen because it's reciprocal is short and its reciprocal does not have vertical asymptotes..

b) Sketch the quadratic and its reciprocal on the same set of axes. See below

c) Compare the vertex (or turning point) of the quadratic and the corresponding feature of the reciprocal. Write down your observation.
The vertex of the Y1 is located at (0, 5) and the corresponding feature on its reciprocal is located at (0, 0), which is the origin.

d) Will the observation made hold for all quadratics of the form $y = ax^2+bx+c$ whose reciprocal has no vertical asymptotes? Give a detailed argument to support the answer.
It is believed that this observation is true because the x-intercepts do not exist. The x coordinate for both turning points will always be the same as long as the x-intercepts do not exist.

e) Discuss both graphs for x approaching infinity.
Y2 will continue to infinity, while not move its position from y=0. However y1 will approach +∞.



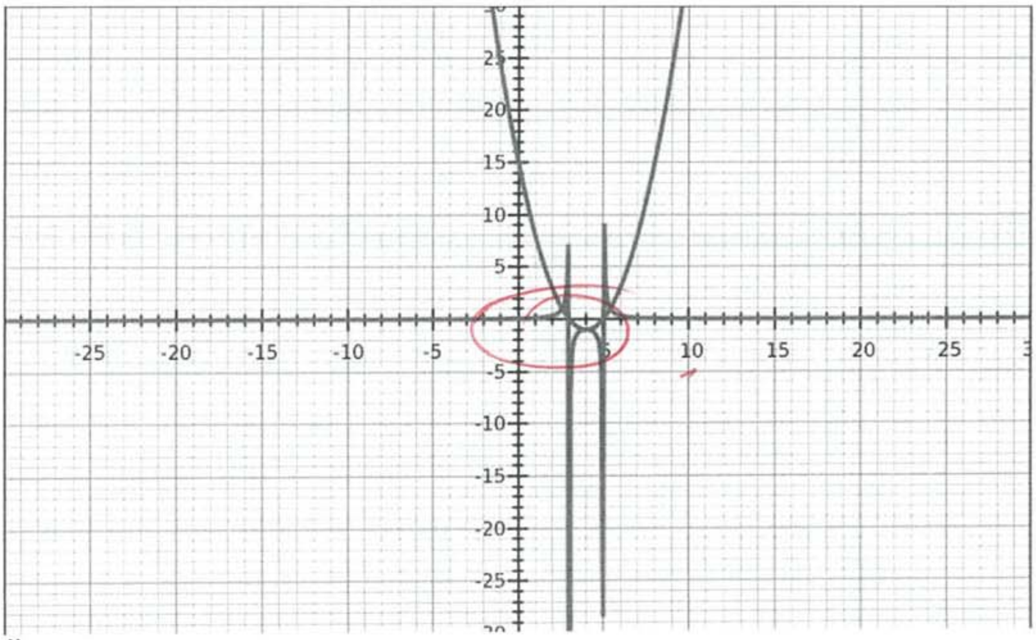
y1 must just not touch the x-axis !!

Task four: Function used is $y_1 = x^2 - 8x + 15$ and its reciprocal $(1/y_1)$.

Details: Below are two functions that have five points collectively, Y1 cuts its reciprocal and the turning points of the functions intersect one another.

The vertical asymptotes occur because of anything over zero is undefined which gives an asymptote.
x^2-8x+15 gives (x-5)(x-3) when it is factorised.
When x = 5 and x = 3, it result in having 1/0x0, which = 0,which means it is undefined, giving us the asymptote
For this function, y_1 has a minimum turning point, and y_2 has a maximum turning point. In the graph, both turning points share the same x and y values which are (4,-1)

As x goes to infinity, Y2 will be 0, and Y1 approaches infinity.



u

In final summation, the graphs and reciprocals contain interesting interactions with their zeros, turning points and asymptotes where available. As the x approached infinity, the asymptote of the reciprocal approaches zero, Y1 approaches infinity.