**Stage 2 mathematics pathways**

**Investigation**

**Under Supervision**

**Investments and Loans**

**TIME: 3 HOURS (Plus 10 Minutes Reading Time)**

**English and Bilingual Dictionaries may be used**

**Scientific Calculators and Casio Graphic Calculators are allowed**

**Students SACE Number:**

**School SACE Number:**

**Instructions to Students**

1. You will have 10 minutes reading and preparation time. Use this time to **READ ALL SECTIONS** of the paper carefully. You may plan or use a calculator during this time but you may ONLY make notes on the scrap paper provided.
2. This paper consists of ‘stepped questions’ all of which are linked to the Introductory Scenario. You may need to refer back to the introduction and diagrams for some steps.
3. You are able to use the information provided on the detachable Formulae Sheet
4. Your answers need to be completed in blue or black pen only.

**PLEASE DO NOT** write in your Investigation booklet until you are instructed to do so.

***HINT: Allow time to check completed work for errors***

*This task has been used with the kind permission of Valley View Secondary School*

**STAGE 2 MATHEMATICS PATHWAYS**

**EXTERNAL ASSESSMENT – INVESTIGATION [Investments and Loans]**

This is a written investigation to be completed under supervision, and consists of a series of connected questions. You will need to draw your own conclusions and complete a report about the investigation.

You will need to demonstrate the ability to use problem-solving strategies, as well as your knowledge, skills, and understanding of the topic of Investment and Loans. You are expected to make use of electronic technology where appropriate.

The total time you will have to complete the series of connected questions and the report is 3 hours (as this is for a 20-credit subject) with an additional 10 minutes reading time at the beginning of the session. The investigation must be completed in one sitting.

The report for this investigation will include:

* an introduction that demonstrates an understanding of the features of the problem or the situation investigated
* evidence that the student has followed instructions
* mathematical calculations and results, and appropriate representations
* a summary of results or findings and conclusions drawn.

**You will need to:**

* **complete each section, setting your work out in the spaces provided on the following pages**
* **show ALL working out (including all calculations needed for entering data into scientific and graphics calculators)**
* **choose which of the equations from the Formulae Sheet to use in each question where necessary**
* **use the evidence from the scenarios to make a decision about what you would recommend Jim does with his money.**

If you need extra room, write your answers on a separate piece of paper with the Step and your name clearly written.

As there is quite a lot of generic information on the first two pages, giving a copy of these to the students in class to familiarise them with the information would assist them in reading through the Introductory Scenario and the task questions in their 10 minutes reading time.

**Introductory Scenario**

Joanne achieved a First Class Honours at UniBA and has won a HECS-free, APA (Australian Postgraduate Scholarship) worth $23, 728 p.a. to study a PhD. The University has given her a ‘top up’ scholarship of $4,987 and she will therefore be earning $28, 715 p.a. for the minimum three years it will take her to complete her PhD. She has no siblings and will continue to live with her widowed mother and pay $1000 board a month to help with her mother’s mortgage, utilities and food bills. Joanne has managed to save $4000 over the last 4 years working at ‘Woleworks Supermarket’ and is not sure what she wants to do after her PhD, whether to get a job as a Researcher in Australia or travel overseas for a while. She is planning to see a financial advisor but wants to investigate different scenarios of what she could do with her money before she talks to them.

This introductory scenario is written in a sophisticated manner, and some students may find it difficult to access the information provided. Tasks should be written so that the students can access and therefore engage with the information.

**Step 1: Introduction**

After reading through the entire paper, write a brief introduction to this investigation in your own words.

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**Step 2: Borrowing – Flat Rate vs Compound [*All equations and solutions need to be shown*]**

Joanne’s old car blew its engine in the final months of her Undergraduate studies and she had to catch 2 buses each morning/night to get to University and home again. She really doesn’t want to repeat this and is looking at getting a good second hand car for around $10, 000. She has $4000 in savings but doesn’t want to spend all of it so would be looking to borrow $8000.

1. Joanne’s Aunt has offered to lend Joanne $8000 at a flat rate of 12.5% interest over the next two years.
	1. What amount of interest would Joanne have to pay?
	2. What would her fortnightly payments be to repay the original loan and the interest back over the 2 years?
2. Joanne checks with her bank and finds she can borrow $8000 as a personal loan at 11% p.a. compounded monthly over two years.
	1. What amount of interest would Joanne have to pay?
	2. What would her fortnightly payments be?
3. Discuss whether Joanne better off borrowing money from her Aunt or the bank.

It is important when designing the task that adequate space is left for the students to provide an appropriate response.

Structuring the questions so that students can respond to them one at a time makes the initial questions more accessible.

**Step 3: The Effect of Inflation**

Joanne is not sure whether her PhD scholarship is indexed for inflation. Predicted inflation rates for the next two years are 1.4% in the first year and 2.8% in the second year.

1. What is inflation and how does it occur?
2. What should Joanne’s scholarship be indexed to in her third year, to overcome the rate of inflation?
3. Joanne finds in her scholarship paperwork that PhD scholarships are indexed at an average inflation rate p.a. of 2.5%. Is she better or worse off in her third year than if the Scholarships Office had used the predicted rates?

**Step 4 : Investing money (Compound interest and the effective Interest Rate).**

Joanne’s savings over the last few years have been kept it in her local ‘ONZSA Bank’, she now wants to compare different savings accounts offered by online banks. Joanne looks at depositing $2000 for three years in four different fee-free, at call online accounts (*Note: assume no leap years*) for a holiday after she finishes all her research work before she writes up her PhD thesis.

As each of these calculations requires the same set of mathematical skills, this question could be modified to investigate at least one less bank.

* Bank Alpha: 5.50% interest compounded bi-annually
* Bank Beta: 5.35% interest compounded quarterly
* Bank Gamma: 5.45% interest compounded monthly
* Bank Delta: 5.33% interest compounded daily
1. Find out how much interest each bank will pay over 3 years.

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| --- | --- | --- | --- | --- |
|  | Bank Alpha | Bank Beta | Bank Gamma | Bank Delta |
| n = |  |  |  |  |
| I% = |  |  |  |  |
| PV = |  |  |  |  |
| PMT = |  |  |  |  |
| FV = |  |  |  |  |
| P/Y =  |  |  |  |  |
| C/Y = |  |  |  |  |
| Interest over 3 years |  |  |  |  |

1. What is the effective interest rate for each Bank?

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| --- | --- | --- | --- | --- |
|  | Bank Alpha | Bank Beta | Bank Gamma | Bank Delta |
| n = |  |  |  |  |
| I% = |  |  |  |  |
| Effective Interest Rate |  |  |  |  |

1. Which bank would you recommend to Joanne and why?
2. What are the pros (positive features) and cons (negative features) of this type of bank account for savings over 3 years?
3. What if Joanne looked at Term Deposits instead, how would this be different to an online bank?

**Step 5: Future Value Annuity**

Joanne has managed to save money from her ‘Woleworks’ job over the last 4 years. With her good savings habits, she feels she could continue saving each fortnight. If she started with the $2000 she was going to put away for her holiday and adds $250 per fortnight to Bank Gamma from Step 4 [5.45% p.a. interest compounded monthly] over the next three years, how much will she be able to save?

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**Step 6: Paying on Term**

Joanne and her mother have been sharing a computer at home, however now she is starting her PhD, she is investigating buying her own laptop. Joanne is currently looking at a laptop with appropriate software included, a Colour Printer/Scanner and Speakers/Professional Recording Microphone from Hervey Marmon’s for $1820 cash or 20% deposit-$85 per month for 2 years Payment Terms.

If she buys it on terms:

1. Calculate the deposit required and hence the amount borrowed.
2. Calculate the total repayments and interest paid*.*
3. Calculate the annual interest rate charged monthly on the reduced balance.

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1. Is this a reasonable rate of interest?
2. Would Joanne be better off paying cash or buying her computer on a payment plan? Explain your answer.

**Step 7: Personal Loan**

Now there will be more income coming into the house, Joanne and her mother discuss Joanne taking out a personal loan of $15, 000 at 8.95% p.a. over three years to renovate their very dated ‘80s’ orange and aqua blue kitchen.

1. Calculate the repayments if interest is charged monthly and payments are made monthly.

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1. What happens to the length of the loan if they change their payments to fortnightly and why?
2. If they decide to pay monthly, what is the outstanding balance after 1 year of the loan?

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1. Explain why after one year, they would still have over two thirds of the loan to pay off.

**Step 8: Home Loan [Effective Rate of Interest]**

Instead of just renovating, Joanne and her mother discuss taking out an entirely new mortgage now they will have extra income coming in.

1. They look at loans from 3 banks for $150 000 over 15 years, compare the following loans and then rank the options:
	1. Option 1: 6.5% p.a. charged monthly with no set up or monthly fees
	2. Option 2: 6.25% p.a. charged monthly with a service fee of $6 per month
	3. Option 3: 6.0 % p.a. charged monthly with an establishment fee of $250 and a service fee of $10 per month.

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|  | Step 8 and 9 involve calculations that are complex in nature due to the requirement of a number of steps in the calculation process. The unstructured manner in which the question is presented in Step 9 also adds to the complexity. Making decisions about how to investigate the given options mathematically may make it difficult for some students to access the question. |
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1. Discuss any assumptions that they are making comparing and ranking these three loans.

**Step 9: Home Loan, Minimizing Interest**

Joanne has started reading financial articles on the net and comes across a website that recommends minimizing the interest on your home loan rather than refinancing.

1. Compare the options mentioned below and explain how each of them works based on a home loan of $350, 000 at 6.5% interest compounded monthly for 25 years, and discuss any issues related to choosing them as an option.
	1. Option A: More frequent payments

The options that the students are required to discuss should be based upon options that have been explicitly taught in the classroom. The options covered in this step are quite extensive. It is important that it is made clear in the question when students are required to carry out calculations to support their responses. In Step 9 it would be appropriate for students to carry out calculations for Options A, B and C. This could be indicated by the comment *(investigate using calculations)* alongside the options for which calculations are required. If calculations are required, time may become an issue, so it is important to be mindful of this.

* 1. Option B: Making larger payments per period
	2. Option C: Making lump-sum repayments of the principal
	3. Option D: Using an offset account
	4. Option E: Using a line of credit account
1. Which option would you recommend for Joanne and her mother and why?

[refer back to the Introductory Scenario if needed].

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**Step 10: Finding the regular payment and Present Value annuity**

Joanne starts thinking about her future and potentially travelling for a year after she finishes her PhD. There are also a number of international Dietetic Research Facilities that she would be able to go and do work placements with for extra experience overseas, however they do not pay very much and she figures she would need to save at least $30, 000 over the next three years to cover her expenses.

1. If she used the Bank Gamma from Step 4:
	1. How much would her regular payments need to be to save the $30, 000 over three years?

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1. How likely is she to make these regular payments based on her income and board?
2. Assuming Joanne did save the $30, 000, how much would she then be able to withdraw each month to live on?

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1. Do you think this would be enough for Joanne to live on as she travelled overseas for a year?

**Step 11: Discussion and conclusion**

Discuss your results from Steps 1-10 above, including their limitations, as well as discussing any other relevant details or investments/loans for Joanne that you feel have been missed.

Draw a conclusion about what Joanne should do in the future and what she should do with her money.

(Discussion should be about ½ to 1 page in length. Refer to figures and answers from Step 1-10 to back up your argument.)

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***Formulae Sheet***

 *I =* $\frac{PRT}{100}$ *Where I = Interest*

 *P = Principal of the investment/loan*

 *R= the annual rate of interest*

 *T = Time ( in years)*

 *Fv = Pv(1 + i)n Fv = is the Future value of the investment/loan*

 *Pv = the principal or present value of the investment/loan*

 *i is the interest rate per period (as a decimal)*

 *n = the number of compound periods*

*Actual Interest earned is I = Fv - Pv*

*Total Interest on a loan is*  *Pmt x n -Pv*

**CASIO Graphics Calculators**

n represents the number of time periods

I% represents the interest rate per year

PV represents the present value of the investment

PMT represents the payment each time period

FV represents the future value of the investment

P/Y is the number of payment periods per year

C/Y is the number of compounding periods per year

For PV and FV a negative value indicates an amount owed or to be paid out.

A positive value indicates money received.

The **effective rate of a loan** is the nominal interest rate plus fees and charges pro rata over the life of the loan

Performance Standards for Stage 2 Mathematics Pathways

|  | **Mathematical Knowledge and Skills and Their Application** | **Mathematical Modelling and Problem-solving** | **Communication of Mathematical Information** |
| --- | --- | --- | --- |
| **A** | Comprehensive knowledge of content and understanding of concepts and relationships.Appropriate selection and use of mathematical algorithms and techniques (implemented electronically where appropriate) to find efficient solutions to complex questions.Highly effective and accurate application of knowledge and skills to answer questions set in applied and theoretical contexts. | Development and effective application of mathematical models.Complete, concise, and accurate solutions to mathematical problems set in applied and theoretical contexts.Concise interpretation of the mathematical results in the context of the problem.In-depth understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. | Highly effective communication of mathematical ideas and reasoning to develop logical arguments.Proficient and accurate use of appropriate notation, representations, and terminology. |
| **B** | Some depth of knowledge of content and understanding of concepts and relationships.Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to complex questions.Accurate application of knowledge and skills to answer questions set in applied and theoretical contexts. | Attempted development and appropriate application of mathematical models.Mostly accurate and completesolutions to mathematical problems set in applied and theoretical contexts.Complete interpretation of the mathematical results in the context of the problem.Some depth of understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. | Effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly accurate use of appropriate notation, representations, and terminology. |
| **C** | Generally competent knowledge of content and understanding of concepts and relationships.Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find mostly correct solutions to routine questions.Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts. | Appropriate application of mathematical models.Some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts.Generally appropriate interpretation of the mathematical results in the context of the problem.Some understanding of the reasonableness and possible limitations of the interpreted results and some recognition of assumptions made.  | Appropriate communication of mathematical ideas and reasoning to develop some logical arguments. Use of generally appropriate notation, representations, and terminology, with some inaccuracies. |
| **D** | Basic knowledge of content and some understanding of concepts and relationships.Some use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to routine questions.Sometimes accurate application of knowledge and skills to answer questions set in applied or theoretical contexts.  | Application of a mathematical model, with partial effectiveness.Partly accurate and generally incompletesolutions tomathematical problems set in or applied or theoretical contexts.Attempted interpretation of the mathematical results in the context of the problem.Some awareness of the reasonableness and possible limitations of the interpreted results. | Some appropriate communication of mathematical ideas and reasoning.Some attempt to use appropriate notation, representations, and terminology, with occasional accuracy. |
| **E** | Limited knowledge of content.Attempted use of mathematical algorithms and techniques (implemented electronically where appropriate) to find limited correct solutions to routine questions.Attempted application of knowledge and skills to answer questions set in applied or theoretical contexts, with limited effectiveness. | Attempted application of a basic mathematical model.Limited accuracy in solutions to one or more mathematical problems set in applied or theoretical contexts.Limited attempt at interpretation of the mathematical results in the context of the problem.Limited awareness of the reasonableness and possible limitations of the results. | Attempted communication of emerging mathematical ideas and reasoning.Limited attempt to use appropriate notation, representations, or terminology, and with limited accuracy. |