STAGE 2 MATHEMATICAL METHODS

SKILLS AND APPLICATIONS TASK 3

Purpose

To demonstrate your ability to:

- accurately apply the mathematical concepts, processes, and strategies that you have learned in class to solve a range of matrices questions set in different contexts
- effectively and appropriately communicate relevant information within your solutions.

Description of assessment

This assessment allows you to show your skills in understanding and appropriate use of the mathematical concepts, processes, and strategies in Subtopic 4.2: Matrices.

Assessment conditions

This is a supervised assessment. Provide complete working for all calculations. Use electronic technology where appropriate. You may bring one side of one A5 page of hand-written notes.

<table>
<thead>
<tr>
<th>Learning Requirements</th>
<th>Assessment Design Criteria</th>
<th>Capabilities</th>
</tr>
</thead>
</table>
| 1. Understand fundamental mathematical concepts, demonstrate mathematical skills, and apply routine mathematical procedures | Mathematical Knowledge and Skills and Their Application  
The specific features are as follows:  
- MKSA1 Knowledge of content and understanding of mathematical concepts and relationships.  
- MKSA2 Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find solutions to routine and complex questions.  
- MKSA3 Application of knowledge and skills to answer questions set in applied and theoretical contexts. | Communication  
Citizenship  
Personal Development  
Work  
Learning |
| 2. Plan courses of action after using mathematics to analyse data and other information elicited from the study of situations taken from social, scientific, economic, or historical contexts | Mathematical Modelling and Problem-solving  
The specific features are as follows:  
- MMP1 Application of mathematical models.  
- MMP2 Development of mathematical results for problems set in applied and theoretical contexts.  
- MMP3 Interpretation of the mathematical results in the context of the problem.  
- MMP4 Understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made and possible new mathematical questions to be investigated.  
- MMP5 Development and testing of conjectures. | |
| 3. Think mathematically by posing questions, making and testing conjectures, and looking for reasons that explain the results | Communication of Mathematical Information  
- CM1 Communication of mathematical ideas and reasoning to develop logical arguments.  
- CM12 Use of appropriate mathematical notation, representations, and terminology. | |
## PERFORMANCE STANDARDS FOR STAGE 2 MATHEMATICAL METHODS

<table>
<thead>
<tr>
<th>Mathematical Knowledge and Skills and Their Application</th>
<th>Mathematical Modelling and Problem-solving</th>
<th>Communication of Mathematical Information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Comprehensive knowledge of content and understanding of concepts and relationships. Appropriate selection and use of mathematical algorithms and techniques (implemented electronically where appropriate) to find efficient solutions to complex questions. Highly effective and accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.</td>
<td>Development and effective application of mathematical models. Complete, concise, and accurate solutions to mathematical problems set in applied and theoretical contexts. Concise interpretation of the mathematical results in the context of the problem. In-depth understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. Development and testing of reasonable conjectures.</td>
<td>Highly effective communication of mathematical ideas and reasoning to develop logical arguments. Proficient and accurate use of appropriate notation, representations, and terminology.</td>
</tr>
<tr>
<td><strong>B</strong> Some depth of knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to complex questions. Accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.</td>
<td>Attempted development and appropriate application of mathematical models. Mostly accurate and complete solutions to mathematical problems set in applied and theoretical contexts. Complete interpretation of the mathematical results in the context of the problem. Some depth of understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. Development and testing of some reasonable conjectures.</td>
<td>Effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly accurate use of appropriate notation, representations, and terminology.</td>
</tr>
<tr>
<td><strong>C</strong> Generally competent knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find mostly correct solutions to routine questions. Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.</td>
<td>Appropriate application of mathematical models. Some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts. Generally appropriate interpretation of the mathematical results in the context of the problem. Some understanding of the reasonableness and possible limitations of the interpreted results and some recognition of assumptions made. Development and testing of one or more reasonable conjectures.</td>
<td>Appropriate communication of mathematical ideas and reasoning to develop some logical arguments. Use of generally appropriate notation, representations, and terminology, with some inaccuracies.</td>
</tr>
<tr>
<td><strong>D</strong> Basic knowledge of content and some understanding of concepts and relationships. Some use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to routine questions. Sometimes accurate application of knowledge and skills to answer questions set in applied or theoretical contexts.</td>
<td>Application of a mathematical model, with partial effectiveness. Partly accurate and generally incomplete solutions to mathematical problems set in applied or theoretical contexts. Attempted interpretation of the mathematical results in the context of the problem. Some awareness of the reasonableness and possible limitations of the interpreted results. Attempted development or testing a reasonable conjecture.</td>
<td>Some appropriate communication of mathematical ideas and reasoning. Some attempt to use appropriate notation, representations, and terminology, with occasional accuracy.</td>
</tr>
<tr>
<td><strong>E</strong> Limited knowledge of content. Attempted use of mathematical algorithms and techniques (implemented electronically where appropriate) to find limited correct solutions to routine questions. Attempted application of knowledge and skills to answer questions set in applied or theoretical contexts, with limited effectiveness.</td>
<td>Attempted application of a basic mathematical model. Limited accuracy in solutions to one or more mathematical problems set in applied or theoretical contexts. Limited attempt at interpretation of the mathematical results in the context of the problem. Limited awareness of the reasonableness and possible limitations of the results. Limited attempt to develop or test a conjecture.</td>
<td>Attempted communication of emerging mathematical ideas and reasoning. Limited attempt to use appropriate notation, representations, or terminology, and with limited accuracy.</td>
</tr>
</tbody>
</table>
1. Consider the following matrices:

\[
A = \begin{bmatrix}
1 & 3 \\
-1 & 0 \\
4 & -2
\end{bmatrix} \quad B = \begin{bmatrix}
1 & -3 \\
4 & 2
\end{bmatrix} \quad C = \begin{bmatrix}
x & 3 \\
-1 & -2
\end{bmatrix} \quad D = \begin{bmatrix}
x \\
y
\end{bmatrix} \quad E = \begin{bmatrix}
0 \\
3
\end{bmatrix}
\]

Evaluate, if possible (otherwise explain why not):

a. \(AB\)  
   \[
   \begin{bmatrix}
   1 & 3 \\
   -1 & 0 \\
   4 & -2
   \end{bmatrix}
   \begin{bmatrix}
   1 & -3 \\
   4 & 2
   \end{bmatrix}
   = \begin{bmatrix}
   -3 \\
   1
   \end{bmatrix}
   \]
   Not Possible

b. \(A - C\)  
   \[
   \begin{bmatrix}
   1 & 3 \\
   -1 & 0 \\
   4 & -2
   \end{bmatrix} - \begin{bmatrix}
x & 3 \\
-1 & -2
\end{bmatrix}
   \]
   Cannot Subtract Different Sizes

c. \(C^2\)  
   \[
   \begin{bmatrix}
x & 3 \\
-1 & -2
\end{bmatrix}
   \begin{bmatrix}
x & 3 \\
-1 & -2
\end{bmatrix}
   = \begin{bmatrix}
x + 3 \\
-x + 2
\end{bmatrix}
   \]
   \[
   \begin{bmatrix}
x^2 + 3x - 6 \\
-x + 2
\end{bmatrix}
   \]
   \[
   x^2 + 3x - 6 = 0
   \Rightarrow
   x = \frac{3 \pm \sqrt{17}}{2}
   \]

\[
\begin{bmatrix}
x - 3y = 0 \\
x + 3y = 3
\end{bmatrix}
   \Rightarrow
   x = 3y
   \]
   \[
   4x + 2y = 3
   \Rightarrow
   y = 3/4
   \]
   \[
   x = \frac{3}{4}
   \]

d. \(BD = E\)  
   \[
   \begin{bmatrix}
   1 & 3 \\
   4 & 2
   \end{bmatrix}
   \begin{bmatrix}
x \\
y
\end{bmatrix}
   = \begin{bmatrix}
0 \\
3
\end{bmatrix}
   \]
   \[
   \begin{bmatrix}
x + 3y = 0 \\
4x + 2y = 3
\end{bmatrix}
   \Rightarrow
   x = \frac{3}{14}
   \]
   \[
   y = \frac{3}{14}
   \]

e. The order of \(X\) in \(BXA\) so it can be multiplied

\[
\begin{bmatrix}
B \\
(X) \\
A
\end{bmatrix}
\begin{bmatrix}
2 \times 2 \\
3 \times 2
\end{bmatrix}
\]
\[
X \text{ has to be } 2 \times 3
\]
2. **Show all headings and matrix labels in the following question**

A canteen sells the following foods over the course of a week.

The matrix of **food sold** is given by matrix \( Q \):

\[
\begin{array}{cccc}
  & M & T & W & T & F \\
\text{Pastie} & 216 & 244 & 220 & 232 & 226 \\
\text{Pie} & 145 & 130 & 152 & 121 & 113 \\
\text{Pizza} & 83 & 102 & 78 & 75 & 94 \\
\text{Hot Dog} & 12 & 14 & 8 & 13 & 18 \\
\end{array}
\]

The **selling price** of the foods is given by matrix \( S \):

<table>
<thead>
<tr>
<th></th>
<th>Pastie</th>
<th>Pie</th>
<th>Pizza</th>
<th>Hot Dog</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>$2.20</td>
<td>$1.80</td>
<td>$1.70</td>
<td>$3.20</td>
</tr>
</tbody>
</table>

The **cost price** to make the foods is given by matrix \( C \):

<table>
<thead>
<tr>
<th></th>
<th>Pastie</th>
<th>Pie</th>
<th>Pizza</th>
<th>Hot Dog</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>$0.60</td>
<td>$0.50</td>
<td>$1.10</td>
<td>$1.80</td>
</tr>
</tbody>
</table>

a. Describe the **element** in position \((4,2)\) in \( Q \) (1)

```
14 Hot Dogs were sold on Tuesday.
```

d. Calculate how many Pizzas were eaten for the week (1)

```
Add Row 3: 83 + 102 + 78 + 75 + 94 = 432
```

c. Calculate \( [1 \ 1 \ 1] \cdot \text{[Thursday]} \) and interpret the result (2)

```
\[
\begin{bmatrix}
1 & 1 & 1
\end{bmatrix} 
\begin{bmatrix}
252
12
75
13
\end{bmatrix} = \begin{bmatrix}
441
\text{Total Foods were Sold on Thursday}
\end{bmatrix}
\]

Question 2 provides the first opportunity to apply the appropriate knowledge and skills to answer a question set in an applied context.
d. Use matrix methods to calculate the **total cost** of the food

\[
\text{Total Cost} = \text{Cost} \times \text{Quantity}
\]

\[
\begin{bmatrix}
\text{PA} & \text{PIE} & \text{PIZZA} & \text{HD}
\end{bmatrix}
\begin{bmatrix}
0.6 & 0.5 & 1 & 1.25
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{M} & \text{T} & \text{W} & \text{T} & \text{F}
\end{bmatrix}
\begin{bmatrix}
3.5 & 54.80 & 303.20 & 305.6 & 727.9
\end{bmatrix}
\]

\[
\text{CT} = \begin{bmatrix}
516.05
\end{bmatrix}
\]


e. Use matrix methods to calculate the **total income** from the sale of food

\[
\text{Total Income} = \text{Selling} \times \text{Quantity}
\]

\[
\begin{bmatrix}
\text{PA} & \text{PIE} & \text{PIZZA} & \text{HD}
\end{bmatrix}
\begin{bmatrix}
1.5 & 1.7 & 3.2
\end{bmatrix}
\]

\[
\begin{bmatrix}
\text{M} & \text{T} & \text{W} & \text{T} & \text{F}
\end{bmatrix}
\begin{bmatrix}
9.15 & 9.30 & 9.6 & 597.3 & 9.6
\end{bmatrix}
\]

\[
\text{SI} = \begin{bmatrix}
4635.80
\end{bmatrix}
\]


f. Hence, calculate the **weekly profit** from the food

\[
\text{Profit} = \text{SI} - \text{CT} = \text{(Selling} - \text{Cost})
\]

\[
\begin{bmatrix}
\text{M} & \text{T} & \text{W} & \text{T} & \text{F}
\end{bmatrix}
\begin{bmatrix}
600.7 & 640.2 & 597.6 & 591.7 & 590.1
\end{bmatrix}
\]

\[
\text{Profit} = \begin{bmatrix}
3090.30
\end{bmatrix}
\]
3. Consider the network diagram below:

![Network Diagram]

a. Draw up a **connectivity matrix** $R$ to describe the network above (headings in alphabetical order) 

$$
R = \begin{pmatrix}
\text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F} \\
\text{A} & 0 & 1 & 0 & 0 & 0 \\
\text{B} & 1 & 0 & 0 & 0 & 0 \\
\text{C} & 0 & 0 & 0 & 0 & 0 \\
\text{D} & 0 & 0 & 0 & 0 & 0 \\
\text{E} & 0 & 0 & 0 & 0 & 0 \\
\text{F} & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
$$

(2)

b. Calculate $R^2$ 

$$
R^2 = \begin{pmatrix}
\text{A} & \text{B} & \text{C} & \text{D} & \text{E} & \text{F} \\
\text{A} & 0 & 1 & 0 & 0 & 0 \\
\text{B} & 1 & 0 & 0 & 0 & 0 \\
\text{C} & 0 & 0 & 0 & 0 & 0 \\
\text{D} & 0 & 0 & 0 & 0 & 0 \\
\text{E} & 0 & 0 & 0 & 0 & 0 \\
\text{F} & 0 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
$$

(1)

c. What do the following entries tell you about the network?

i. $R^2_{(1,3)}$?

There are 2 ways to go from A to C by a two-stage route (jump)
ii. $R^2(2, 4)$?

There is no way to get from $B$ to $D$ by a 2-stage route.

\[ \text{d. Describe what happens to anything arriving at } F? \]

You cannot leave after arriving.

\[ \text{e. What is the } \text{minimum number of stages} \text{ it takes to travel from } B \text{ to } D? \]

Looking at diagram:

\[ B \rightarrow D: B \rightarrow C \rightarrow D = A \rightarrow D \text{ 4 stages (jumps)} \]

f. Calculate the matrix: $Q = R + R^2 + R^3$.

What is the meaning of $Q_{(4, 2)}$?

\[
\begin{bmatrix}
A & B & C & D & E \\
1 & 4 & 2 & 1 & 3 \\
\end{bmatrix}
\]

There are 4 ways to go from $D \rightarrow B$ by either $A$, $2$, or $3$ stage route.
4. It is known that tennis players, Ace (A), Best (B), Champion (C), Dux (D) and Excel (E) have played each other before, with the results being:

- A has defeated B, C and D
- B has defeated D
- C has defeated B
- D has defeated C and E
- E has defeated A, B and C

a. Draw a network diagram to illustrate the results of these contests

b. Show the victories as a results matrix $R$ with the players listed alphabetically
c. Calculate the Supremacy Matrix: \( S = R + \frac{1}{2} R^2 \)  

\[ \begin{array}{cccc} 
A & B & C & D & E \\
\begin{array}{ccccc}
A & 0 & 1.5 & 1.5 & 5 \\
B & 0 & 0 & 1.5 & 2 \\
C & 0 & 1 & 0.5 & 0.5 \\
D & 0 & 0 & 0.5 & 0 \\
E & 0 & 0 & 1.5 & 1.5 & 0.5 \\
\end{array}
\end{array} \]

\[ S = \begin{pmatrix} 
5 \\
2 \\
0.5 \\
0 \\
0.5 \\
\end{pmatrix} \]

\[ S^2 = \begin{pmatrix} 
5.5 \\
1.5 \\
1 \\
1 \\
5.5 \\
\end{pmatrix} \]

\[ \frac{1}{2} R^2 = \begin{pmatrix} 
2.5 \\
0.75 \\
0.25 \\
0.25 \\
2.75 \\
\end{pmatrix} \]

\[ R = \begin{pmatrix} 
2.5 \\
1.75 \\
0.75 \\
0.75 \\
2.75 \\
\end{pmatrix} \]

\[ R + \frac{1}{2} R^2 = \begin{pmatrix} 
5.5 \\
3.25 \\
1.5 \\
1.5 \\
5.25 \\
\end{pmatrix} \]

\[ S = R + \frac{1}{2} R^2 \]

(1)

d. What is the significance of the **coefficient** of \( R^2 \) in (c.)?  

(1)

You get an **extra 3/2 point if you defeated a winner**.

e. List the **supremacy vector** and **rank the players** in descending order

(2)

\[ \text{Add up all the rows in matrix } S. \]

\[ \begin{array}{cccc} 
A & B & C & D & E \\
\begin{array}{ccccc}
A & 5 \\
B & 2 \\
C & 1.5 \\
D & 4 \\
E & 1.5 \\
\end{array}
\end{array} \]

\[ S = \begin{pmatrix} 
5 \\
2 \\
1.5 \\
4 \\
1.5 \\
\end{pmatrix} \]

\[ S^2 = \begin{pmatrix} 
5.5 \\
1.5 \\
1 \\
1 \\
5.5 \\
\end{pmatrix} \]

\[ \frac{1}{2} R^2 = \begin{pmatrix} 
2.5 \\
0.75 \\
0.25 \\
0.25 \\
2.75 \\
\end{pmatrix} \]

\[ R = \begin{pmatrix} 
2.5 \\
1.75 \\
0.75 \\
0.75 \\
2.75 \\
\end{pmatrix} \]

\[ R + \frac{1}{2} R^2 = \begin{pmatrix} 
5.5 \\
3.25 \\
1.5 \\
1.5 \\
5.25 \\
\end{pmatrix} \]

\[ S = R + \frac{1}{2} R^2 \]

(1)

**Ranking:**

\[ \begin{array}{cccc} 
\text{1st} & \text{2nd} & \text{3rd} & \text{4th} & \text{5th} \\
E & A & D & B & C \\
\end{array} \]

**Order:**

E, A, D, B, C
5. Jordan processes insurance claims and he flies between the three major cities: Chicago, New York and Boston. The city that Jordan will be in tomorrow depends only on which city he is in today. His movements between cities are represented mathematically by the following matrix $T$:

\[
(tomorrow) \quad T = \begin{bmatrix}
C & N & B \\
\frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\
\frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\
0 & \frac{5}{6} & \frac{1}{6}
\end{bmatrix}
\]

For example, if Jordan was in New York today, the probability that he will be in Boston tomorrow is $\frac{2}{3}$.

a. Assign the probabilities to the diagram below

b. Interpret the values of $T_{(1, 1)}$, $T_{(2, 2)}$, $T_{(3, 3)}$ (1)

For all cities there is a \( \frac{1}{6} \) probability of staying there until tomorrow.

c. Explain the meaning of the zero in the matrix $T$ (1)

There is no chance of going from Boston to Chicago.
\( T^2 \) represents the matrix of the probabilities that if Jordan is in a particular city today, he will be in Chicago, New York and Boston in two days time.

d. Determine the matrix \( T^2 \)

\[
T^2 = \begin{bmatrix}
1 & .3 & .7 \\
.11 & .44 & .45 \\
.05 & .67 & .28 \\
.14 & .28 & .58
\end{bmatrix}
\]

(1)

e. The entry for travelling from Boston to Chicago in Matrix \( T^2 \) is different from matrix \( T \). Explain why.

[Black and white matrix showing transitions between cities]

The row matrix \( X_0 \) represents the probability that he is in Chicago, New York and Boston on Thursday.

\[
X_0 = \begin{bmatrix}
1 \\
2 \\
2 \\
0
\end{bmatrix}
\]

f. Find the matrix \( X_1 \) if \( X_1 = X_0T \) and explain where and when he is likely to be.

\[
X_0T = \begin{bmatrix}
.16 \\
.53 \\
.33 \\
.5
\end{bmatrix}
\]

BA

(2)

g. Find the matrix \( X_2 \) if \( X_2 = X_0T^2 \) and explain where and when he is likely to be.

\[
X_0T^2 = \begin{bmatrix}
.08 \\
.56 \\
.36 \\
.58
\end{bmatrix}
\]

\(58\%\) chance in New York - Saturday

(2)

Question 5 provides several opportunities to demonstrate application of knowledge and skills to answer questions in an applied context related to transition matrices. Therefore together the responses may be indicative of highly effective communication of mathematical ideas and reasoning to develop logical arguments.
h. In which city is Jordan likely to be on Monday?

\[ X_1 T^4 = 50.77^\circ. \]

To be in New York.

ii. What is the probability that Jordan will be processing insurance claims in that city on Monday?

\[ 50.77^\circ. \]

iii. In which city is Jordan likely to be in one months time?

\[ \text{Send: St. Louis} \]

\[ 0.1 \quad 0.4 \quad 0.5 \]

\[ \text{In New York.} \]

6. Algebraically, calculate the steady state of matrix \( B \) if \( B = \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix} \)

\[ \begin{bmatrix} x \\ y \end{bmatrix} \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & 0.4 \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} \]

\[ 2x + 6y = x \quad \quad 8x + 4y = 10x \]
\[ 6y = 8x \quad \quad 8x = 6y \]
\[ 3y + 4x \quad \quad 4x = 3y \quad \quad \Rightarrow x = \frac{3y}{4} \]

\[ x + y = 1 \]
\[ \frac{3}{4} y + y = 1 \]
\[ \frac{7}{4} y = 1 \quad \quad \Rightarrow y = \frac{4}{7} \]
\[ x = \frac{3}{7} \]

\[ \text{Steady state} \begin{bmatrix} 0.5 \\ 0.7 \end{bmatrix} \]
7. The ‘36’ers’ is Adelaide’s long serving NBL basketball team. To create a good team image, a local snack food company, Sporting Munchies, has produced three healthy bar products to be sold in the NBL matches and local basketball stadiums.

There has been a rapid increase in business and you have been assigned to calculate the weekly raw material requirements, costs of production, selling price of products and ordering of raw materials.

You have been supplied with the following information:

‘Sporting Munchies’ produces three different snack bars, using different proportions of the following ingredients: sultanas, apricots, sugar, sesame seeds and butter.

To make one batch of each health food requires:

‘Slam Dunks’
3 units of sultanas
2 units of apricots
2 units of sugar
4 units of sesame seeds
2 units of butter

‘Three Pointers’
4 units of sultanas
2 units of apricots
3 units of sugar
1 unit of sesame seeds
5 units of butter

‘Time Outs’
2 units of sultanas
3 units of apricots
2 units of sesame seeds
1 unit of butter

a. Represent the above information in the form of a single 3x5 matrix

\[
\begin{array}{cccccc}
S & A & Su & Ss & B \\
Sp & 3 & 2 & 2 & 4 & 2 \\

St & 4 & 2 & 3 & 1 & 5 \\
To & 2 & 3 & 0 & 2 & 1
\end{array}
\]
During the basketball season the normal weekly order is 15 batches of Slam Dunks, 12 batches of Three Pointers and 9 batches of Time Outs.

b. Represent the normal weekly order in the form of a matrix

\[
\begin{bmatrix}
15 & 12 & 9 & 7
\end{bmatrix}
\]

Each bar is made differently and involves the following labour, which costs the company $8.00 per unit.

- **Slam Dunks**: 3 units of labour per batch
- **Three Pointers**: 5 units of labour per batch
- **Time Outs**: 4 units of labour per batch

c. Represent this information in the form of a matrix

\[
\begin{bmatrix}
5 & 3 & 24 \\
5 & 10 & 40 \\
4 & 10 & 32
\end{bmatrix}
\]

The costs of raw materials are as follows:

- **Sultanas**: $4 per unit
- **Apricots**: $6 per unit
- **Sugar**: $2 per unit
- **Sesame Seeds**: $3 per unit
- **Butter**: $5 per unit

d. Represent this information in the form of a matrix

\[
\begin{bmatrix}
4 \\
6 \\
2 \\
3 \\
5
\end{bmatrix}
\]
Calculate the following using matrix methods:

e. The total raw material requirements for one week’s production during the basketball season

\[
\begin{bmatrix}
3 & 5 & 7 & 9 \\
4 & 6 & 8 & 10 \\
5 & 7 & 9 & 11 \\
6 & 8 & 10 & 12
\end{bmatrix}
\]

\[
= N_W.O. \begin{bmatrix}
11 \\
6 \\
9 \\
9
\end{bmatrix}
\]

f. The costs of producing one batch of each bar

\[
\begin{bmatrix}
1 & 2 & 3 \\
4 & 5 & 6 \\
7 & 8 & 9 \\
10 & 11 & 12
\end{bmatrix}
\]

\[
\begin{bmatrix}
50 \\
62 \\
37 \\
99
\end{bmatrix}
\]

\[
= N_W.O. \begin{bmatrix}
74 \\
62 \\
34 \\
69
\end{bmatrix}
\]

\[
= N_W.O. \begin{bmatrix}
74 \\
62 \\
34 \\
69
\end{bmatrix}
\]

g. The total costs for the week’s order

\[
\begin{bmatrix}
\text{Week 1} & \text{Week 2} & \text{Week 3} & \text{Week 4}
\end{bmatrix}
\]

\[
= N_W.O. \begin{bmatrix}
50 \\
62 \\
37 \\
99
\end{bmatrix}
\]

\[
= N_W.O. \begin{bmatrix}
2955 \\
\end{bmatrix}
\]
8. Scientists are studying the changes in the number of female frogs living along a creek bed. The scientists claim that the following shows how female frogs change over time. They are currently feeling the effects of a virus. 

- The females are dying after 3 years
- The stages of the frogs development are tadpoles, frogteens and adults
- Females less than a year old are immature and cannot reproduce.
- Only a $\frac{1}{2}$ of these frogs reach the age of one.
- Frogteens produce an average of 1.2 female tadpoles in that year.
- Only $\frac{1}{5}$ of frogteens reach adulthood because of the virus.
- Adult frogs produce on average, two female tadpoles in that year.
- Very few frogs reach the age of three, those that do are past the point of producing tadpoles.

a. Using the information above complete the matrix below

\[ T \quad F \quad A \]
\[ \begin{bmatrix} 0 & 1.2 & 2 \\ 0 & 0 & 0 \\ 0 & 0.5 & 0 \end{bmatrix} \]

Let \( P = \begin{bmatrix} 0.5 \\ 0 \\ 0.5 \end{bmatrix} \).

The number of 0 (tadpoles), 1 (frogteens) and 2 year old adult frogs at the time of the study are represented by:

\[ O = \begin{bmatrix} 200 \\ 80 \\ 20 \end{bmatrix} \]

b. Find \( PO \) and explain what this gives

\[ PO = \begin{bmatrix} 136 \\ 100 \\ 16 \end{bmatrix} \]

After 1 year there are 136 T, 100 F and 16 A.
c. Find $P^2O$ and explain what this gives

$$P^2O \frac{152}{8} = \frac{T}{20} A.$$ 

(2)

d. Find $P^3O$ and $P^5O$

$$P^3O \int \frac{121.6}{76} = \frac{T}{18.6} A$$

$$P^5O \int \frac{103.36}{59.2} = \frac{T}{12.16} A.$$ 

(2)

e. Calculate the percentage increase of decrease of frogs along the creek bed between year two and three

$$\begin{array}{c|c|c|c|c}
\text{Year} & \text{Total} & \text{Increase} & \% \text{Change} \\
\hline
2 & 240 & 271.2 & 271.2 - 240 \\
3 & 211.2 & 12.6 & \frac{12.6}{240} \times 100 = 5\% \text{ DECREASE} \\
\end{array}$$ 

(2)

f. What do you think is likely to happen with the frog population along the creek as time goes on?

$$P^{\infty} \left[ \begin{array}{c}
0 \\
0 \end{array} \right] \} \text{FROGS DIE OUT.}$$ 

(1)

g. If frogteens now produce on average, four females tadpoles, explore what happens to the frog population as time goes on?

$$A(1, 2) = 4$$

$$P^{\infty} \left[ \begin{array}{c}
0 \\
0 \end{array} \right] \} \text{FROG EXPLOSION!}$$ 

(2)

TOTAL [78]