STAGE 2 MATHEMATICAL METHODS

SKILLS AND APPLICATIONS TASK 1

Purpose

To demonstrate your ability to:

- accurately apply the mathematical concepts, processes, and strategies that you have learned in class to solve a range of linear programming questions set in different contexts
- effectively and appropriately communicate relevant information within your solutions.

Description of assessment

This assessment allows you to show your skills in understanding and appropriate use of the mathematical concepts, processes, and strategies in Subtopic 4.1: Linear Programming.

Assessment conditions

This is a supervised assessment. Provide complete working for all calculations. Use electronic technology where appropriate. You may have one side of one A5 page of hand-written notes.

Learning Requirements	*Assessment Design Criteria*	*Capabilities*
1. Understand fundamental mathematical concepts, demonstrate mathematical skills, and apply routine mathematical procedures 2. Plan courses of action after using mathematics to analyse data and other information elicited from the study of situations taken from social, scientific, economic, or historical contexts 3. Think mathematically by posing questions, making and testing conjectures, and looking for reasons that explain the results 4. Make informed and critical use of electronic technology to provide numerical results and graphical representations 5. Communicate mathematically and present mathematical information in a variety of ways 6. Work both individually and cooperatively in planning, organising, and carrying out mathematical activities	Mathematical Knowledge and Skills and Their Application The specific features are as follows: ▪ MKSA1 Knowledge of content and understanding of mathematical concepts and relationships. ▪ MKSA2 Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find solutions to routine and complex questions. ▪ MKSA3 Application of knowledge and skills to answer questions set in applied and theoretical contexts. Mathematical Modelling and Problem-solving The specific features are as follows: ▪ MMP1 Application of mathematical models. ▪ MMP2 Development of mathematical results for problems set in applied and theoretical contexts. ▪ MMP3 Interpretation of the mathematical results in the context of the problem. ▪ MMP4 Understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. ▪ MMP5 Development and testing of conjectures. Communication of Mathematical Information The specific features are as follows: ▪ CMI1 Communication of mathematical ideas and reasoning to develop logical arguments. ▪ CMI2 Use of appropriate mathematical notation, representations, and terminology.	Communication Citizenship Personal Development Work Learning

PERFORMANCE STANDARDS FOR STAGE 2 MATHEMATICAL METHODS

	Mathematical Knowledge and Skills and Their Application	Mathematical Modelling and Problem-solving	Communication of Mathematical Information
A	Comprehensive knowledge of content and understanding of concepts and relationships. Appropriate selection and use of mathematical algorithms and techniques (implemented electronically where appropriate) to find efficient solutions to complex questions. Highly effective and accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Development and effective application of mathematical models. Complete, concise, and accurate solutions to mathematical problems set in applied and theoretical contexts. Concise interpretation of the mathematical results in the context of the problem. In-depth understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. Development and testing of reasonable conjectures.	Highly effective communication of mathematical ideas and reasoning to develop logical arguments. Proficient and accurate use of appropriate notation, representations, and terminology.
B	Some depth of knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to complex questions. Accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Attempted development and appropriate application of mathematical models. Mostly accurate and complete solutions to mathematical problems set in applied and theoretical contexts. Complete interpretation of the mathematical results in the context of the problem. Some depth of understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. Development and testing of some reasonable conjectures.	Effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly accurate use of appropriate notation, representations, and terminology.
C	Generally competent knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find mostly correct solutions to routine questions. Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Appropriate application of mathematical models. Some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts. Generally appropriate interpretation of the mathematical results in the context of the problem. Some understanding of the reasonableness and possible limitations of the interpreted results and some recognition of assumptions made. Development and testing of one or more reasonable conjectures.	Appropriate communication of mathematical ideas and reasoning to develop some logical arguments. Use of generally appropriate notation, representations, and terminology, with some inaccuracies.
D	Basic knowledge of content and some understanding of concepts and relationships. Some use of mathematical algorithms and techniques implemented electronically where appropriate) to find some correct solutions to routine questions. Sometimes accurate application of knowledge and skills to answer questions set in applied or theoretical contexts.	Application of a mathematical model, with partial effectiveness. Partly accurate and generally incomplete solutions to mathematical problems set in applied or theoretical contexts. Attempted interpretation of the mathematical results in the context of the problem. Some awareness of the reasonableness and possible limitations of the interpreted results. Attempted development or testing a reasonable conjecture.	Some appropriate communication of mathematical ideas and reasoning. Some attempt to use appropriate notation, representations, and terminology, with occasional accuracy.
E	Limited knowledge of content. Attempted use of mathematical algorithms and techniques (implemented electronically where appropriate) to find limited correct solutions to routine questions. Attempted application of knowledge and skills to answer questions set in applied or theoretical contexts, with limited effectiveness.	Attempted application of a basic mathematical model. Limited accuracy in solutions to one or more mathematical problems set in applied or theoretical contexts. Limited attempt at interpretation of the mathematical results in the context of the problem. Limited awareness of the reasonableness and possible limitations of the results. Limited attempt to develop or test a conjecture.	Attempted communication of emerging mathematical ideas and reasoning. Limited attempt to use appropriate notation, representations, or terminology, and with limited accuracy.

STAGE 2 MATHEMATICAL METHODS

SKILLS AND APPLICATIONS TASK 1 – LINEAR PROGRAMMING TEST

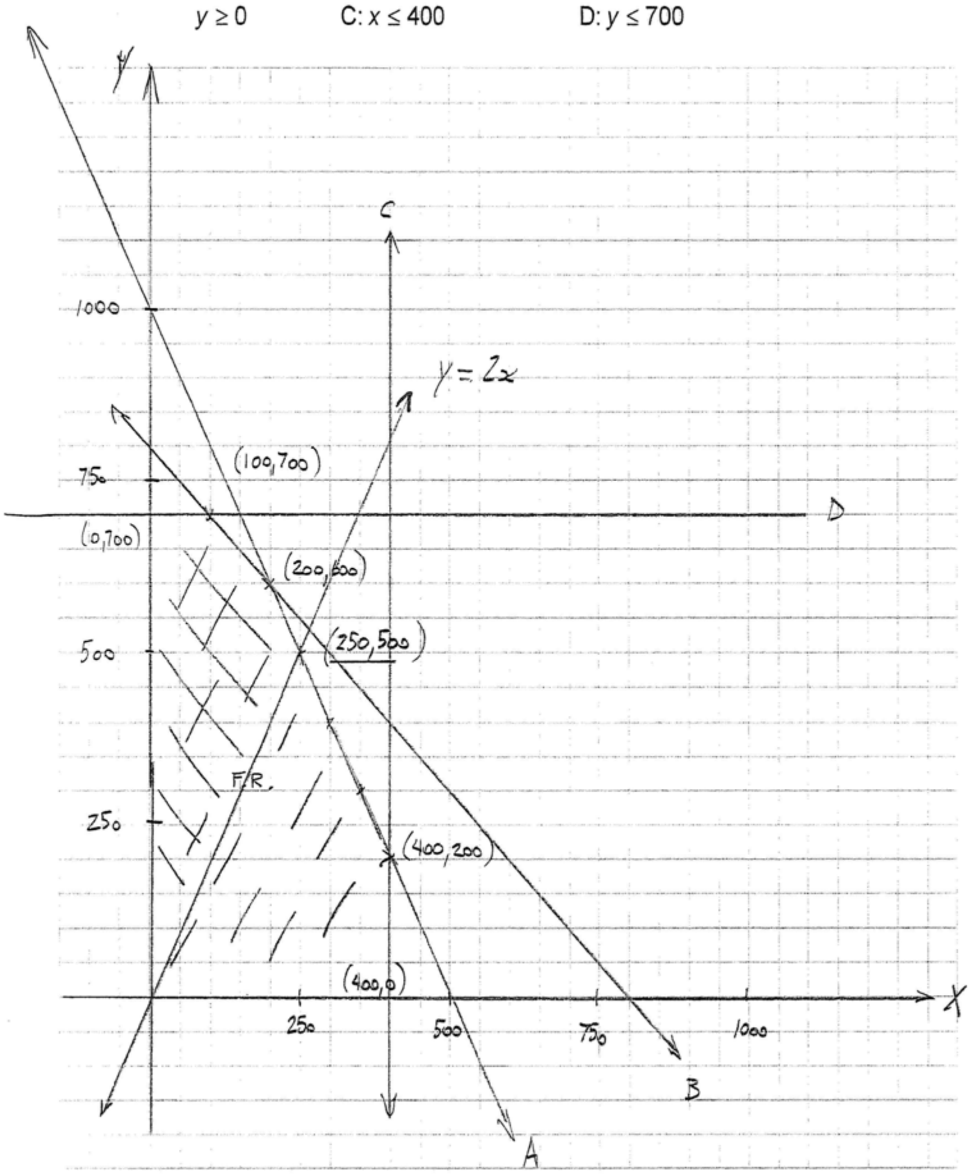
1.

a. Sketch the graph of the constraints below, clearly identifying the coordinates of each vertex, and shade the feasible region.

Let 'x' represent desks and 'y' represent chairs (4)

$x \geq 0$ A: $4x + 2y \leq 2000$ B: $x + y \leq 800$

$y \geq 0$ C: $x \leq 400$ D: $y \leq 700$



Mathematical Knowledge and Skills and Their Application
Provides an opportunity to demonstrate knowledge of graphing of linear inequalities as constraints, and the appropriate use of electronic technology.

b. Given that the objective function is $F = \$80x + \$150y$, design a table to calculate the maximum solution to the problem (2)

	Point	$F
*	(100,700)	$113 000
	(200,600)	$106 000
	(400,600)	$62000

c. Describe your optimal solution (1)

100 DESKS AND 700 CHAIRS
CREATING PROFIT OF \$113 000

d. Based on the feasible region you sketched in part (a.), construct an objective function equation that gives the most number of optimal solutions. List at least three optimal solutions. (3)

MOST OPTIMAL SOLUTIONS - LONGEST LINE

$M = \frac{600-200}{200-400}$

$= \frac{400}{-200}$

$= \frac{-2}{1}$

SOME SOLUTIONS

(200, 600), (250, 500), (300, 400)
(350, 300) AND (400, 200)

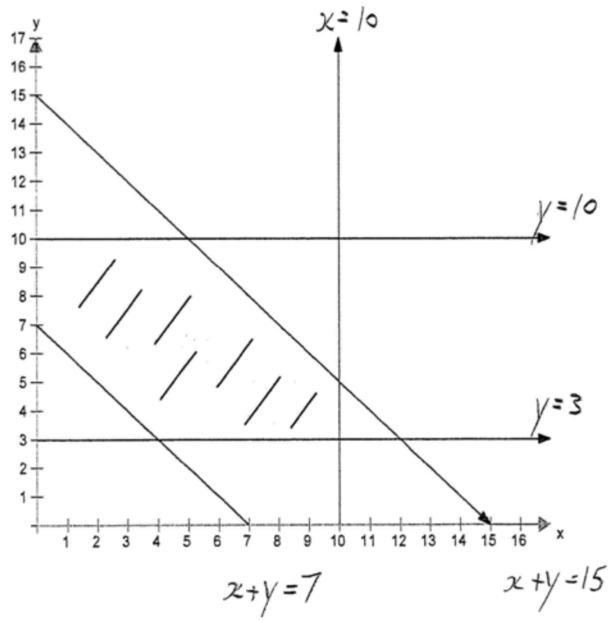
NEW \$F = 2x + y.

e. Consider the new constraint E: $y \geq 2x$. Add this to your graph and discuss the impact upon your feasible region. Shade the new feasible region and the new vertex. (3)

EXPLAIN

NEW VERTEX (250, 500). MUST SELL TWICE AS MANY CHAIRS AS DESKS. FEASIBLE REGION ↓

2. Consider the graph below. Tick three boxes to indicate which inequalities together determine some of the feasible region shaded in the graph above (3)

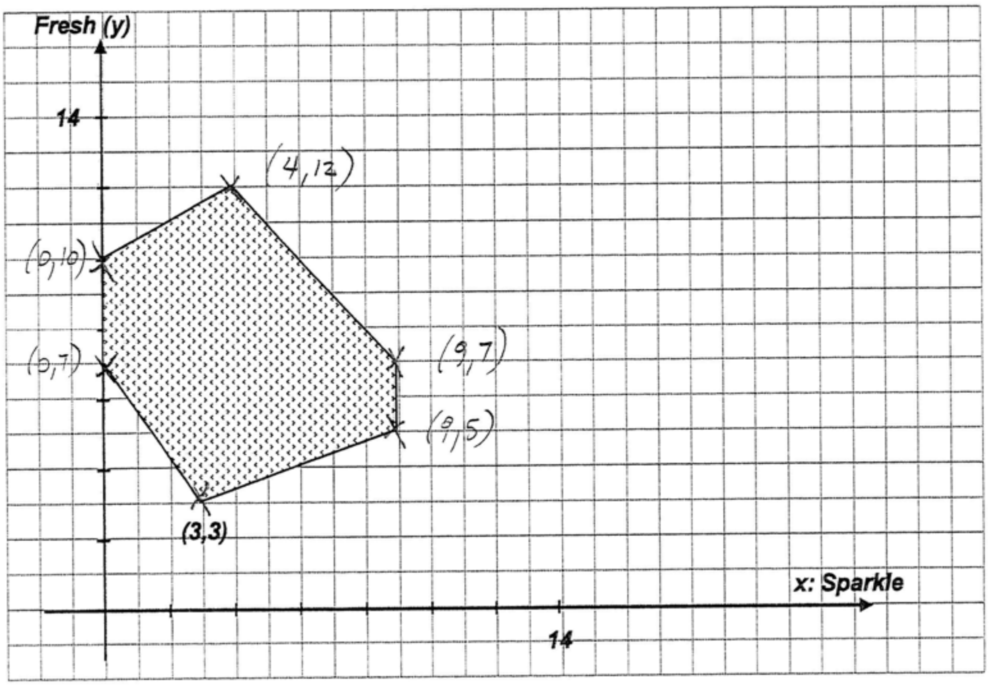


☑ $x + y \geq 7$	☐ $x + y \geq 15$
☐ $x \geq 10$	☑ $y \leq 10$
☑ $y \geq 3$	☐ $x + y \leq 7$
☑ $x + y \leq 15$	☑ $x \leq 10$
☐ $y \geq 10$	☐ $y \leq 3$

Mathematical Knowledge and Skills and Their Application
Part d) provides an opportunity to demonstrate accurate application of knowledge and skills in an applied context, therefore also demonstrating knowledge of content and understanding of concepts.

3. Based on unknown constraints a feasibility region has formed for two different types of cleaning products: *Sparkle* and *Fresh*.

   a. Use the feasible region and the grid to show key vertices (1)



   b. Using the feasible region shown in the graph, determine the maximum value for the profit equation: $P = x + y$. (2)

   YIELD A PROFIT OF $16

   c. At what point(s) on the region does the maximum occur? (2)

   (4,12) AND (9,7)

   d. If the objective function changed to represent costs: $C = x + y$, what optimal solution now exists, so that your house is cleaned using one product only? Explain. (2)

   (0,7) - HITS THE Y-AXIS.
   ONLY USE 7 BOTTLES OF FRESH.

   USING ONE PRODUCT ONLY

   e. Create an additional constraint that would make the feasible triangular in shape (1)

   $5x-4y$ $3x+9y$ $y=5$
   $4x-6y$ $2x+9y$. $y=10$

   f. Discuss the impact on the feasible solutions (1)

   FEASIBLE SOLUTIONS HAVE REDUCED

4. A sick child needs at least 240 units of vitamin A, 36 units of vitamin B and 300 units of vitamin C each week. The child's parents put mashed pears and mangoes on their breakfast cereal each morning to ensure they get these vitamins.

- A tin of pears has 20 units of vitamin A, 6 units of vitamin B and 150 units of vitamin C.
- A tin of mangoes has 60 units of vitamin A, 6 units of vitamin B and 30 units of vitamin C.

A tin of pears costs $2.60 and a tin of mangoes costs $3.00.

	Vitamin A	**Vitamin B**	**Vitamin C**
Pears	20	6	150
Mangoes	60	6	30
	240	36	300

Question 4 provides opportunities to demonstrate application of knowledge and skills to answer questions set in an applied context from the linear programming subtopic 4.1. Evidence of appropriate selection of mathematical algorithms and techniques (implemented electronically where appropriate) would also be demonstrated.

Let x be the number of tins of pears used each week.
Let y be the number of mangoes used each week.

a. Complete the table above (2)

b. The inequalities below summarise the information about a sick child's daily requirements of each vitamin listed above.

Complete the inequalities by filing in the spaces. (3)

Vitamin A: $20x + 60y \geq$ 240 Y_1

Vitamin B: $6x +$ 6y ≥ 36 Y_2

Vitamin C: $150x + 30y$ $\geq$ 300 Y_3

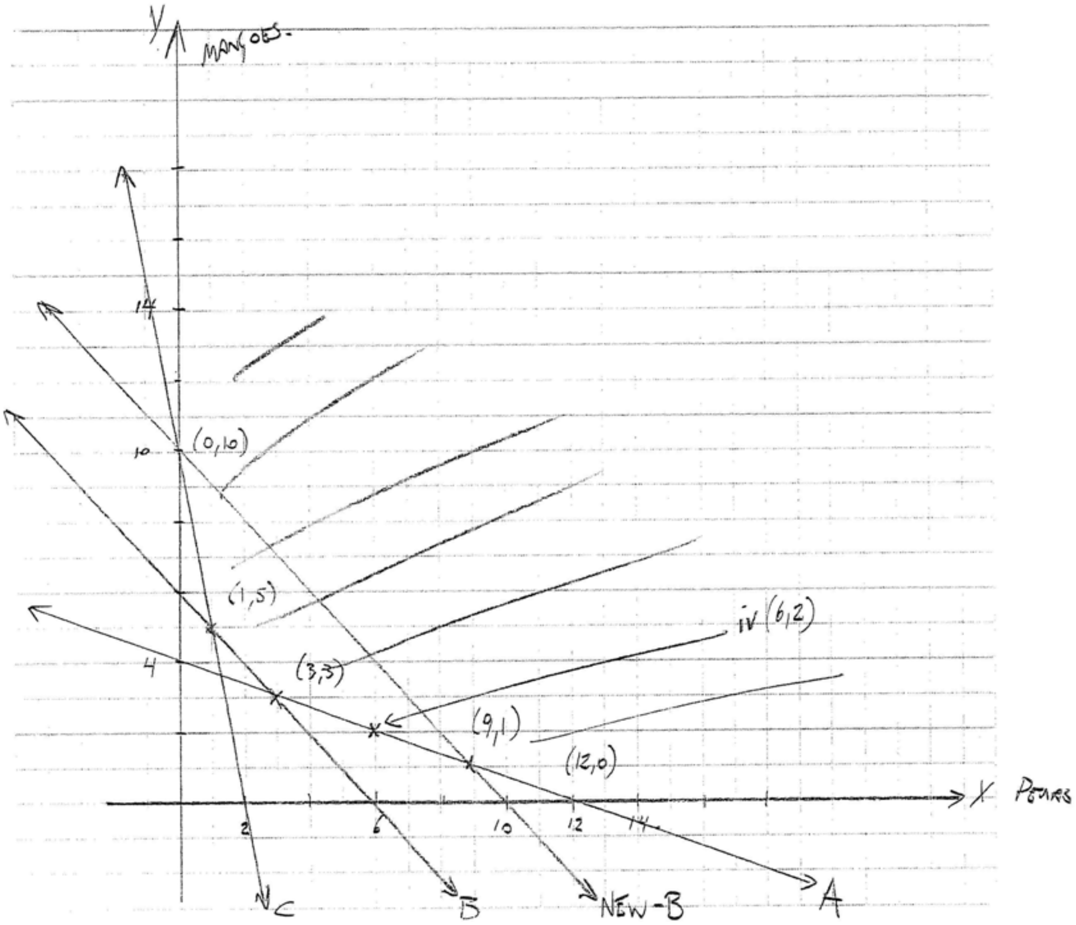
Non-negativity constraints: $x \geq 0$, $y \geq 0$

c. State the objective function that will minimise the cost of the daily requirement of vitamins. (1)

$C = $2.6x + $3y

d.

i. Draw the constraints and shade the feasible region for the three vitamins, clearly labelling each. (5)



Communication of Mathematical Information
Provides the opportunity to demonstrate the use of appropriate representations, including correctly labelled lines, vertices, and axes.

ii. Determine the optimal value for the cost and state the number of tins of each fruit they should give the child each week. (3)

Point	Cost	
(0,10)	$30	3 TINS OF PEARS
(1,5)	$17·60	3 TINS OF MANGOES
(3,3)	$16·80 *	MINIMUM COST OF $16·80
(12,0)	$31·20	

iii. What is the minimum number of tins needed to satisfy the requirement if only tins of pears are used? (1)

12 TINS OF PEARS $31·20

iv. Consider ~~six~~ 4 tins of pears and ~~two~~ 2 tins of mangoes.

Calculate how much the child is receiving (more or less than) for each vitamin type. Show working for all vitamin types. (3)

POINT. (4, 2)

A	B	C
$20x + 60y \geq 240$	$6x + 6y \geq 36$	$150x + 30y \geq 300$
$20(4) + 60(2)$	$6(4) + 6(2)$	$150(4) + 30(2)$
$200 \leq 240$	$36 = 36$	$660 \geq 300$
40 LESS	EXACT AMOUNT.	360 MORE

e.

i. Calculate the impact on the optimal solution if the cost of a tin of mangoes decreased to \$2.60? (2)

NEW FUNCTION : \$2.6x + \$2.6y

POINT	COST.	
(0,10)	\$26	
(1,5)	\$15.60	*
(3,3)	\$15.60	*
(12,0)	\$31.20	

3 SOLUTIONS

(1,5) (3,3) AND. (2,4).

ii. By how much would this change affect the optimal value for the cost? (1)

PRICE CHANGE HAS CAUSED COSTS TO DECREASE BY \$1.20 / TIN.

f. Further research has shown that the daily requirements of vitamin B is 60 units instead of 36 units previously stated.

i. Rewrite the constraint that is affected by this change. (1)

$6x + 6y \geq 60$

OR

$x + y \geq 10$

ii. Draw this constraint on the graph in part (d. i.) and label it 'new vitamin B'. (1)

iii. Determine the new optimal solution (by a table) using the objective function in part (c.) stating the cost and number of tins of each. (3)

Vertex	$F = 2.6x + 3y$
(0,10)	\$30
(9,1)	\$26.40
(12,0)	\$31.20

iii) if x/y is fixed (6x or 6y) rewrite the expression to create many solutions.

Additional comments

Communication of Mathematical Information

- Each question has been structured using multiple parts providing opportunities to develop logical arguments through communication of mathematical ideas and reasoning.
- The questions provide opportunities to use appropriate notation (e.g. defining the variables), representations (e.g. graphing of constraints to shade the feasible region), and terminology (e.g. profit, optimal solution).

Mathematical Modelling and Problem-solving

- Each question is structured to produce or provide a suitable graph to use as a model for further directed investigation. Therefore there are opportunities to apply the model but not independently develop a model. A folio task in which the student develops and applies a mathematical model would be preferable for assessing MMP1.