STAGE 2 MATHEMATICAL METHODS

SKILLS AND APPLICATIONS TASK 3 – MATRICES TEST

1. Consider the following matrices:

$$A=\begin{bmatrix}1 & 3\\ -1 & 0\\ 4 & -2\end{bmatrix} \quad B=\begin{bmatrix}1 & -3\\ 4 & 2\end{bmatrix} \quad C=\begin{bmatrix}x & 3\\ -1 & -2\end{bmatrix} \quad D=\begin{bmatrix}x\\ y\end{bmatrix} \quad E=\begin{bmatrix}0\\ 3\end{bmatrix}$$

3×2 2×2 2×2 1×1 1×1

Evaluate, if possible (otherwise explain why not):

a. AB b. $A-C$ (2)

$AB=\begin{bmatrix}1 & 3\\ -1 & 3\\ -4 & -16\end{bmatrix}$ $A-C=$

−1

c. C^2 d. $BD=E$ ~~(2)~~ 4

$C^2=\begin{bmatrix}x & 3\\ -1 & -2\end{bmatrix}\begin{bmatrix}x & 3\\ -1 & -2\end{bmatrix}$

$=\begin{bmatrix}x^2-3 & 3x-6\\ -x+2 & -1\end{bmatrix}$

$BD=\begin{bmatrix}1 & -3\\ 4 & 2\end{bmatrix}\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}0\\ 3\end{bmatrix}$

$=\begin{bmatrix}x-3y\\ 4x+2y\end{bmatrix} \Rightarrow \begin{matrix}x-3y=0\\ 4x+2y=3\end{matrix}$

$\therefore x-3y=0 \Rightarrow y=\frac{-x}{-3}$

$4x+2y=3 \Rightarrow y=\frac{3-4x}{2}$

$\therefore x=-6 \quad y=-2$

−2

Mathematical Knowledge and Skills and Their Application
The evidence provided in question 1 is indicative of basic knowledge of the operation of multiplication of matrices.

e. The order of ***X*** in ***BXA*** so it can be multiplied (1)

−1

2. ***Show all headings and matrix labels in the following question***

A canteen sells the following foods over the course of a week.

The matrix of ***food sold*** is given by matrix ***Q***:

$$Q = \begin{array}{c} \\ \textit{Pastie} \\ \textit{Pie} \\ \textit{Pizza} \\ \textit{Hot Dog} \end{array} \begin{array}{c} \begin{array}{ccccc} M & T & W & T & F \end{array} \\ \begin{bmatrix} 216 & 244 & 220 & 232 & 226 \\ 145 & 130 & 152 & 121 & 113 \\ 83 & 102 & 78 & 75 & 94 \\ 12 & 14 & 8 & 13 & 18 \end{bmatrix} \end{array} \quad 4\times5$$

The ***selling price*** of the foods is given by matrix ***S***:

$$S = \begin{array}{c} \\ \$ \end{array} \begin{array}{c} \begin{array}{cccc} \textit{Pastie} & \textit{Pie} & \textit{Pizza} & \textit{Hot Dog} \end{array} \\ \begin{bmatrix} \$2.20 & \$1.80 & \$1.70 & \$3.20 \end{bmatrix} \end{array} \quad 1\times4$$

The ***cost price*** to make the foods is given by matrix ***C***:

$$C = \begin{array}{c} \begin{array}{cccc} \textit{Pastie} & \textit{Pie} & \textit{Pizza} & \textit{Hot Dog} \end{array} \\ \begin{bmatrix} \$0.60 & \$0.50 & \$1.10 & \$1.80 \end{bmatrix} \end{array} \quad 1\times4$$

a. Describe the ***element*** in position $(4,2)$ in ***Q*** (1)

This shows the number of hotdogs sold on Tuesday

b. Calculate many Pizzas were eaten for the week (1)

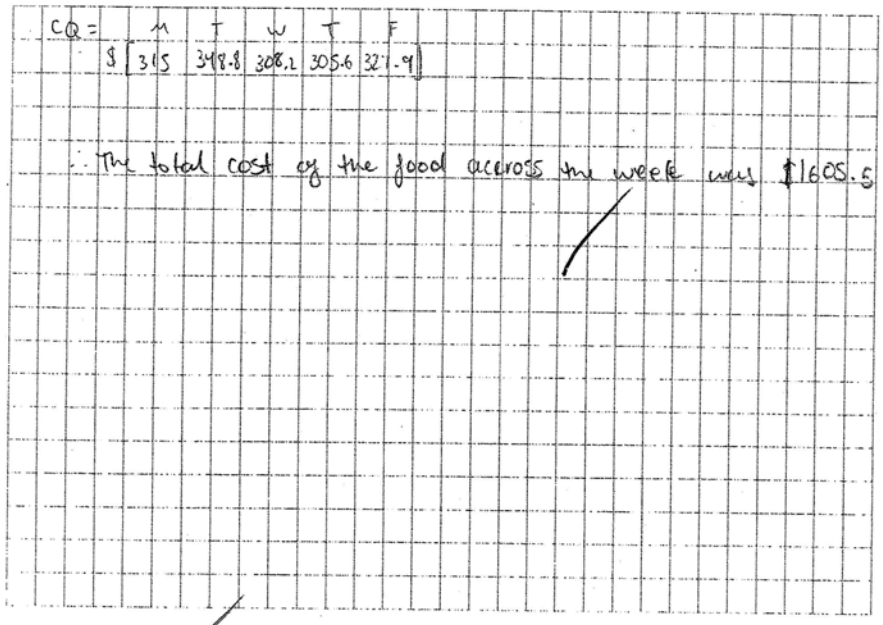
432 pizzas were eaten during the week.

c. Calculate $[1 \quad 1 \quad 1 \quad 1][Q]$ and interpret Thursday's result (2)

$$= \begin{array}{cccccc} M & T & W & T & F \\ [456 & 490 & 458 & 441 & 451] \end{array}$$

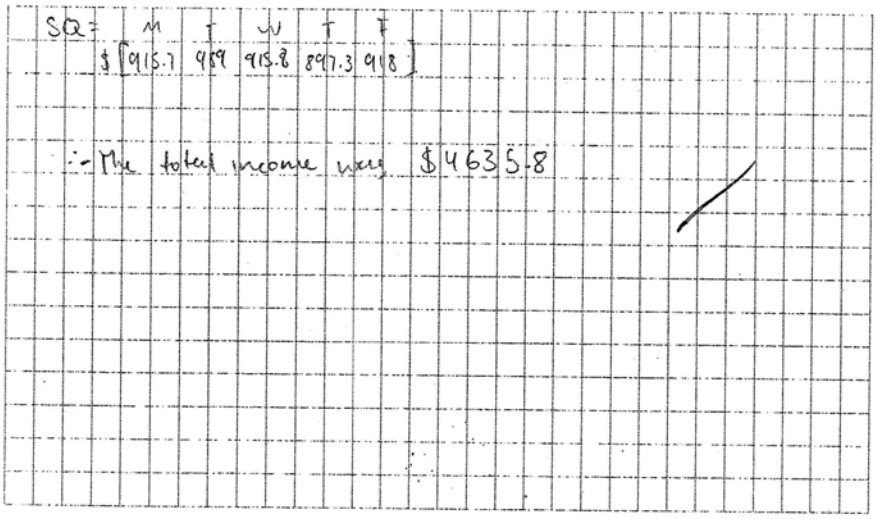
∴ There were 441 total items of food sold on Thursday.

d. Use matrix methods to calculate the ***total cost*** of the food (2)

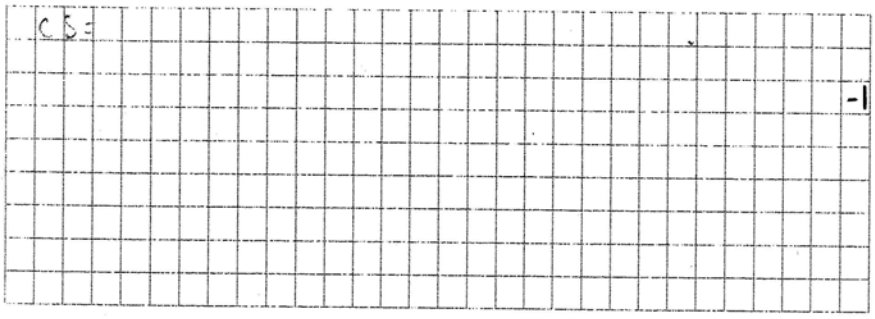


Communication of Mathematical Information
The evidence provided in question 2 is illustrative of mostly accurate use of appropriate notation, representations, and terminology.

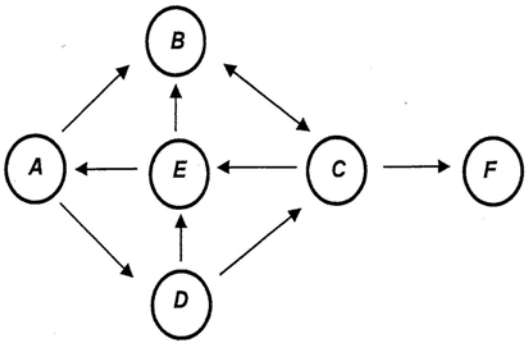
e. Use matrix methods to calculate the ***total income*** from the sale of food (2)



f. Hence, calculate the ***weekly profit*** from the food (1)



3. Consider the network diagram below:



a. Draw up a ***connectivity matrix R*** to describe the network above (headings in alphabetical order) (2)

$$R = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

b. Calculate R^2 (1)

$$R^2 = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

c. What do the following entries tell you about the network?

i. $R^2_{(1,3)}$? (1)

$R^2(1,3) = 2$ This shows there are 2 ways from A to C.

ii. $R^2_{(2,4)}$? (1)

$R^2_{(2,4)} = 1$ This shows there is 0 ways from B to D. ✓

d. Describe what happens to anything arriving at ***F***? (1)

They would not be able to go back as there are no routes. ✓

e. What is the ***minimum number of stages*** it takes to travel from ***B*** to ***D***? (1)

minimum of 4 stages ✓

f. Calculate the matrix: $Q = R + R^2 + R^3$.

What is the meaning of $Q_{(4,2)}$? (2)

$Q =$

	A	B	C	D	E	F
A	1	4	2	1	3	2
B	1	2	2	0	1	1
C	1	4	2	1	2	2
D	2	4	3	1	2	1
E	1	3	3	1	2	1
F	0	0	0	0	0	0

✓

$Q_{(4,2)}$ shows there are 4 ways from D to B. ✓

~✓

4. It is known that tennis players, *Ace (A), Best (B), Champion (C), Dux (D) and Excel (E)* have played each other before, with the results being:

A has defeated B, C and D

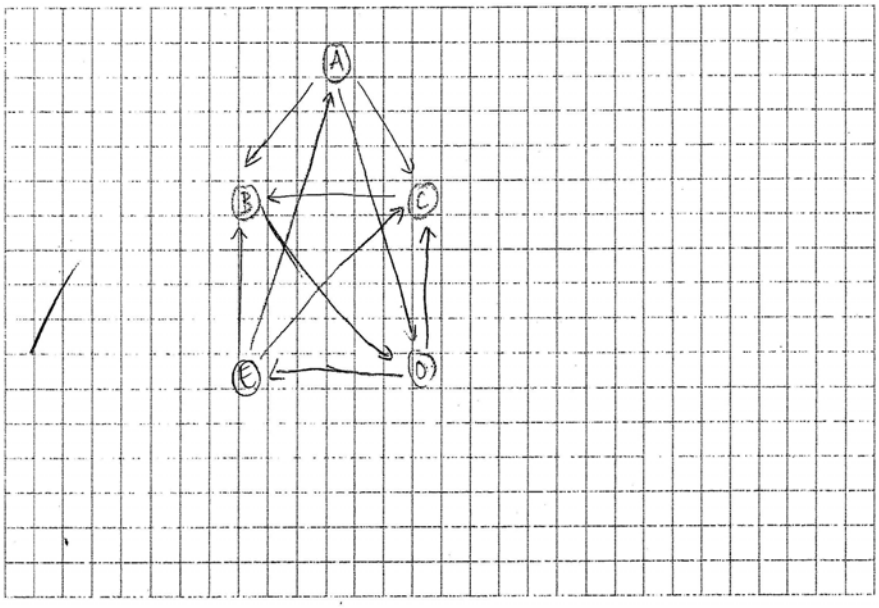
B has defeated D

C has defeated B

D has defeated C and E

E has defeated A, B and C

a. Draw a network diagram to illustrate the results of these contests (2)



b. Show the victories as a results ***matrix R*** with the players listed ***alphabetically*** (2)

$$R = \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \end{array} \begin{array}{c} \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

c. Calculate the Supremacy Matrix: $S = R + \frac{1}{2}R^2$ (1)

	A	B	C	D	E
A	0	1.5	1.5	1.5	.5
B	0	0	.5	1	.5
C	0	1	0	.5	0
D	.5	1	1.5	0	1
E	1	2	1.5	1	0

d. What is the meaning of the $\frac{1}{2}$ in R^2 in (c.)? (1)

The $\frac{1}{2}$ in R^2 means the certain victory is only worth $\frac{1}{2}$ point.

e. List the ***supremacy vector*** and ***rank the players*** in descending order (2)

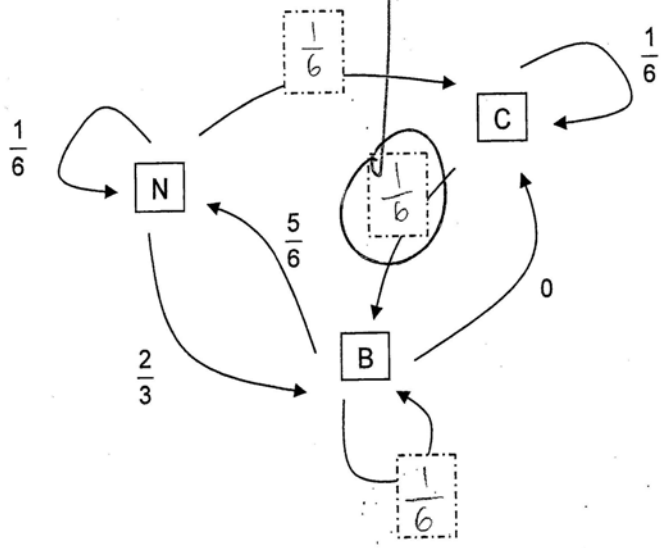
A	5	2nd
B	2	4th
C	1.5	5th
D	4	3rd
E	5.5	1st

5. Jordan processes insurance claims and he flies between the three major cities: Chicago, New York and Boston. The city that Jordan will be in tomorrow depends only on which city he is in today. His movements between cities are represented mathematically by the following matrix 'T':

$$\text{(today)}\quad T = \begin{array}{c} \\ C \\ N \\ B \end{array} \begin{array}{c} \begin{array}{ccc} C & N & B \end{array} \\ \begin{bmatrix} \frac{1}{6} & \frac{1}{2} & \frac{1}{3} \\ \frac{1}{6} & \frac{1}{6} & \frac{2}{3} \\ 0 & \frac{5}{6} & \frac{1}{6} \end{bmatrix} \end{array} \quad \text{(tomorrow)}$$

For example, if Jordan was in New York today, the probability that he will be in Boston tomorrow is $\frac{2}{3}$.

a. Assign the three probabilities to the diagram below (1)



b. Interpret the values of $T_{(1,1)\ (2,2)\ (3,3)}$ (1)

These values show that there is a $\frac{1}{6}$ probability of being back in that same city the following day

c. Explain the meaning of the zero in the matrix T (1)

There is no chance to be in Chicago tomorrow if you left today.

T^2 represents the matrix of the probabilities that if Jordan is in a particular city today, he will be in Chicago, New York and Boston in two days time.

d. Determine the matrix T^2 (1)

$T^2 =$

	C	N	B
C	.1	.4	.4
N	.05	.6	.3
B	.14	.3	.6

e. The entry for travelling from Boston to Chicago in Matrix T^2 is different from matrix T. Explain why. (1)

Because this shows the probability of being in Chicago in 2 days time instead of 1 days time.

The row matrix X_0 represents the probability that he is in Chicago, New York and Boston on Thursday.

$$X_0 = \begin{matrix} C & N & B \end{matrix} \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & 0 \end{bmatrix}$$

f. Find the matrix X_1 if $X_1 = X_0 T$ and explain where and when he is likely to be. (2)

$X_1 =$

C	N	B
.16	.33	.5

He is likely to be in Chicago or New York on Thursday

g. Find the matrix X_2 if $X_2 = X_0 T^2$ and explain where and when he is likely to be. (2)

$X_2 =$

C	N	B
.083	.5	.361

83%

There is a 83% chance he will be in Chicago
50% chance he will be in New York
36.1% chance he will be in Boston.

h.

i. In which city is Jordan likely to be on Monday? (1)

New York

ii. What is the probability that Jordan will be processing insurance claims in that city on Monday (1)

-1

50% chance

iii. In which city is Jordan likely to be in one months time? (1)

New York ✱ 48% chance

6. Fill in the missing element and algebraically calculate the steady state of matrix B if $B = \begin{bmatrix} 0.2 & 0.8 \\ 0.6 & \end{bmatrix}$ (3)

-3

Mathematical Knowledge and Skills and Their Application
No attempt to find a correct solution to a complex question by the use of appropriate mathematical algorithms and techniques is evident.

7. The '36'ers' is Adelaide's long serving NBL basketball team. To create a good team image, a local snack food company, Sporting Munchies, has produced three healthy bar products to be sold in the NBL matches and local basketball stadiums.

There has been a rapid increase in business and you have been assigned to calculate the weekly raw material requirements, costs of production, selling price of products and ordering of raw materials.

You have been supplied with the following information:

'Sporting Munchies' produces three different snack bars, using different proportions of the following ingredients: sultanas, apricots, sugar, sesame seeds and butter.

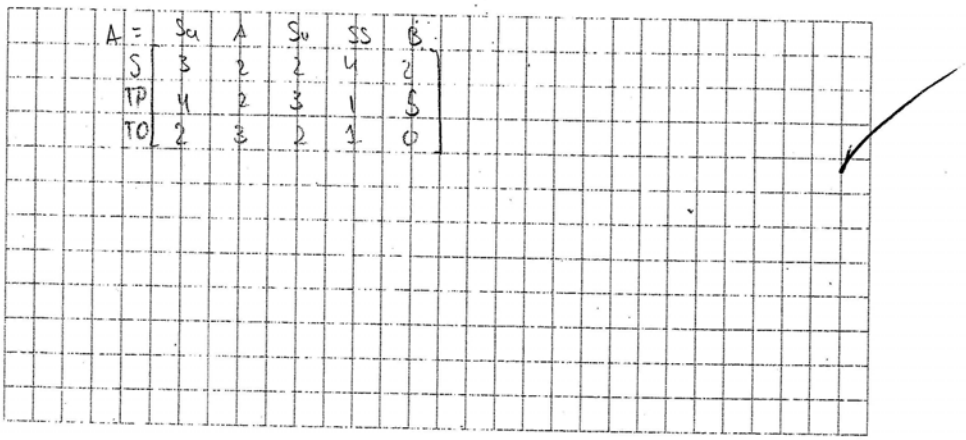
To make one batch of each health food requires:

'Slam Dunks'
3 units of sultanas
2 units of apricots
2 units of sugar
4 units of sesame seeds
2 units of butter

'Three Pointers'
4 units of sultanas
2 units of apricots
3 units of sugar
1 unit of sesame seeds
5 units of butter

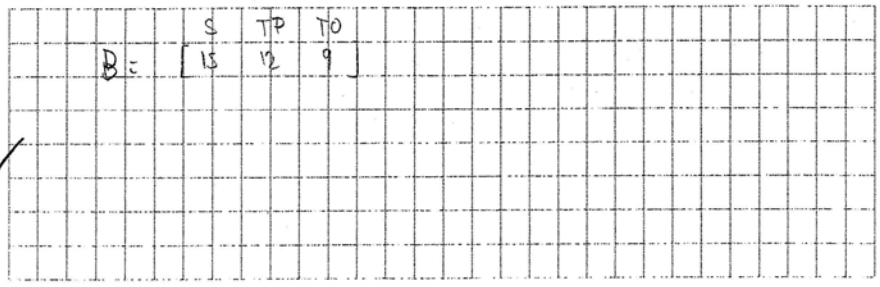
'Time Outs'
2 units of sultanas
3 units of apricots
2 units of sesame seeds
1 unit of butter

a. Represent the above information in the form of a single 3×5 matrix A (3)



During the basketball season the normal weekly order is 15 batches of Slam Dunks, 12 batches of Three Pointers and 9 batches of Time Outs.

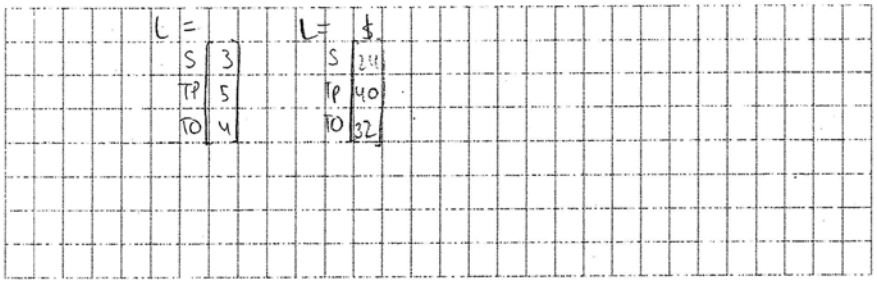
b. Represent the normal weekly order in the form of a matrix B (2)



Each bar is made differently and involves the following labour, which costs the company $8.00 per unit.

Slam Dunks	*3 units of labour per batch*
Three Pointers	*5 units of labour per batch*
Time Outs	*4 units of labour per batch*

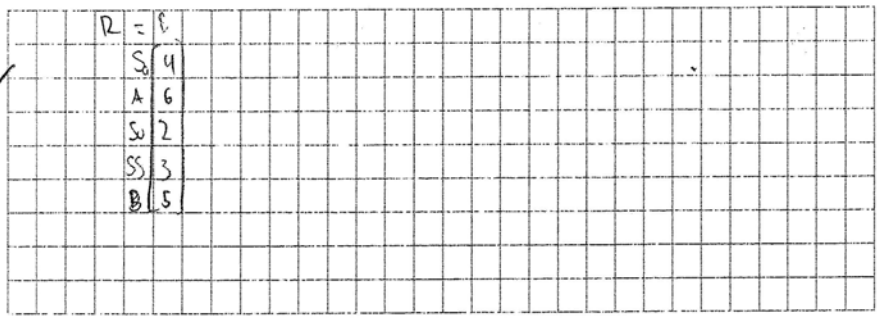
c. Represent this information in the form of a column matrix (2)



The costs of raw materials are as follows:

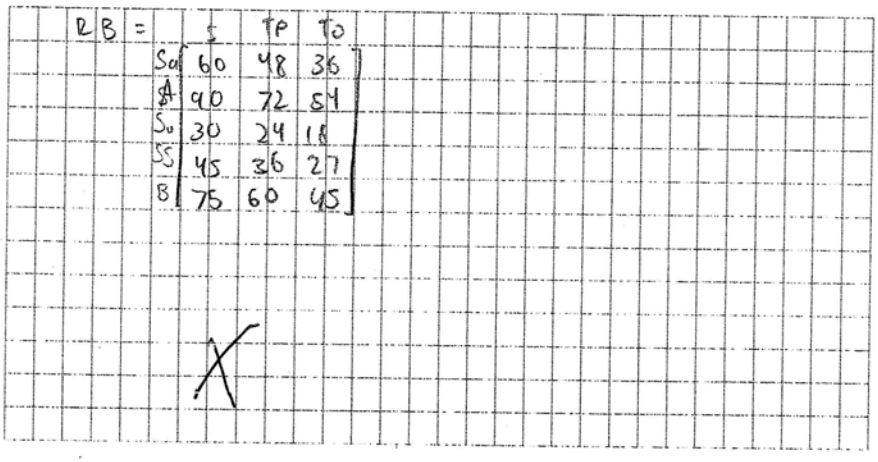
Sultanas	*$4 per unit*
Apricots	*$6 per unit*
Sugar	*$2 per unit*
Sesame Seeds	*$3 per unit*
Butter	*$5 per unit*

d. Represent this information in the form of a matrix (2)



Calculate the following using matrix methods:

e. The total raw material requirements for one week's production during the basketball season (2)



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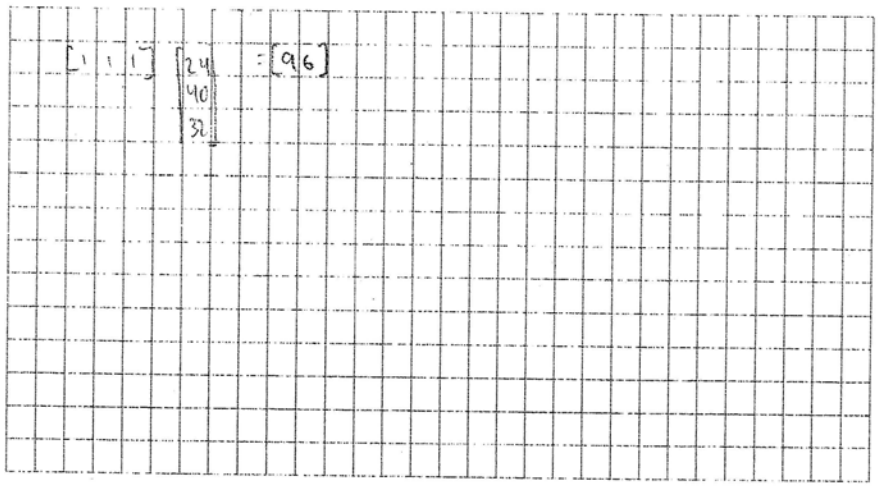
f. The costs of producing one batch of each bar (2)



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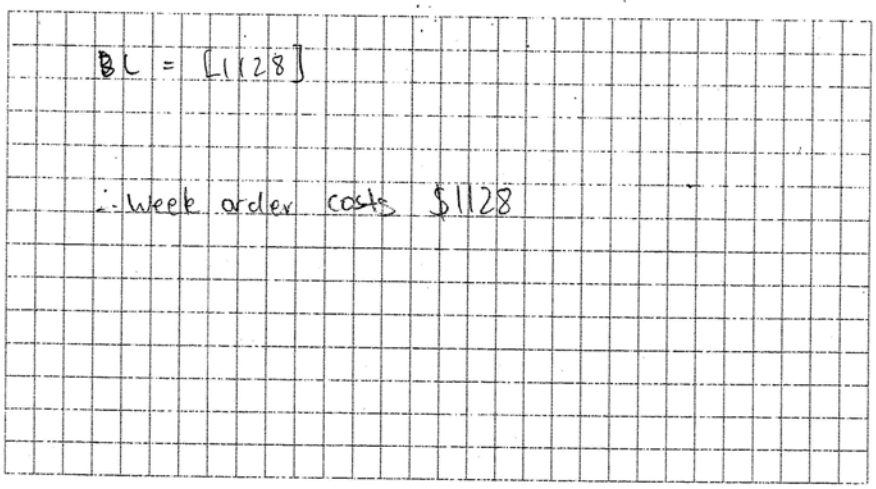
g. The total costs for the weeks order (2)



-2

Mathematical Knowledge and Skills and Their Application
In parts a) to d) there is evidence of knowledge of appropriately representing data in matrix form. However, there is no evidence in later parts to support the application of mathematical knowledge and skills to answer questions in the context of this question.

8. Scientists are studying the changes in the number of female frogs living along a creek bed. They are currently feeling the effects of a virus.

- The females are dying after 3 years.
- The stages of the frogs development are tadpoles, frogteens and adults
- Females less than a year old are immature and cannot reproduce.
- Only a $\frac{1}{2}$ of these frogs reach the age of one.
- Frogteens produce an average of 1.2 female tadpoles in that year.
- Only $\frac{1}{5}$ of frogteens reach adulthoold because of the virus
- Adult frogs produce on average, two female tadpoles in that year
- Very few frogs reach the age of three, those that do are past the point of producing tadpoles

a. Using the information above complete the matrix below (3)

-1

1:2 or 1/2 ✓

$$P = \begin{matrix} \\ T \\ F \\ A \end{matrix} \begin{matrix} \begin{matrix} T & F & A \end{matrix} \\ \begin{bmatrix} 0 & 1.2 & 2 \\ 1.2 & 0 & 0 \\ 0 & .2 & 0 \end{bmatrix} \end{matrix}$$

The number of Tadpoles, Frogteens and Adult frogs at the time of the study are represented by:

$$O = \begin{matrix} \\ T \\ F \\ A \end{matrix} \begin{bmatrix} 200 \\ 80 \\ 20 \end{bmatrix}$$

b. Find *PO* and explain what this gives (2)

$$PO = \begin{matrix} \\ T \\ F \\ A \end{matrix} \begin{bmatrix} 136 \\ 240 \\ 16 \end{bmatrix}$$

✓

This gives the number of Tadpoles, frogteens and adults are produced in 1 lifecycle

c. Find P^2O and explain what this gives (2)

P^2O =

T [320]
F [163.2]
A [48]

Shows the number of tadpoles, frogteens and adults produced in 2 yrs.

-1

d. Find P^3O and P^5O (2)

P^3O =

T [291.84]
F [384]
A [32.64]

P^5O =

T [573.8496]
F [631.296]
A [70.0416]

-1

e. Calculate the percentage increase of decrease of frogs along the creek bed between year two and three (2)

T = 28.46 % decrease
F = 220.8 % decrease
A = 15.36 % increase

-2

f. What do you think is likely to happen with the frog population along the creek as time goes on? (1)

The frog population will decrease and then increase.

g. If frogteens now produce on average, four females tadpoles, explore what happens to the frog population as time goes on? (2)

The Frog population will increase to large amounts.

52

Additional comments
A review of the student's response provides evidence of:

Mathematical Knowledge and Skills and Their Application

- generally competent knowledge of content and understanding of concepts and relationships due to some inconsistiencies being evident. Questions 2, 3, and 4 are mostly complete and accurate but this is not reflected in later questions
- mostly correct solutions to routine questions which required the use of mathematical algorithms and techniques (implemented electronically where appropriate)
- generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.

Communication of Mathematical Information

- appropriate communication that supports the development of some logical arguments according to the context of the question
- mostly accurate use of appropriate notation (e.g. naming of matrices, including R^2, X_1, and P^3O), representations (e.g. matrix form, network diagrams), and terminology (e.g. probability).

PERFORMANCE STANDARDS FOR STAGE 2 MATHEMATICAL METHODS

	Mathematical Knowledge and Skills and Their Application	Mathematical Modelling and Problem-solving	Communication of Mathematical Information
A	Comprehensive knowledge of content and understanding of concepts and relationships. Appropriate selection and use of mathematical algorithms and techniques (implemented electronically where appropriate) to find efficient solutions to complex questions. Highly effective and accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Development and effective application of mathematical models. Complete, concise, and accurate solutions to mathematical problems set in applied and theoretical contexts. Concise interpretation of the mathematical results in the context of the problem. In-depth understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. Development and testing of reasonable conjectures.	Highly effective communication of mathematical ideas and reasoning to develop logical arguments. Proficient and accurate use of appropriate notation, representations, and terminology.
B	Some depth of knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to complex questions. Accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.	Attempted development and appropriate application of mathematical models. Mostly accurate and complete solutions to mathematical problems set in applied and theoretical contexts. Complete interpretation of the mathematical results in the context of the problem. Some depth of understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made. Development and testing of some reasonable conjectures.	Effective communication of mathematical ideas and reasoning to develop mostly logical arguments. Mostly accurate use of appropriate notation, representations, and terminology.
C	Generally competent knowledge of content and understanding of concepts and relationships. Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find mostly correct solutions to routine questions. Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts	Appropriate application of mathematical models. Some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts. Generally appropriate interpretation of the mathematical results in the context of the problem. Some understanding of the reasonableness and possible limitations of the interpreted results and some recognition of assumptions made. Development and testing of one or more reasonable conjectures.	Appropriate communication of mathematical ideas and reasoning to develop some logical arguments. Use of generally appropriate notation, representations, and terminology, with some inaccuracies.
D	Basic knowledge of content and some understanding of concepts and relationships. Some use of mathematical algorithms and techniques implemented electronically where appropriate) to find some correct solutions to routine questions. Sometimes accurate application of knowledge and skills to answer questions set in applied or theoretical contexts.	Application of a mathematical model, with partial effectiveness. Partly accurate and generally incomplete solutions to mathematical problems set in applied or theoretical contexts. Attempted interpretation of the mathematical results in the context of the problem. Some awareness of the reasonableness and possible limitations of the interpreted results. Attempted development or testing a reasonable conjecture.	Some appropriate communication of mathematical ideas and reasoning. Some attempt to use appropriate notation, representations, and terminology, with occasional accuracy.
E	Limited knowledge of content. Attempted use of mathematical algorithms and techniques (implemented electronically where appropriate) to find limited correct solutions to routine questions. Attempted application of knowledge and skills to answer questions set in applied or theoretical contexts, with limited effectiveness.	Attempted development of a basic mathematical model. Attempted development of the mathematical results for problems set in familiar contexts. Limited attempt at interpretation of the mathematical results in the context of the problem. Limited awareness of the reasonableness and possible limitations of the results. Limited attempt to develop or test a conjecture.	Attempted communication of emerging mathematical ideas and reasoning. Limited attempt to use appropriate notation, representations, or terminology, and with limited accuracy.