2021 Mathematical Methods Subject Assessment Advice

Overview

Subject assessment advice, based on the 2021 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

School Assessment

Teachers can improve the moderation process and the online process by:

* thoroughly checking that all grades entered in school online are correct.
* ensuring the uploaded tasks are legible, all facing up (and all the same way), and remove blank pages, student notes and formula pages.
* ensuring the uploaded responses have pages the same size and in colour so teacher marking and comments are clear.
* for SATs and Mathematical Investigations responses, clearly marked answers showing which mathematical calculations are fully or partially correct and which are incorrect is a requirement of moderation. Showing marks and totals for SATs is also helpful.
* uploading the SATs as a single scanned file.
* preferably providing a summary of student results in each of the SATs on the first page of the uploaded SAT’s file.
* using the same tasks when combining with another school or schools where possible to support comparable assessments. When combining classes across schools, teachers should be involved in moderation activities prior to uploading materials for the actual moderation to ensure that the rank order of the students within the combined assessment group is appropriate.

Assessment Type 1: Skills and Application Tasks

Students complete six skills and application tasks under the direct supervision of the teacher. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* including tasks that provided a mix of routine and more complex problems that effectively differentiated student mathematical knowledge and understanding of concepts and relationships across the grade bands.
* allowing multiple opportunities to demonstrate conjecture development and proof.
* including several questions that allow students to demonstrate their interpretation of concepts and results.
* including tasks that provided the opportunity for interpretation of the mathematical results in the context of the problem.
* including questions that allowed for multiple entry points into extended questions by including some ‘show that‘ parts to enable students to access the following sections of the question even if they do not manage to calculate the correct answer in the ‘show that’ part.
* providing students with axes and grids when asking them to sketch graphs.

The more successful responses commonly:

* provided detailed evidence of the development and proof of one or more conjectures.
* consistently demonstrated mathematical knowledge and understanding of concepts and relationships across all topics.
* showed all algebraic working by providing relevant steps, particularly for the ‘show that’ type questions.
* used correct notation and terminology.

The less successful responses commonly:

* were seen in tasks that did not provide opportunity for students to solve more complex problems.
* did not demonstrate an appropriate balance of understanding the key questions and concepts across the topics in the subject outline.
* did not provided enough attention to detail in presenting their solutions.
* lacked an understanding of how technology could be used effectively.
* did not often attempt questions that asked for interpretation or logical conclusions of results.
* were based on tasks that used questions directly from a textbook or from past exams.
* were based on tasks that provided questions that were repetitive which either disadvantaged students with poor knowledge and understanding, or advantaged students with highly effective knowledge and understanding.

Assessment Type 2: Investigation

Students complete one mathematical investigation with minimal teacher direction. The task should be written in a format that allows the student to conduct their own open-ended investigation. It must be completed in report format (if written) and must be no longer than 15 single-sided A4 pages. Appendices should be used for repetitive calculations only.

Teachers can elicit more successful responses by:

* providing some structure of an initial problem leading to a more open-ended problem to investigate.
* ensuring that the investigation is at an appropriate level of complexity and aligns well with the subject outline.
* ensuring that the design of the task allows for the discussion of limitations and reasonableness. This is best done by providing a task in the context of a situation.
* presenting the response in report format. Communication of mathematical information is best done by using appropriate headings, labelling graphs and tables, referring to them in the main body of the response, and using appendices for repetitive calculations.

The more successful responses commonly:

* included detailed development and application of a mathematical model.
* require mathematics beyond just modelling an equation using a calculator for a set a data.
* demonstrated a comprehensive understanding of the reasonableness and limitations of the model developed, which can more easily be done in the context of a situation.
* effectively communicated mathematical reasoning through the use of appropriate and well-labelled graphs and the inclusion of relevant calculations and notation.
* made appropriate use of the appendices to avoid repetition in the report.
* were based on tasks that included an element of student directed learning that differentiated student understanding of the model or problem being investigated.
* effectively and efficiently used technology to solve problems.

The less successful responses commonly:

* included excessive routine and often repetitive calculations that were prescribed in the task design.
* did not effectively select and apply appropriate mathematical techniques or use appropriate electronic technology when solving problems.
* lacked depth in the interpretation of the mathematical results, often summarising what was calculated rather than analysing the results in the context of the problem.
* were difficult to follow, lacking in explanation of choices made in developing a model.
* relied on proofs that are easily accessible through the textbook or online.
* were based on tasks that had a limited open-ended component that students found difficult to further develop a mathematical model for beyond fitting a curve.

External Assessment

Assessment Type 3: Examination

Examination markers aim to award marks for evidence of student understanding in responding to examination questions wherever possible, however, students should be advised not to cross out their responses or attempted responses to questions in the examination booklet, unless they are confident that no part of their response should be considered by the marker. Mathematical Methods external examinations will not award any marks to crossed-out work, so care should be taken by students to cross out work that they do not wish the marker to consider only.

Overall, students appeared to find the examination to be approachable and they had success in accessing each question to some degree.

Some noteworthy things to consider discussing with students in preparation for the 2022 exam are:

* Although one trivial error within a question is not penalised, students need to ensure that their answer makes sense in the context of the problem. If an unrealistic answer is obtained through error, a student should write a comment addressing this.

Students need to pay closer attention to the wording of questions. Words or phrases such as ‘exact’, ‘hence’, ‘show’ or ‘using an algebraic process’ are used to help guide a student’s approach in finding a solution.

* Students need to take note of the allocated marks given in a question (and the space provided) to determine if they can use technology or simply state the answer (rather than show any mathematical process). Unless a particular approach is suggested by the wording of the question, students should always use the optimal method to find a solution
* Great care is taken during the writing process to allow multiple entry points into questions wherever possible; hence, it is worth reminding students that even if they do not successfully solve one part of a question, they should continue to attempt to solve the following sections.

Question 1

This question provided an opportunity for students to demonstrate their routine calculus skills with 85% of students receiving 6 or 7 marks out of 7.

The more successful responses commonly:

* correctly implemented the chain rule, product rule and quotient rule when required
* used the quotient rule in part (c) instead of rearranging the equation to allow the use of the product rule.

The less successful responses commonly:

* did not include appropriate brackets in their answers
* tried to express the derivative of  in surd form without interim steps in part (b)
* incorrectly differentiated the function inside of the brackets in part (b). Students often:
* forgot to differentiate the leading *x* term in the expression  resulting in a derivative of
* misread the function as 
* forgot to differentiate it at all when using the chain rule.

Question 2

This question provided further opportunity for students to demonstrate their knowledge of Calculus. Students needed to show how the functions  and  relate to the stationary points and points of inflection of the graph of the function . Most students successfully completed the first routine part of the question; however, many did not know the required conditions of the that result in the graph of having a point of inflection. This one common error led to only 20% of students achieving more than 6 marks out of 8.

The more successful responses commonly:

* read the question carefully to find both the *x*-coordinate and *y*-coordinate of the stationary point requested in part (a)(ii)
* used a sign diagram to justify that there was no point of inflection of the graph of the function in part (b)(ii); or, commented that  for all values of *x*, hence there were no points of inflection.

The less successful responses commonly:

* incorrectly solved the resulting equation of  to get solutions of either  in part (a)(ii)
* substituted the value of  into  instead of  to state the coordinates of the stationary point as (1,0) in part (a)(ii).
* relied only on the visual aspects of the graph in Figure 1 to discern if there was a point of inflection in part (b)(ii), despite the question specifically requesting ‘mathematical evidence’.
* assumed that if , there must be an inflection point in part (b)(ii).

Question 3

This question contained some routine statistical computations and concepts that students should be familiar with from past examinations. Students needed to apply properties of the central limit theorem throughout the question in addition to using their graphics calculator to state a requested probability. Overall, students did well in this question, as evident by it having the second highest mean percentage of marks gained in the paper (73%).

The more successful responses commonly:

* used concise reasoning, referencing the ‘shape’ of the data in Figure 2 in part (a), stating reasons such as ‘not bell-shaped’
* used concise reasoning when explaining their choice of histogram in part (b). Answers that gained the mark for the justification of their choice most often involved comments regarding the fact that Figure 4 was more bell shaped as a result of the Central Limit Theorem (as a result of the larger sample size to generate each sample sum) or that Figure 4 had a larger spread.

The less successful responses commonly:

* were not concise in their worded response to part (a), using vague or colloquial words such as ‘spikey’ to describe the shape of the distribution; or, stated unrelated facts such as ‘the sample was not large enough’
* restated the question in part (a) when giving their answer, only stating that it was not appropriate to model *V* with a normal distribution, instead of addressing why this is the case
* incorrectly stated that the spread of the histogram in Figure 3 is greater than that in Figure 4 to justify their decision in part (b) (this error results from not considering the scale on the horizontal axis)
* stated that  was normally distributed as  in part (c)(i).

Question 4

This question is centred around how the values of k and *c*, in the graph of the function  determine the nature of the transformation of the graph of  This question should be becoming familiar to students, as it has been asked in many exams since the curriculum change in 2017. Students generally found the algebraic processes in this question approachable; however, they found it challenging to express the relationships between graphs in a concise manner leading to only 18% of students achieving full marks.

The more successful responses commonly:

* showed all clear working in part (a) as a result of being directed to ‘show’ the equality between the two functions
* were able to successfully factorise the quadratic contained within the argument of the natural logarithm in part (c)(i)
* used concise wording to describe the transformations in part (b) and part (c)(ii). Students who used words such as translation and dilation in conjunction with a corresponding direction (horizontal or vertical) more regularly achieved full marks for these questions.

The less successful responses commonly:

* used inadequate/incorrect language in part (b) and part (c)(ii) to describe the transformation. Examples of student responses that did not gain marks are:
*  is parallel to 
*  is stretched (without stating a direction).
* misused the laws of logarithms to express as in part (c)(i).

Question 5

This contextual application of discrete probability distributions required students to use probabilities from a table and the Binomial Distribution to calculate values. Students generally demonstrated their knowledge well in the more routine parts of the question (parts (a) to (d)), however, evidence suggest they found part (e) challenging. Almost 60% of the students achieved 5 marks or greater in this question. Additionally, this question contained the second highest percentage of students achieving full marks with 26% of the cohort achieving this (second only to Question 1 which had 65% of students achieving full marks).

The more successful responses commonly:

* used technology to calculate part (b) and (d) (evident by their correct answer and lack of working)
* used a well-structured ‘trial-and-error’ approach in part (e), clearly showing the ‘borderline’ cases with evidence of correctly calculated corresponding probabilities (i.e. that when , and if , hence the number of selected guests much be at least 38
* demonstrated an excellent understanding of the laws of logarithms and inequalities in finding a solution for values of *n* when using an algebraic approach in part (e).

The less successful responses commonly:

* calculated  rather than  in part (d)(i)
* only considered ‘non-borderline’ cases for in part (e) when attempting to construct a ‘trial-and-error’ approach
* did not round their value for *n* appropriately in part (e) when implementing an algebraic process.

Question 6

This is the first question of the paper which involved the application of Calculus within a context. Several sections of this question were written to allow students to make use of their graphics calculator to find answers quickly. Additionally, students needed to interpret calculated values in context and solve complex equations. I encourage students to always use a pencil when drawing graphs to allow for easier correction of mistakes. Approximately 70% of students achieved half marks or more in this question.

The more successful responses commonly:

* used technology to carefully draw the graph of the function required in part (a). Students who showed the graph passing through the origin, placed the local maximum in approximately the correct position and demonstrated the asymptotic behaviour or the curve were able to achieve all marks.
* used technology to calculate part (b) and part (c)(i)
* provided a comprehensive interpretation of the value of  in part (c)(ii). Students who achieved well in this question addressed the following in their answer:
*  represents ‘2 hours after consuming a 150 mg dose of caffeine’
*  relates to the ‘rate of concentration of caffeine in the blood plasma’, or that the ‘concentration of caffeine in the blood plasma is increasing’ as a result of its positive value
* the units of the rate of concentration (mgL–1 per hour).
* successfully removed a common factor (either  or ) in part (d)(ii) or manipulated the equation (when ) to be written as  or similar
* used a correct and thorough algebraic process in part (d)(ii) (laws of logarithms, null factor law) to find the given answer
* used an inequality in their solution to part (e) which showed an understanding that if the dose was greater than 600, the maximum concentration would be greater than 15 mgL–1 resulting in serious side effects.

The less successful responses commonly:

* contained an imprecise graph in part (a). Several students’ graphs had the local maximum in the incorrect position or sketched a graph with a heavily truncated domain (i.e. there was no curve drawn past values such as )
* attempted to estimate the value for  from their sketched graph in part (b)
* attempted to find a value for  in part (c)(i) using an algebraic approach, despite hints suggesting the use of technology was the optimal way to find the answer (i.e. there was minimal space provided and only one mark allocated to the answer)
* did not present a clear set of algebraic steps leading to the given answer in part (d)(ii).

Question 7

This question involved the concept of the average rate of change and the derivative of an unfamiliar type of function. Students were also asked to find the exact equation of a tangent. This question rewarded students who had good understanding of how to differentiate exponential functions. Additionally, as part (c) requested the exact equation of the tangent, students needed to implement careful algebraic processes to arrive at a fully correct answer. Again, as with Question 6, approximately 70% of students achieved half marks or more.

The more successful responses commonly:

* substituted their chosen points from part (a)(i) correctly into the slope formula in part (a)(ii), representing their answer either as a decimal or in rational form
* used clear and concise reasoning to show the given relationship in part (b)(i)
* used one of many possible valid methods to find the exact equation of the tangent in part (c). It is an expectation that when students are finding the equation of a line their final answer should be expressed in either slope-intercept form or general form.

The less successful responses commonly:

* found  instead of finding the average rate of change in part (a)(ii)
* forgot the negative sign in their calculation in part (a)(ii) or their slope in part (c)
* did not take note of the italicised word ‘exact’ used in the wording of part (c). These students frequently used technology to find their equation of the tangent, or alternatively listed the slope and *y*-intercept to three significant figures.

Question 8

This question contained no context, however, it combined both the probability density function of a continuous random variable and a conjecture. Early parts of the question involved routine concepts and allowed for the use of technology; however, as the question progressed, its complexity sharply increased, culminating with students using the given conjecture to find a requested value. 11% of students achieved full marks, with a mean percentage of marks gained being approximately 60%. Oddly, this question also contained the highest percentage of students achieving no marks in the paper (11%). [closely followed by Question 9]

The more successful responses commonly:

* used technology in parts (a) and (b) to state the answers
* showed clear substitution of  and  into their correctly integrated expression in part (c)
* proved the conjecture, rather than assuming it was true in their working in part (d)
* took note of the domain for *m* and *n* in conjunction with the given conjecture when finding the maximum value of *m* in part (e).

The less successful responses commonly:

* did not use the correct formula for standard deviation of a continuous random variable in part (a) (e.g. often answers given were the variance, or students forgot to subtract the expected value squared away
* if using the formula ).
* considered the random variable to be normally distributed when finding the probability in part (b)
* only substituted in an alternative numerical value for *m* and *n* in part (d) (e.g. some students only found  to *show* that the probability is  which does not *prove* it for all values of *n* and *m* for the stated domain)
* incorrectly linked the word ‘maximum’ used in the question in part (e) with the concept of a local maximum resulting in the incorrect use of calculus.

Question 9

This question focused on the inadequacies of the method used within our course to calculate confidence intervals for a population proportion when a sample proportion is close to zero. Students were required to demonstrate excellent algebraic skills to manipulate a complex equation followed by a correct substitution within this equation to find a requested value. Evidence suggests that students found this question challenging. It had the lowest percentage of correct marks gained across the entire paper (43%).

The more successful responses commonly:

* used technology to calculate the confidence interval in part (a)
* showed clear and concise algebraic steps in their working in part (b). Some common steps in a successful solution were:
* after letting the given equation equal to zero, expressing it as 
* removal of the radical symbol through the difference of two squares or by squaring both sides (when in the form given above)
* taking out a common factor of 
* dividing through by  (as ) to simplify the equation.
* successfully rounded the calculated value of *x* up in part (c) as a result of its discrete nature. The conditions of the question would not have been satisfied if *x* was rounded down.

The less successful responses commonly:

* calculated the confidence interval algebraically leading to incorrect upper and lower limits in part (a) due to errors
* used incorrect notation in their confidence interval in part (a) such as  rather than the required *p*
* did not show appropriate steps of logic in part (b)
* used a trial-and-error approach in part (c), despite the question stating that the given equation for must be used
* only found the value for and not *x* in part (c)

Question 10

This question assessed students’ knowledge of underestimates and overestimates for an area under a function. Students also needed to show understanding of how the shape of the graph can be used to discern which of the two estimates (underestimate or overestimate) would be closer to the exact area under a function for a given domain. Additionally, students needed to have a good understanding of the structure of the product rule to differentiate an unfamiliar function and show clear and concise mathematical reasoning. Over 60% of students were able to achieve half of the allocated marks or more.

The more successful responses commonly:

* successfully chose the correct width and height of the rectangles to calculate an underestimate of the area in part (a)
* used the product correctly in conjunction with the given statement to successfully find  in part (c)(i)
* used clear and concise reasoning to demonstrate that  for  to show that the graph of  is always convex in part (c)(ii). Students who achieved well in this question commonly stated that:
* 
* 
* 
* used a diagram to demonstrate that the error of an underestimate was less than an overestimate in part (c)(iii)

The less successful responses commonly:

* did not take note of the requested rounding in part (a)
* calculated an overestimate of the area instead of the underestimate in part (a)
* read , resulting in an incorrect derivative in part (c)(i)
* incorrectly stated that  when  in part (c)(ii)
* used calculated values for an underestimate, overestimate, and exact area to justify their answer in part (c)(iii), ignoring the ‘hence’ stated in the wording of the question.

Question 11

This question covered a diverse range of topics within the Subject Outline. The initial parts of the question involving inferential statistics. These questions, although set in a complex context, were considered routine. The question from part (c) onward focused on integral calculus and trigonometric functions. In order to solve this section of the question successfully, students needed comprehensive understanding of these topics in conjunction with excellent algebraic skills. As several aspects of this question were routine, students were able to be reasonably successful in this question overall (the mean percentage of marks gained was approximately 50%), however, only 7% of students were able to gain 14 or more marks out of 15.

In a small number of papers, the final page containing part (d) was left blank suggesting that perhaps some students found it challenging to complete the paper in the allocated time.

The more successful responses commonly:

* used technology to calculate the confidence interval in part (a)
* correctly stated ‘no’ in part (a)(ii) followed by a clear reason which avoided the use of double negatives
* calculated a 10% increase by multiplying 30 by 1.1 in part (b)
* showed clear logical reasoning to explain why is the *only* solution in part (d)(i). This reasoning often involved graphical evidence involving the period of a trigonometric function
* correctly found the exact value for *a* in part (d)(ii). Students who achieved well in this question commonly
* equated the integral of  rather than 1
* correctly integrated , with correct substitution of the definite integral limits of  and 
* took out a common factor of *a* after substituting in the limits  and .

The less successful responses commonly:

* calculated the confidence interval algebraically in part (a)(ii) leading to incorrect upper and lower limits as a result of errors
* used incorrect notation in their confidence interval in part (a)(ii) such as  rather than the required 
* did not read the question carefully in part (b) and missed the ‘10% longer’ statement, hence, only comparing the given confidence interval to a value of 30
* were not concise enough in their reasoning in part (b). For example, merely stating that 33 was outside of the confidence interval does not imply that the perceived completion time has increased, as if 33 happened to be above the confidence interval, the evidence would support a claim that the perceived completion time has decreased.
* did not consider that several solutions of the form , also pause at  in part (d)(i). Hence, they did not show that  is the only value of *b* to satisfy the conditions. (if , it would also pause at other values).
* treated *a* in part (d)(ii) as a variable in the integral of  rather than a constant
* were unable to isolate the *a* value in part (d)(ii) after evaluating the definite integral of 
* stated a value of as a decimal expansion in part (d)(ii) rather than the exact value requested in the question.