**Stage 2 Mathematical Methods**

**Assessment Type 2: Mathematical Investigation**

**Topic 1: Further Differentiation and Applications**

**Surge and Logistic Models**

**Part 1: The Surge Function**

A surge function is in the form $f\left(x\right)=Axe^{-bx}$ where *A* and *b* are positive constants.

* On the same axes, graph $y=f(x) $and $y=f’(x)$ for the case where $A=10$ and $b=4$

 $ i.e.$ $f\left(x\right)=10xe^{-4x}$

* Determine the coordinates of the stationary point and point of inflection and label these on the graph.
* Repeat the investigation for three different values of $A$ while maintaining $b=4$.
* Include your graphs in the report and summarise the findings in a suitable table.
* State the effect of changing the value of$ A$ on the graph of $y=Axe^{-bx}$.
* Using a similar process investigate the effect of changing the value of $b$ on the graph of $y=Axe^{-bx}$.
* Make a conjecture on how the value of b effects the x-coordinates of the stationary point and the point of inflection of the graph of $y=Axe^{-bx}$.
* Prove your conjecture.
* Comment on the suitability of the surge function in modelling medicinal doses by relating the features of the graph to the effect that a medicinal dose has on the body.

 Discuss any limitations of the model.

 *At least four key points should be made.*

**Part 2: The Logistic Function**

A logistic function is in the form $P\left(t\right)=\frac{L}{1+Ae^{-bt}}$ where $L, A $and $b $are constants and the independent variable t is usually time; $t\geq 0$.

This model is useful in limited growth problems, that is, when the growth cannot go beyond a particular value for some reason.

* Investigate the effect that the values of $L, A $and $b$ have on the graph of the logistics function.
* Discuss your findings on the logistic model.
* Relate the specific features of the logistic graph to a limited growth model.

 *At least three key points should be made.*

**Part 3: Modelling using Surge and Logistic Functions**

Using either a surge or a logistic function (or both) develop a model to investigate one of the following scenarios.

* Movements of students into the school building at the end of lunch.
* A crowd leaving a sports venue.
* The limited growth of a population.
* pH levels in a titration.
* Repeat doses of a medicine.
* The spread of information in a group of people.
* Traffic density during peak hour.
* The acceleration of a car.
* A suitable alternative of your choosing.

Select a suitable function that would model your chosen scenario with the dependent and independent variables clearly defined.

* State the values of any constants for this model with evidence to support your choices.
* Draw a sketch of the graph of the function showing as much detail as known.
* Discuss the significance of the key features of the graph including the reasonableness of the model and of your conclusions.
* Justify all your decisions and discuss any limitations of your model.

**For this section write your response as a report which includes an introduction, a main body and a conclusion.**

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| Your investigation will be assessed using the following assessment design criteriaConcepts and TechniquesCT1 Knowledge and understanding of concepts and relationships.CT2 Selection and application of mathematical techniques and algorithms to find solutions to problems in a variety of contexts.CT3 Application of mathematical models.CT4 Use of electronic technology to find solutions to mathematical problems.Reasoning and CommunicationRC1 Interpretation of mathematical results.RC2 Drawing conclusions from mathematical results, with an understanding of their reasonableness and limitations.RC3 Use of appropriate mathematical notation, representations, and terminology.RC4 Communication of mathematical ideas and reasoning to develop logical arguments.RC5 Development, testing, and proof of valid conjectures. |

Performance Standards for Stage 2 Mathematical Methods

| - | Concepts and Techniques | Reasoning and Communication |
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| A | Comprehensive knowledge and understanding of concepts and relationships.Highly effective selection and application of mathematical techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.Successful development and application of mathematical models to find concise and accurate solutions.Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | Comprehensive interpretation of mathematical results in the context of the problem.Drawing logical conclusions from mathematical results, with a comprehensive understanding of their reasonableness and limitations.Proficient and accurate use of appropriate mathematical notation, representations, and terminology.Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.Effective development and testing of valid conjectures, with proof. |
| B | Some depth of knowledge and understanding of concepts and relationships.Mostly effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine and some complex problems in a variety of contexts.Some development and successful application of mathematical models to find mostly accurate solutions.Mostly appropriate and effective use of electronic technology to find mostly accurate solutions to routine and some complex problems. | Mostly appropriate interpretation of mathematical results in the context of the problem.Drawing mostly logical conclusions from mathematical results, with some depth of understanding of their reasonableness and limitations.Mostly accurate use of appropriate mathematical notation, representations, and terminology.Mostly effective communication of mathematical ideas and reasoning to develop mostly logical arguments.Mostly effective development and testing of valid conjectures, with substantial attempt at proof. |
| C | Generally competent knowledge and understanding of concepts and relationships.Generally effective selection and application of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts.Successful application of mathematical models to find generally accurate solutions.Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | Generally appropriate interpretation of mathematical results in the context of the problem.Drawing some logical conclusions from mathematical results, with some understanding of their reasonableness and limitations.Generally appropriate use of mathematical notation, representations, and terminology, with reasonable accuracy.Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.Development and testing of generally valid conjectures, with some attempt at proof. |
| D | Basic knowledge and some understanding of concepts and relationships.Some selection and application of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts.Some application of mathematical models to find some accurate or partially accurate solutions.Some appropriate use of electronic technology to find some accurate solutions to routine problems. | Some interpretation of mathematical results.Drawing some conclusions from mathematical results, with some awareness of their reasonableness or limitations.Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.Some communication of mathematical ideas, with attempted reasoning and/or arguments.Attempted development or testing of a reasonable conjecture. |
| E | Limited knowledge or understanding of concepts and relationships.Attempted selection and limited application of mathematical techniques or algorithms, with limited accuracy in solving routine problems.Attempted application of mathematical models, with limited accuracy.Attempted use of electronic technology, with limited accuracy in solving routine problems. | Limited interpretation of mathematical results.Limited understanding of the meaning of mathematical results, their reasonableness or limitations.Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.Attempted communication of mathematical ideas, with limited reasoning.Limited attempt to develop or test a conjecture. |