2022 Specialist Mathematics Subject Assessment Advice

Overview

Subject assessment advice, based on the 2022 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

School Assessment

Teachers can improve the moderation process and the online process by:

Overall comments

* thoroughly checking that all grades entered in schools online are correct. Errors in entered grades cannot usually be fixed through the moderation process, particularly if the error means a change in the rank order of results
* ensuring the uploaded tasks are legible, all facing up (and all the same way), and remove blank pages, student notes and formula pages
* ensuring the uploaded tasks also have pages the same size and in colour so teacher marking, and comments are clearly distinguishable from student work
* using the same tasks where possible when combining with another school or schools to ensure standards are equitable. When combining classes across schools, teachers should be involved in moderation activities prior to up-loading materials for the actual moderation to ensure that the rank order of the students within the combined assessment group is appropriate.

SAT comments

* ensuring the uploaded student SATs have been clearly marked showing which mathematical calculations are fully or partially correct and which are incorrect is a requirement of moderation. Showing marks and totals is also helpful
* preferably providing a summary of student results in each of the SATs at the start of the uploaded SAT’s file
* uploading the SATs as a single scanned file rather than six separate files.

Investigation comments

* for investigations, comments and clearly marked mathematical calculations are a requirement for the moderation process
* ensuring uploaded investigations are the final work and not the draft. However, a draft can be assessed and uploaded if a student does not submit a final response.

Assessment Type 1: Skills and Applications Tasks (50%)

Students complete six skills and applications tasks under the direct supervision of the teacher. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes. This year there may have been one task removed due to Covid but this should have been the same task for all students in the assessment group.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring the SAT is at a good level that allows for both routine questions, and questions that include enough complexity to allow achievement at the higher-grade bands
* designing some complex questions to allow students to progress one step at a time through a process, using the ‘Show that …’ style of question
* structuring questions with multiple parts that begin with ‘access’ points to elicit C grade evidence and subsequently increase in complexity, with the potential to elicit A grade evidence
* providing a marks scheme and working space reflective of the cognitive demand of the question
* providing students with appropriate feedback, including marks, to help them improve their work
* assessing conjecture and proof through this assessment type as they can be difficult to assess within a mathematical investigation:
* please ensure that your LAP accurately indicates where RC5 is being assessed
* induction per se is not RC5, completing proofs using Mathematical Induction does not on its own achieve RC5. Students must be given the opportunity to form their OWN conjecture and to then prove it. Teachers may choose to assess this in the Induction SAT, for example by including a question of the form:

(a) Given the matrix  find (i)  (ii)  (iii) .

(b) On the basis of your answers to (a) make a conjecture about the matrix .

(c) Prove your conjecture using the principle of mathematical induction for all positive integers *n*.

* referring closely to the key questions and key concepts in the subject outline when designing assessment tasks
* while including material that is outside the subject outline can be considered an extension (e.g. Inequalities in Induction, Summation and Product notation in Induction, Euler’s form or exponential form of a complex number and more complex integration by substitution), it should not be included to the detriment of including content required to be known, such as polar form for example, in the summative SAT
* setting a variety of SATs that include limited questions drawn directly from past examinations. Schools can use a mid-year examination to provide students with a formative experience of examination style assessment; however, it is recommended that examinations are not included among the Assessment Type 1 SATs
* not marking crossed out work as work the student has crossed out will not be marked in the final examination
* preferably not awarding half marks as these are not awarded in the final examination and can inflate results and student expectations
* making students fully aware of the capabilities of their graphics calculator so they can make informed choices as to when and where to use it in completing SATs, particularly in graph work
* providing clear feedback on the appropriate use of mathematical notation, with particular attention needed for questions using vectors and integration which were identified as more common problem areas during moderation and examination marking.

The more successful responses commonly:

* provided clear and logical reasoning with correct mathematical notation
* displayed evidence that aligned with the question requirements (e.g. hence, exact)
* provided solutions that were efficient and demonstrated clear, logical and comprehensive understanding and interpretation of the question/problem
* used both algebraic and geometric approaches to solve problems in the topics Complex numbers and 3D Vectors
* showed all algebraic working by providing all relevant steps, particularly for the ‘Show that …’ style of question
* stated any theorems and/or properties that were being applied to support answers
* used mathematically correct notation, particularly in questions using vectors and integration
* labelled axes and scales of graphs correctly and indicated in Argand diagrams when vectors drawn were equal in length or perpendicular
* used the graphics calculator efficiently to draw both cartesian and parametric functions by plotting sufficient points, paying attention to correctly labelling and representing asymptotes and correctly showing shape and behaviour of curves near asymptotes
* paid close attention to all details given in questions and the detail required in answering by showing conceptual thinking in their responses no matter how simple
* included appropriate steps in applying algorithms and did not miss vital steps, especially in ‘Show that…” questions where the answer is given in the question

The less successful responses commonly:

* often did not attempt to answer questions, particularly more complex style questions
* displayed incorrect mathematical notation and/or limited communication of reasoning (i.e. the solution did not successfully ‘flow’ to a logical end)
* included many arithmetic and algebraic mistakes that complicated the nature of the solutions (e.g. an error causing the student to have polynomials that did not factorise easily)
* do not follow instructions that directed the student to use a particular method such as “implicit differentiation” or to use a previous result, either by instructing students to use specified parts of the question or using the word hence
* did not read questions carefully and clearly spent too much time on some, leaving no time to complete other questions
* lacked the appropriate detail; where several marks have been allocated, all relevant conceptual steps are required
* did not communicate a good knowledge of the algorithms covered by the course, often evident through incorrect application of techniques to solve questions
* seemed unfamiliar with the capability of their graphics calculator.

Assessment Type 2: Mathematical Investigation (20%)

Students complete one mathematical investigation.

The subject of the investigation may be derived from one or more subtopics and should have minimal teacher direction. The task must afford students the opportunity to extend the investigation in an open-ended context. Students are encouraged to use a variety of mathematical (and other) software to enhance their investigation. It must be completed in a report format and must be no longer than 15 single-sided A4 pages with minimum font size 10. Appendices may be used to support the report but are not part of the assessment decision unless they are part of the 15 pages. Teachers should provide feedback where appropriate on the suitability of the direction a student may take with their investigation, especially where the investigation topic was chosen by the student and provide feedback on one draft. In draft feedback the teacher may direct the student’s attention to errors but must not explicitly correct these for the student.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring that the format of the investigation allows for an open-ended exploration of a problem where the student can show the development and application of mathematical models through individual choices, refinements/improvements with justification for their rationale
* providing examples in the task sheet of what could be modelled, and structuring the investigation to encourage students to focus on different models, extend their interests and explore more complex models
* ensuring that the investigation is at an appropriate level of complexity, aligns well with the subject outline and does not limit student’s ability to achieve at the highest level
* avoid using question-and-answer style investigations, which limit student success ensuring the design of the task allows for and explicitly encourages the discussion of limitations and reasonableness of the modelling process:
* tasks that are designed to look at the generation of curves or shapes by altering values within formulae is not likely to result in individual work that is sufficiently open ended or allowing deep discussions concerning the reasonableness of solutions or limitations encountered
* examples of types of investigations that may limit student success are Bezier curves with a conjecture and proof that is not unique to the student, graphs of various rational functions, or graphs of other relations that may give altering shapes depending on values chosen. Some Bezier curve investigations may not reach a complex level of modelling and may limit the discussion of reasonableness and limitations
* the most recently updated wine glass investigation on the SACE website allows an open-ended approach after initially being directed. Students need to execute a significantly open-ended section to produce an investigation at a complex level. For example, the modelling of pathways with parametric curves provides no direction and allows students to develop their own modelling and as such is an excellent exemplar.
* encouraging the correct use of notation and labelling of graphs, axes, scales etc.
* assisting students with unfamiliar software so that they can represent graphs etc. with appropriate information attached
* providing feedback through drafting and/or discussing the direction taken to ensure that what students plan to do will provide them with the opportunity to achieve at the higher-grade bands and the teacher may direct the student’s attention to errors but must not explicitly correct these for the student
* explaining clearly the 15-page (single-sided) limit and the corresponding appropriate use of appendices. For example, important or initial mathematical calculations should be provided in the main body of the report; however, repetitions of a calculation with variations to the figures can be provided in the appendices with the results clearly provided in the main body of the investigation (within a table or using some other concise manner of presentation)
* not using investigations that have published solutions such as those provided by MASA to ensure that student work is unique and authentic. Examples of several such investigations not to use include the Tennis application, and De Moivre’s Theorem application, both which present as question/answer style

The more successful responses commonly:

* provided detailed information in their introduction about the investigation and the context in the real world
* were student driven, and included both mathematical calculations and the use of technology with a focus on interpretation and evaluation of models that had been developed and applied in the context chosen
* read as a complete report, with sentences of explanation, not a series of dot-point-like ‘answers’ to an ‘assignment’
* included detailed explanations of all algebra, choices of values, and graphical work produced
* included graphical representations appropriately labelled to enhance the discussion within the investigation
* successfully developed a modelling situation with clear explanation of the decisions made throughout the mathematical investigation justified with reference to the real-life context and/or cited research and references as appropriate. This included mathematical calculations for each stage of development of the model that were commensurate with the cognitive demands of Stage 2 Specialist Mathematics
* demonstrated understanding of the reasonableness of the mathematical results and the limitations of the modelling process used, with attempts to improve, expand on, and develop based on these reflections, as appropriate
* used appropriate mathematical software to enhance the quality of the investigation
* used mathematical notation, representations, and terminology appropriately
* effectively communicated mathematical ideas and reasoning to develop logical arguments
* formatted their document so the mathematical notation flowed properly, and headings didn’t appear at the bottom of one page and the content at the top of the next page
* used appendices appropriately for repeated algebraic calculations to arrive at results.

The less successful responses commonly:

* had a limited introduction to the investigation, giving the reader little insight into the nature of the problem and the investigation to be undertaken
* had limited supporting evidence of how the models were derived (e.g. trial and error, Geogebra, researched and adapted)
* provided little evidence of effective use of technology. The investigation is an ideal assessment to implement a range of technologies to represent and solve problems leading to the development of the model
* read like a series of dot-point-answers as if the student just listed responses to an assignment or worksheet
* did not provide explanations or reasoning for the decisions made throughout the investigation
* made poor use of notation and often did not fully identify graphs
* included little or no labelling of diagrams
* followed the early direction given, but did not achieve much more, often failing to attempt the open-ended part of the investigation or sometimes spending too much time on the directed part and too little on the open-ended part
* appeared to not have submitted their draft to the teacher for feedback.

Operational Advice

If students present their responses in oral or multimodal form, 6 minutes is the equivalent of 1000 words. Students should not speed-up the recording of their videos excessively in an attempt to condense more content into the maximum time limit.

From 2023, if a video is flagged by moderators as impacted by speed, schools will be requested to provide a transcript and moderators will be advised to moderate based on the evidence in the transcript, only considering evidence up to the maximum word limit.

If the speed of the recording makes the speech incomprehensible, it affects the accuracy of transcriptions and it also impacts the ability of moderators to find evidence of student achievement against the performance standards.

External Assessment

Assessment Type 3: Examination

General

The examination consists of two booklets. Book one is worth 55 and book two has longer questions with a total of 45 marks. As in past years the cohort who undertook the examination was made up of those students who knew their work and produced good to very good results, but there were a proportion of students who struggled to respond successfully.

Students found Book one with the shorter questions, worth 55 marks, more accessible than the longer questions in Book 2. Students found the 10 minutes of reading time useful for time to work on the problems.

General comments worth stressing:

* The ‘Show that …’ style of question requires students to show full working, displaying all steps of logic, for maximum marks. The style of solution here should be approaching one side of the given information and working towards developing the other side. The two sides should not be used together.
* An ‘exact answer’ means the answer should be in rational or irrational form without approximations to decimal values.
* Students need to be reminded that if the answer is stated in the question, marks are awarded for providing the working steps needed to reach this answer.
* Knowledge of, and the use of, a graphics calculator is assumed.
* Poor notation was often seen in student responses. Two areas of concern are the poor use of vector notation and integration notation.
* Students should also be mindful of using the variables in the question. For instance, if a function of *t* is stated then student responses should be in terms of not for example).
* Students should recognise that earlier parts of a question are often relevant to the later parts of a longer question. Some questions for instance may state ‘hence’ or ‘using part (a)(i)’ instructing students to follow on from previous work.
* Students should be aware of algebraic language. Some students did not use the brackets required to show a logical flow of their algebraic reasoning. This leads to errors in their mathematics.
* Students must set out mathematical induction proof appropriately to gain full marks. In the detail below there are important comments made concerning this setting out.

Examination markers aim to award marks for evidence of student understanding in responding to examination questions wherever possible, however, students should be advised not to cross out their responses or attempted responses to questions in the examination booklet, unless they are confident that no part of their response should be considered by the marker.

If a student crosses out a response and then decides that it was the correct (or the most correct) answer, then the student should indicate clearly to the marker which part of their response should be considered. This could be done by circling or highlighting all or part of the response that the student wants to be considered and write, for example, “please mark this work”. Students do not need to rewrite their answers in this case unless the crossing out has rendered the response unreadable.

It is advisable that students indicate in the space for an answer if they are also using the extra page for more working. For example, “see page x”. The work on the extra pages must be labelled clearly. Students should ensure they answer a question on extra pages in the correct booklet.

Specific comments for the questions within booklets 1 and 2 follow:

Booklet One

Question 1

Many students started off in a positive manner with this question.

(a) Some students approached the integration by parts very well. Lack of appropriate notation was apparent at times – the lack of the constant *c* and poor integral notation with the ‘d*x*’ omitted.

(b) This is a ‘show that’ style question so it is expected that students set up the problem with logically fluent steps. Many students did this problem well.

Question 2

Many students gained success in the initial parts of this question, but careful consideration of the wording in the final part of the question was missed by some.

(a) (i) (1) and (2) were well done by most who used their calculators to assist with the conversion of Cartesian to polar form of a complex number.

(ii) was well done by most – recognising the properties of .

(b) (i) Recognising how to use De Moivre’s theorem was well done by many.

(ii) This part of the question was difficult for some who did not adhere to the requirement that *n* is positive.

Question 3

Some students achieved very well in this polynomial question and some reasonably well. Those who scored well clearly set their work out with good reasoning.

(a) Many knew that when  is a factor then  but some students tried to undertake polynomial division which led nowhere. It is very important in the next step of substituting  into the polynomial that correct notation is used. That is, brackets around the  is crucial for the logical reasoning of the work that follows.  is the correct substitution which then leads to a sensible discussion of even or odd values of resulting in two possible values for *c*.

(b) Substitution of  into  to find the only possible value of positive integer *n* is 5 when  was generally well done, but it was important for students to investigate the case when  and show that the resulting *n* is not an integer.

(c) Stating the resulting polynomial with values of *n* and *c* was generally well done.

Question 4

Most students who were successful in this problem had good knowledge of calculator capabilities for graphing parametric curves and for evaluating a definite integral.

(a) Students should be mindful of the given interval for the parameter *t* and to set their calculator accordingly, as well as using the scale on the axes supplied to assist the drawing of the curve. To score well in this question students should also be conscious of finding points along the curve to assist drawing the curve more precisely.

(b) (i) Many students successfully found , but when the answer is given students need to show steps of logic.

(ii) Another ‘show that’ style question that requires clear steps leading to the final statement. The most successful responses stated the trigonometric identities used in the reasoning.

(iii) When accuracy is stated in a question students must be aware to follow that direction. Some students found it difficult to use the calculator to find this definite integral.

Question 5

In this problem students who set out the mathematical induction proof appropriately were the most successful. The following points are very important for part (a):

* the proposition must be defined to initiate the proof. Students need to define  initially so that working during the proof with  stated is valid
* some students tried to convert the proposition statement to an inappropriate form using summation notation. This either led to errors in notation or mistakes in the proof. It is not necessary to use this notation
* considering the , and using the previously stated assumed  in the reasoning towards the right-hand side version of , is the process that should be followed. Students should not follow through both sides of 
* clear algebraic steps must be shown and using the inductive step of the proof should be clearly seen.

(b) Students who realised that  in this ‘show that’ style question made some progress. To follow further into this problem students needed to recognise that  due to  for all positive *n*.

Question 6

This question was one of the areas where mathematical notation was very poor. As mentioned previously the lack of correct notation for vectors can lead to poor understanding of vector algebra.

(a) Many students were able to find , but some did not find  as required for (ii).

(b) Many students were not successful in showing the sides PQ and CB are parallel. This also led to errors in the second part of the problem.

(c) Although most students struggled with earlier parts of the question, this part was very well done overall.

Question 7

Many students approached this question with success.

(a) Drawing the solution curve on the given slope field was generally good, with many students paying attention to the direction of the slope fields. Those who were less successful missed the asymptotic trend and/or crossed slope field lines.

(b) Some students managed the separation of variables well and had good knowledge of integration techniques. Those who were not as confident in this area struggled.

(c) In part (i) many misinterpreted the information and substituted  rather than considering the case . The next two parts were generally well done.

Overall, the first booklet saw many students gaining good scores in total for the seven questions.

Booklet 2

Question 8

This vector question was well done overall.

(a) (i) to (iii) were quite well done. Students who did not perform well failed to show row operations in part (i) and some did not give evidence to show that .

(b) In part (i) it is recommended that students use a different parameter than the *t* used in . In part (ii) many students did not supply enough evidence to show that the point C is on both lines. Some students did not find the midpoint of AB correctly. In part (iii) those students who remembered to enter the components of the distance formula correctly were successful. That is, .

(c) Some good work here. Students who found values for CQ and CT were the most successful.

Question 9

This question considered the graphing and area between curves components of the course.

(a) Was generally well done, but a reminder that students must show clear reasoning for a ‘show that’ style question. Most students approached this question by working algebraically with the given right-hand side and developed the single algebraic fraction.

(b) Students are required to read the question carefully and supply the asymptotes and axes intercepts. It is also necessary to display the behaviour of the graph near the asymptotes. Some students were not careful with their drawing of the graph and bent away from asymptotes or were not aware of points on the curve. It is recommended that the calculator be used to set the axes scales as given in the question and to use the functionality of the calculator to find some points to assist the drawing of the curves. Connecting the work in part (a) should have alerted students that there is an oblique asymptote.

(c) Using the calculator to sketch  was important here. Different colours may be used to clearly indicate the new graph when it is drawn on the same set of axes as the graph drawn in part (b). Care must be taken with labelling both branches so that it is clear all parts of the new curve are able to be seen. In part (ii) many students missed the strict inequality requirement and allowed equality.

(d) Students who noticed that part (c) led them to realise that there is a gap between the graphs between  and  managed this question well. Some did not use correct algebra and used  instead of . Those who approached the problem using the form found in part (a) were more successful if brackets were used appropriately. That is, . As mentioned previously students should be mindful of correct notation for integration in their working. Part (ii) was generally well done when students noticed they could use the given information in the previous part. Students must use modulus notation when integrating the form . That is,  .

Question 10

Many students found this question challenging.

(a) In (i) some students dealt with placing their complex number *z* on the Argand diagram at any position well but labelling a point at  sometimes was drawn 2 units to the left rather than to the right. Part (ii) was testing knowledge of the triangle inequality. Some students answered this well but a diagram with labelled lengths on the sides would be useful and enhances the explanation. Similarly with part (iii), students needed to explain that *z* and  are parallel or that the three vertices of the triangle 0, *z* and  are collinear for equality to hold.

(b) This question had varied responses from students. For (i) (1) the line  was required for the set of complex numbers that satisfy , whereas some students tried to draw two circles. For (2) a point P needed to be placed on the  line where the Imaginary scale is positive and hence in (3) Q should be two units to the right of P.

In part (ii) showing that  was able to be approached using different methods but the most successful approach was to use geometry in an isosceles triangle (with ).

(c) Part (i) was very well done with most students using a calculator with the result from part (b)(ii). Showing that  in part (ii) proved difficult for many students. The most successful attempts used the Cartesian form found in part (i). Similarly, many found part (iii) difficult and relied on using the calculator without explanation leading to the given result. Some students did supply excellent work.