2020 Specialist Mathematics Subject Assessment Advice

Overview

Subject assessment advice, based on the 2020 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

In 2020 the assessment specifications had temporary changes introduced to reduce assessment demands due to COVID-19. The flexibilities in the School Assessment component were approved by the COVID-19 response governance. These flexibilities required a total of seven or eight assessments to be undertaken across the year, including the external examination. The school assessment component included:

* five or six skills and applications tasks
* one mathematical investigation task (with a maximum page limit of 15 A4 pages).

Teachers should refer to the subject outline for specifications on content and learning and assessment requirements, and to the subject operational information for operational matters and key dates.

Operational Advice

To support the quality assurance processes during online moderation teachers are requested to:

* thoroughly checking that all grades entered in schools on-line are correct
* ensuring the uploaded tasks are legible and all facing the same way and preferably removing blank pages, student notes and formula pages
* ensuring the uploaded student SATs have been clearly marked showing what is correct and what is incorrect. Providing marks and totals is appropriate and more helpful than just a grade
* comments, ticks and crosses on mathematical working is appropriate and helpful for moderators for investigations
* preferably uploading all the SATs as a single file
* preferably providing a summary of student results in each of the SATs at the start of the uploaded SAT’s file
* ensuring uploaded investigations are the final work and not the draft.

School Assessment

Assessment Type 1: Skills and Applications Tasks (50%)

Students complete six skills and applications tasks under the direct supervision of the teacher. The equivalent of one skills and applications task must be undertaken without the use of either a calculator or notes. This year it has been possible to remove one SAT, being the same for all students in the class where COVID-19 adversely affected the students’ learning.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring the standard of each SAT is at an appropriate standard, allowing for both routine questions, and questions that include enough complexity to allow achievement at the higher-grade bands
* designing some complex questions that allow students to progress one step at a time through a process, using the ‘Show that …’ style of question to ensure that students do not get locked out of the extended questions
* providing students with appropriate feedback, including marks, to help them to identify their strengths and weaknesses and to support them to improve their work in future assessments
* assessing conjecture and proof through this assessment type, as they can be difficult to assess using a mathematical investigation:
* RC5 — Development, testing, and proof of valid conjectures is often indicated for assessment in Assessment Type 1, however when the materials are viewed during moderation there is no appropriate evidence of RC5 found where the assessment opportunity is indicated on the LAP. Please ensure that the LAP is updated to clearly and correctly indicate the assessment in which all specific features are assessed.
* Topic 1 – Mathematical induction questions do not provide evidence of RC5 through the completion of proofs using the induction process. Students must be given the opportunity to form their OWN conjecture and proceed through to a proof of this conjecture. An example of an appropriate question which allows students to be assessed for RC5 in Topic 1 – Mathematical induction SAT is provided below:

(a) Given the matrix  find

(i)  (ii)  (iii) 

(b) On the basis of your answers to (a) make a conjecture about the matrix 

(c) Prove your conjecture using the principle of mathematical induction for all positive integers n.

* referring closely to the key questions and key concepts in the subject outline when designing assessment tasks. Including questions assessing mathematical ideas and concepts that are outside of the subject outline (e.g. such as inequalities in Mathematical Induction, Euler’s form or exponential form of a complex number and more complex integration by substitution), is not advised in SATs. The focus of each SAT should clearly be informed by the relevant key ideas and key concepts outlined in the subject outline (e.g. such as polar form for example)
* using SATs that are not ‘exam like’ in length or mid-year examinations to provide evidence for Assessment Type 1. Schools can use a mid-year examination to provide students with a formative experience of examination style assessment; however, it is recommended that examinations are not included among the Assessment Type 1 SATs
* preferably not marking crossed out work as work the student has crossed out will not be marked in the final examination. Students should be advised of this practice prior to the initial assessment so that they are not penalised for a process that they were not aware of. They should also be provided with appropriate strategies to indicate their correct answer when they have made multiple attempts to solve part of a problem
* preferably not awarding half marks as these are not awarded in the final examination and can inflate results and student expectations
* making students fully aware of the capabilities of their graphics calculator so they can make informed choices as to when and how to use their calculator effectively and efficiently when completing SATs
* provide clear feedback on the appropriate use of mathematical notation, with particular attention needed for questions using vectors and integration which were identified as more common problem areas during moderation and examination marking.

The more successful responses commonly:

* showed all algebraic working by providing all relevant steps, particularly for the ‘show that …’ style of question
* stated any theorems and/or properties that were being applied in order to support answers
* used mathematically correct notation
* labelled axes and scales of graphs correctly and used their graphics calculator efficiently to draw both cartesian and parametric functions, paying attention to correct labelling and representation of asymptotes and accurate representation of shape and behaviour of curves near asymptotes
* carefully read questions to detect all important details given in questions and provided appropriate detail in the mathematical procedures used to answer questions, showing conceptual thinking in their responses no matter how simple the steps required are
* included evidence of appropriate steps in applying algorithms, not missing vital steps, especially in ‘show that…’ questions where the answer was given.

The less successful responses:

* often did not attempt to answer questions, particularly more complex style questions
* included many arithmetic and algebraic mistakes that complicated the nature of the solutions, causing the student to instead have problems such as polynomials that did not factorise easily
* didn’t follow instructions provided in questions. For example, directions that indicate that the student is required to use a particular method such as ‘implicit differentiation’ or the need for the students to use a previous result by giving instructions that include ‘using parts (a) and (b)’ or using the word ‘hence’
* lacked the appropriate detail; where several marks have been allocated for an answer, all relevant conceptual steps are required
* used incorrect notation and did not communicate a good knowledge of the algorithms covered by the course
* seemed unfamiliar with the capability of their graphics calculator.

Assessment Type 2: Mathematical Investigation

Students complete one mathematical investigation.

The subject of the investigation may be derived from one or more subtopics and should have minimal teacher direction. The task must afford students the opportunity to extend the investigation in an open-ended context. Students are encouraged to use a variety of mathematical (and other) software to enhance their investigation. It must be completed in a report format and must be no longer than 15 single-sided A4 pages with minimum font size 10. Appendices may be used to support the report but are not part of the assessment decision unless they are part of the 15 pages. Teachers should provide feedback where appropriate on the suitability of the direction a student may take with their investigation, especially where the investigation topic was chosen by the student and provide feedback on one draft. In draft feedback the teacher may direct the student’s attention to errors but must not explicitly correct these for the student.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* ensuring that the format of the investigation allows for an open-ended exploration of the problem where the student can show individual choices, refinements/improvements to models, justifying their rationale for these developments
* providing investigations that do not overly scaffold the response by providing a question-and-answer style of task. Task such as these limit student success and should not be used
* ensuring the design of the task allows for the discussion of limitations and reasonableness of the modelling process. More specific examples are provided in the following:
* tasks that are designed to look at the generation of curves or shapes by altering values within formulae are not likely to result in individual work that is sufficiently open ended or allow deep discussions concerning the reasonableness of solutions or limitations encountered
* investigations that may limit student success include investigations of Bezier curves with a conjecture and proof that is not unique to the student, graphs of various rational functions, or graphs of other relations that may give altering shapes depending on values chosen. Some Bezier curve investigations may not reach a complex level of modelling and may limit the discussion of reasonableness and limitations
* the wine glass investigation allows an open-ended approach after initially being directed. Students need to extend their response to the task to ensure that significant evidence is provided in the open‑ended section to produce an investigation with sufficient complexity in the response
* encouraging the correct use of notation and labelling of graphs, axes, scales etc.
* assisting students with unfamiliar software so that they can represent graphs etc. with appropriate information/notation
* providing feedback through drafting and/or discussing the direction taken in the open-ended section of the response to ensure that what students plan to do will provide them with the opportunity to achieve at the higher-grade bands
* explaining clearly the 15-page (single-sided) limit and the corresponding appropriate use of appendices. For example, important or initial mathematical calculations should be provided in the main body of the report; however, repetitions of a calculation with variations to the figures can be provided in the appendices with the results clearly provided in the main body of the investigation (within a table or using some other concise manner of presentation)
* using investigations that do not have published solutions to ensure that student work is unique and authentic
* providing students with appropriate feedback to help them to improve their drafts. In draft feedback the teacher may direct the student’s attention to errors but must not explicitly correct these for the student.

The more successful responses commonly:

* provided detailed information about the investigation and the context in the real world
* read as a complete report, with sentences of explanation, not a series of dot-point-like ‘answers’ to an ‘assignment’
* included detailed explanations of all algebra, choices of values, and graphical work produced
* included graphical representations appropriately labelled to enhance the discussion within the investigation
* successfully developed a modelling situation with clear explanation of the decisions made throughout the mathematical investigation justified with reference to the real-life context and/or cited research and references as appropriate
* demonstrated understanding of the reasonableness of the mathematical results and the limitations of the modelling process used, with attempts to improve, expand on, and develop the model based on these reflections, as appropriate
* used appropriate mathematical software to enhance the quality of the investigation
* used mathematical notation, representations, and terminology appropriately
* effectively communicated mathematical ideas and reasoning to develop logical arguments
* formatted their document so the mathematical procedures flowed properly, and the communication of the mathematical processes was clear and easy to follow (e.g. headings that appear at the bottom of one page and the processes following the heading at the top of the next page)
* used appendices appropriately for repeated algebraic calculations to arrive at results and provided the results in a table in the main body of the report.

The less successful responses commonly:

* had a limited introduction to the investigation, giving the reader little insight into the nature of the problem and the investigation to be undertaken
* read like a series of dot-point-answers as if the student just listed responses to an assignment or worksheet
* did not provide explanations or reasoning for the decisions made throughout the investigation
* made poor use of notation and often did not fully identify graphs
* included little or no labelling of diagrams
* followed the early direction given, but did not achieve much more, often failing to attempt the open-ended part of the investigation
* did not submit their draft to the teacher for feedback.

External assessment

Assessment Type 3: Examination

This year saw the introduction of the two-hour examination presented in two question booklets. The students had 130 minutes to complete the examination, with the 10-minute reading time incorporated into the total examination time. Students were able to choose to use the 10 minutes traditionally allocated for reading of the paper to familiarise themselves with the questions in the question booklets or to start work on questions immediately. It is still recommended that students use this time to read the paper to support their knowledge of the paper and their planning towards responding to the paper within the 130 minutes.

The second question booklet contained the extended questions mirroring the design of the previous three-hour examinations. As in past years, the cohort who undertook the examination produced a range of results, from those who were very successful in their demonstration of their skills and knowledge, to those who were unable to engage with the questions and therefore showed limited skills or knowledge had been developed.

Students found the shorter questions contained in Booklet one to be more accessible than the extended questions in Booklet 2. Those students who had a sound level of skills evident in their responses were able to complete the paper, indicating that the length of the examination was appropriate for the time available.

General comments worth emphasising are:

* The ‘show that …’ style of question requires students to show full working, displaying all steps of logic, for maximum marks.
* An ‘exact answer’ means the answer should be in rational or irrational form without approximations to decimal values. Students need to be reminded that the answer is stated in the question, therefore marks are awarded for justifying the working steps needed to reach this answer.
* Knowledge of, and the use of, a graphics calculator is assumed.
* Poor notation was often seen in student responses. Two particular areas of concern were the poor use of vector and integration notation.
* Students should recognise that earlier parts of a question are often relevant to the later parts of extended questions.
* Students should be aware of algebraic language. Some students did not use the brackets required to show a logical flow of their algebraic reasoning, which lead to errors in their mathematics.

Examination markers aim to award marks for evidence of student understanding in responding to examination questions wherever possible, however, students should be advised not to cross out their responses or attempted responses to questions in the examination booklet, unless they are confident that no part of their response should be considered by the marker.

If a student crosses out a response and then decides that it was the correct (or the most correct) answer, then the student should indicate clearly to the marker which part of their response should be considered. This could be done by circling or highlighting all or part of the response that the student wants to be considered and write ‘please mark this work’. Students do not need to rewrite their answers in this case, unless the crossing out has rendered the response unreadable.

It is advisable that students indicate in the space for an answer if they are also using the grid page(s) provided for additional working space. For example, “see page x” written in the question responses box would support the marker to quickly find a response that had to be extended to one of the grid pages. Any work placed on the grid pages must be labelled clearly to support markers to identify the question being attempted there.

Specific comments for the questions within question booklets 1 and 2 follow:

Booklet 1

Question 1

This question was approached well by the majority of students, which made a positive start to the examination for the majority of students.

(a) The most common approach was to work with the right-hand side to produce the left, but some students undertook synthetic division or long division successfully.

(b) (i) and (ii) some students scored full marks, whereas others did not take care to show the behaviour near the  asymptote or the oblique asymptote of  Some only drew the top half of the curve. The modulus graph was generally drawn correctly. An attempt to show both branches of the graph should be made.

As written on the front of the exam booklet, it is indicated that students may use pencil for graphs. Using pencil does make the drawing of graphs easier.

A good technique to employ when using graphics calculators for drawing graphs is to use the scale on the set of axes in the question to set-up the calculator.

(c) Although this question clearly asked for a statement about whether solutions exist, and to find them if they do, many students did not read the question thoroughly.

(i) The answer of “no solutions” was all that was required here. Some attempts to use algebra were unsuccessful.

(ii) Using the calculator was the most efficient method for this question. Some students chose an algebraic approach with success, but others unfortunately chose to solve  instead of  An exact answer was not required for this question, but was accepted.

Question 2

This question also provided evidence of a good level of understanding with the question being attempted well by many.

(a) (i) Most students correctly stated  as the other zero. There were few students who acknowledged this to be the case due to the polynomial being real.

(ii) Most students successfully considered the sum and product of zeros to find the quadratic factor, although some incorrectly wrote the factor as  Some tried to find the quadratic factor by multiplying out two linear factors with the two zeros.

(b) (i), (ii) and (iii) Many successful solutions were presented however students must show clear steps of working to attain full marks for the ‘show that’ style questions in parts (i) and (ii). For instance, some students did not equate the expression to zero in part (ii). Some students managed to answer (iii) correctly even though earlier parts were incorrect, and many used their graphics calculator well.

This question was well done, although there was evidence that some students did not read the question correctly.

Question 3

Many students found this question a challenge.

(a) This question was worth 3 marks, so students should realise there is some work to be shown. A lot of students only attained 2 marks due to not applying De Moivre’s theorem. The graphics calculator is best used here to produce the polar form requested. Some students found Euler’s form for the complex numbers which is not part of the Specialist Mathematics course as described in the subject outline.

(b) (i) A ‘show that’ style question indicting to students that reasoning was required. Some equated the moduli with success.

(ii) Very poorly done. Students had to equate the arguments of the complex numbers, but there is a need to add in  to one of the arguments and find  This results in the solutions required for n and m.

Question 4

Some students gained 7 marks out of the 9, but some made very little progress.

(a) (i), (ii) and (iii). Parts (i) and (ii) were mostly well done but some students reduced their vectors which led to an incorrect area for part (iii). In part (ii) a requirement for an ‘exact’ value was stated which some students did not address. However, some students did not simplify the expression  any further.

(b) (i), (ii) and (iii). Parts (i) and (ii) were attempted by most students. Part (iii) was poorly attempted with some students finding  successfully, however not many continued from there to find the coordinates of P.

Question 5

This PMI proof was approached well by the majority of the students in part (a) but the connected parts (b) and (c) were not successfully completed by many students.

Many students are not setting up the initial proposition, before beginning the process of showing the proposition is true (e.g. by letting ,  and ). One mark is gained through providing the initial proposition so it is an easy mark to earn, and disappointing when students do not complete the statement and lose the mark.

(a) It should be stressed it is incorrect to write  or  as  is defined to be the stated proposition, not an expression or equation which might be part of this proposition. The final statement together with the initial statement are important inclusions to the proof.

(b) and (c) The direction to use part (a) is given via the word “hence” so a separate PMI proof was not required. Students should also be aware of the marks allocated and use them as a guide to the work required. Given that there is one mark for the question, it should have been evident that another proof was not required. Similarly, in part (c), students are clearly advised to use the previous results. These parts of the question were not well done.

It is advisable for teachers to include this style of further questioning after a PMI proof within their SATs so that students become familiar with this connected style of questioning.

Question 6

This question was mostly well done with a few marks lost by some as outlined below.

(a) Most students attempted the integration by following the instructions to use the ‘by parts’ method. The most common loss of mark was due to not showing clearly the development of the integral



(b) (i) Again, there is the need for the students to show how the given equation results. For instance, students could have written 

(ii) Evaluating the volume was successfully completed by most. An exact value was required, and it is expected that students evaluate  and 

Question 7

Most students found this question challenging.

(a) Most followed the instruction to use  Students who then tried to use integration by parts were not often successful. Using  and expanding led to a more easily managed integral.

(b) (i) Was successfully completed by most students.

(ii) Some students thought it sufficient to state that either the vertical line test is passed, or the horizontal line test is passed. Students need to state that the function is one-to-one.

(iii) Students were generally successful in drawing the inverse function using the given  line and symmetry.

(c) This part of the question was poorly done. Many did not ‘see’ the area to be found and many did not follow the instruction to use parts (a) and (b). To gain full marks students needed to be clear with their reasoning due to the ‘show that’ instruction provided in the question. Recognising the rectangular area as  and subtracting  using symmetry was the approach successful students made.

Booklet 2

The three questions in this booklet are longer and worth 45 marks in total. The parts within the questions are connected.

Question 8

The vector question seems to provide many students with a good opportunity to display their knowledge.

(a) (i) – (iii). Part (i) was mostly well done although some students did not follow the instructions to use and state row operations. Since the parametric equations are given students are required to show their working.

In parts (ii) and (iii) it would be beneficial to see that students recognise they have actually answered the question by stating for example, “Hence A and B are on line 1”, or “Hence C is on plane 2”, after their substitutions.

(b) (i) – (iv). These parts were well done by most of the student cohort.

In part (i) some stated the equation of a plane instead of a line, and some did not connect that the direction of the line is the direction of the normal to the plane.

In part (ii), students needed to show the method/process used to find the coordinates of point D as the coordinates are given in the question. The most successful approach was to substitute the parametric equations from part (b)(i) into plane 1 to solve for the parameter (which is ). This leads to finding D as required.

Part (iii) required knowledge that the shortest distance from a point to a line is the perpendicular distance and hence using the dot product was the best approach. Some students chose to find an expression for the distance between two points and used the calculator to establish when this was a minimum. The last part of this question was successfully attempted by most students who attempted it. However, issues seen were that students did not see the connection from previous parts or did not follow the requirement to calculate the difference in distances.

Question 9

Most students were successful in the earlier parts of this question, but many struggled as the question developed. The need to make connections between complex numbers, trigonometry and geometry seems to be difficult for the majority of students.

(a) (i) – (v).

Part (i) was generally answered well with students recognising the trigonometric identity, but as previously commented on in Question 8, care must be taken to set out work logically.

Drawing the curve in part (ii) was successfully completed by most, however students must be aware that their calculator screen is not square. Hence the shape is a circle not an ellipse. Some drew part circles, due to not setting the parameter values correctly.

The direction to use implicit differentiation was followed by the majority of students in part (iii) and those who attempted part (iv) were generally successful.

Part (v) involved finding the equation of a tangent at a point A. Many found the slope of the tangent and then stopped, therefore only achieving one out of two marks for this question.

(b) (i) – (iv).

Part (i) saw many answers missing the  having an inequality, or having 2 instead of  These errors resulted in the loss of marks that should have been quite easy for students to earn.

Those students who did not achieve success in part (ii) generally did not follow the ‘exact’ requirement or the ‘polar form’ requirement.

The explanation for the argument of z in part (iii) was very poorly completed. Students need to be able to explain themselves either using a diagram or with a symmetry discussion. Re-state the given information is not sufficient. There was mixed success for part (iv).

(c) Some students realised the need to equate the tangent line through P with the tangent line through point X. Not many students realised that the tangent of an angle can be used to find the slope of a line.

Question 10

Some students managed this question very well initially. Surprisingly, a significant number found the integration of a standard differential equation difficult.

(a) Care needs to be taken in drawing slope fields. It is recommended that students attempt solution curves for slope fields using a pencil. Some students plotted (0,1) instead of (1,0), some started from the initial point too steeply (crossing many slope fields), and some drew above the horizontal asymptote of 

(b) This part was well done by most, although the most successful students set their work out clearly showing an intermediary step providing evidence of the LHS becoming the RHS of the equation. The use of equal signs down both sides of the work is very poor setting out.

(c) Many students struggled with this integration. After separation, some students recognised the need for a factor of  to be used. Some students who succeeded to this point then made errors such as not using modulus symbols around natural logarithm arguments or having an addition of terms instead of subtraction. Substituting in the values for x and y was best done early on for the most efficient development of the solution.

(d) (i) Good use of the graphics calculator was evident in this part of the question, although some students started their graph before the requirement of 

(ii) Some answered this correctly, however more attention to the detail in the question needs to be taken as the requirement for three significant figures was missed by some.