# Government of South Australia LogoSACE Board Logo2023 Mathematical Methods Subject Assessment Advice

Overview

Subject assessment advice, based on the 2023 assessment cycle, gives an overview of how students performed in their school and external assessments in relation to the learning requirements, assessment design criteria, and performance standards set out in the relevant subject outline. They provide information and advice regarding the assessment types, the application of the performance standards in school and external assessments, and the quality of student performance.

Teachers should refer to the subject outline for specifications on content and learning requirements, and to the subject operational information for operational matters and key dates.

# School Assessment

Teachers can improve the moderation process and the online process by:

* thoroughly checking that all grades entered in Schools Online are correct
* ensuring the uploaded files are a reasonable scan quality, the work has the correct orientation, and blank pages and student notes have been removed
* uploading the Skills and Application Tasks (SATs) as a single scanned file
* preferably providing a summary of student results in each of the SATs on the first page of the uploaded SATs file
* filling in the variation form if a student did not complete one or more skills and applications tasks or mathematical investigation(s)
* for SATs and Mathematical Investigations responses, clearly marked answers showing which mathematical calculations are fully or partially correct and which are incorrect is a requirement of moderation. Showing marks and totals for SATs is also helpful.

Assessment Type 1: Skills and Applications Tasks (50%)

Teachers can elicit more successful responses by:

* designing tasks that provide a mix of routine and more complex problems that effectively differentiate student mathematical knowledge and understanding of concepts and relationships across the grade bands
* providing multiple opportunities for students to demonstrate their interpretation of concepts and results in the context of the problem, including discussion of the assumption and limitations of the results in all skills and applications tasks
* using appropriate verbs, such as state, explain, and interpret to guide students to form an appropriate response
* strategically placing ‘show’ questions that allow students back into a question if they were not able to complete a previous part successfully. An example of a ‘show’ question is providing the approximate answer to an annuity problem so that students who are not able to find the value can use the provided figure to continue through the following questions.

In a ‘show’ question, students are required to not only present the final value but also provide evidence of the method used to determine that value, with marks not awarded for simply stating the given value in the question stem

* guiding students with information on the allocated marks for each question and the provided space for writing the answers, helping them understand the expected level of detail in their answers.

*The more successful responses commonly:*

* used technology appropriately, using the space allocated proportionately to the marks allocated
* provided detailed evidence of the development and proof of one or more conjectures
* consistently demonstrated mathematical knowledge and understanding of concepts and relationships across all topics
* showed all algebraic working by providing relevant steps, particularly for the ‘show that’ type questions
* used correct notation and terminology.

*The less successful responses commonly:*

* ignored what the question was asking and attempted full algebraic working when technology could have been used
* provided a decimal approximation instead of presenting the exact solution as requested
* did not demonstrate an appropriate balance of understanding the key questions and concepts across the topics in the subject outline
* did not provide enough attention to detail in presenting their solutions
* lacked an understanding of how technology could be used effectively
* did not often attempt questions that asked for interpretation or logical conclusions of results
* were based on tasks that used questions directly from a textbook or from past exams
* were based on tasks that provided questions that were repetitive which either disadvantaged students with poor knowledge and understanding, or advantaged students with highly effective knowledge and understanding.

Assessment Type 2: Investigation (20%)

Students complete one mathematical investigation with minimal teacher direction. The task must afford students the opportunity to extend the investigation in an open-ended context. The task should be written in a format that allows the student to conduct their own open-ended investigation. It must be completed in report format (if written) and must be no longer than 15 single-sided A4 pages. Appendices should be used for repetitive calculations only.

Students provide evidence of their learning in relation to the following assessment design criteria:

* concepts and techniques
* reasoning and communication.

Teachers can elicit more successful responses by:

* providing some structure of an initial problem leading to a more open-ended problem to investigate, to allow the student to develop the model at the higher-grade bands
* ensuring that the investigation is at an appropriate level of complexity and aligns well with the subject outline
* ensuring that the design of the task allows for the discussion of limitations and reasonableness. This is best done by providing a task in the context of a situation
* avoiding the use of question-and-answer style investigations, which limit student success ensuring the design of the task allows for and explicitly encourages the discussion of limitations and reasonableness of the modelling process:
* tasks that are designed to look at the generation of curves or shapes by altering values within formulae is not likely to result in individual work that is sufficiently open ended or allowing deep discussions concerning the reasonableness of solutions or limitations encountered
* presenting the response in report format. Communication of mathematical information is best done by using appropriate headings, labelling graphs and tables, referring to them in the main body of the response, and using appendices for repetitive calculations.

The more successful responses commonly:

* included detailed development and application of a mathematical model
* requires mathematics beyond just modelling an equation using technology for a set of data
* demonstrated a comprehensive understanding of the reasonableness and limitations of the model developed, which can more easily be done in the context of a situation
* effectively communicated mathematical reasoning through the use of appropriate and well-labelled graphs and the inclusion of relevant calculations and notation
* made appropriate use of the appendices to avoid repetition in the report
* were based on tasks that included an element of student directed learning that differentiated student understanding of the model or problem being investigated
* effectively and efficiently used technology to solve problems.

The less successful responses commonly:

* included excessive scaffolding with routine and often repetitive calculations that were prescribed in the task design
* had limited supporting evidence of how the models were derived (e.g. trial and error, Desmos, researched and adapted)
* did not effectively select and apply appropriate mathematical techniques or use appropriate electronic technology when solving problems
* lacked depth in the interpretation of the mathematical results, often summarising what was calculated rather than analysing the results in the context of the problem
* were difficult to follow, lacking in explanation of choices made in developing a model
* relied on proofs that are easily accessible through the textbook or online
* were based on tasks that had a limited open-ended component that students found difficult to further develop a mathematical model for beyond fitting a curve.

# External Assessment

Assessment Type 3: Examination

1. Data suggests that students found his year’s exam slightly more approachable on average than last year’s exam with a slightly higher average overall (3% increase). Once again, it was pleasing to see students successfully applying their knowledge in unfamiliar contexts and successfully solving questions of considerable complexity. One notable difference in student’s exam attempts this year was that it was more likely that parts of the last question were left blank. This could be a result of time not being managed successfully by students; however, it is suspected that this is due to the considerable complexity of the question.

The markers of the examination endeavour to ensure students are awarded marks for evidence of understanding in responding to questions wherever possible. However, the following dot points are given below to allow students to achieve improved results overall more consistently. It is noteworthy that the majority of the bullet points listed below have been previously highlighted in feedback for prior examinations. Nonetheless, it is deemed necessary to reiterate them, as the team of markers consistently observe instances where students fall short of securing marks that they appear to be capable of attaining.

When completing their examination, students should:

* not cross out their responses or attempted responses to questions in the examination booklet unless they are confident that no part of their response should be considered by the marker
* clearly let markers know if they complete a question on one of the blank pages available to ensure that it is considered as a possible response to a question.
* ensure that their answer makes sense in the context of the problem. Although one trivial error within a question is not penalised, if an unrealistic answer is obtained through error, a student should write a comment addressing this. An example of an unrealistic answer would be when calculating a probability through an integral, resulting in a value that is negative
* pay closer attention to the wording of questions. Words or phrases such as ‘exact’, ‘hence’, ‘show’ or ‘using an algebraic process’ are used to help guide a student’s approach in finding a solution
* take greater note of the allocated marks given in a question (and the space provided) to determine if they can use technology or simply state the answer (rather than show any mathematical process). Unless a particular approach is suggested by the wording of the question, students should always use the optimal method to find a solution
* understand that even if they do not successfully solve one part of a question, they can generally still continue to attempt to solve the following sections. Great care is taken during the writing process to allow multiple entry points into questions wherever possible. In the situation when a student does require a previous answer which they were unable to find, to continue with the next part of the question, I encourage them to ‘make up’ a reasonable result and use this to continue with all future parts if possible.
* be more careful when rounding numbers appropriately. It is an expectation of the course that answers are given to 3 significant figures (unless otherwise stated). In order to not unfairly penalise a student multiple times throughout both booklets, I often pick a specific question to penalise rounding.
* improve their understanding of sign diagrams. Please encourage students to only include the necessary critical points on their sign diagrams, and no extras.
* not make definitive statements about the population parameter when performing analysis using a confidence interval. Although it is possible to say ‘yes’ when asked if a given ‘claim’ can be supported (with % confidence) under correct conditions it cannot be definitively stated that the population parameter has increased/decreased/not changed.

Question 1

After a slightly different Question 1 in the 2022 paper (involving First Principles), this paper returned to the norm, opening with a purely algebraic derivative question allowing students to demonstrate routine calculus skills. Overall students were very successful in this question, with approximately 80% of students achieving 6 or 7 out of the 7 marks on offer.

The more successful responses commonly:

* correctly implemented the chain rule, product rule and quotient rule when required
* converted fractions and surds into index form in part (b) before differentiating, leading to less careless errors
* used the quotient rule correctly in part (c) being careful of
	+ the order of terms in the numerator
	+ not forgetting the denominator.

The less successful responses commonly:

* did not include appropriate brackets when required, thus leading to incorrect derivatives
* applied incorrect log laws in part (a) to ‘simplify’ the given equation before differentiating
* misplaced the index of or did not multiply by in the second term of their derivative in part (b)
* used the product rule in part (c), often expressing the denominator when ‘shifted’ to the numerator as instead of

Question 2

This question required students to demonstrate their proficiency in the application of probability density functions and integrals within a context. Students’ accurate use of technology was required, along with comprehension of their answers within a context.

The final part of the question presented a significant challenge, with students having to solve a definite integral, while maintaining attention to more complex algebraic operations involving negative values and logarithms. Students did very well in the routine parts of this question, however, the complex finish was polarising. This was evident in the data, as although there were a large number of students (around 50%) who achieved 7 or 8 out of the 8 marks on offer, there was a sharp drop off in the number of students achieving 5 or 6 marks.

The more successful responses commonly:

* correctly used technology in part (a)(i) and part (b)(i)
* correctly interpreted their answer in part (a)(ii) (perhaps taking advantage of later parts of the question) with the inclusion of the appropriate units (minutes)
* contained correct and concise algebraic processes in part (c) and (d).

This was evident by

* + the inclusion of the arbitrary constant in part (c)
	+ correctly substituting and into their result from part (c) in part (d)
	+ correct selection of inverse operations to isolate in part (d)
	+ the use of the guidance given in part (d) to use the answer from part (c).

The less successful responses commonly:

* interpreted the variable time as a discrete random variable (which takes only integer values), resulting in in part (b) (i)
* made errors with the exponent of when determining in part (c). Many students forgot to divide by in their answer (i.e. ), or on occasion multiplied by (i.e. )
* attempted to ‘take the logarithm’ of both sides when each side (or often one side due to an error) was negative in part (d)
* did not ensure that their final answer to part (d) was positive. A more successful student would have realised their answer resulted in a negative number and self-corrected their error
* showed a lack of understanding of the requirements in part (d).

Some incorrect answers involved:

* + setting the as the upper bound of the definite integral, and then making the integral equal to
	+ setting the original probability density function to be equal to , i.e.
	+ using technology to obtain a decimal answer, rather than the exact answer requested.

Question 3

Although this question only required the derivative of a quadratic function using first principles, it was considered slightly more complex and unfamiliar for students as a result of the inclusion of a constant, , within the function. The present of the constant was maintained through the second part of the question when finding the equation of the tangent to the graph of , before a value of being found in the final section when given a -intercept. This question was well done by students, with the second highest average in Booklet 1 with approximately 42% of students achieving all marks on offer.

The more successful responses commonly:

* used limit notation correctly throughout part (a). This involves including when *h* was present and removing it when letting *h* tend towards zero
* were able to find and in terms of in part (b)
* implemented one among several correct methods to correctly verify the equation of the tangent given in part (b).

The less successful responses commonly:

* did not correctly substitute into (or equivalent) in part (a) or did not expand the brackets correctly after substituting in and
* tried to show the cancellation of like terms, and the ‘taking out’ of a common factor of in a single line of working-out in part (a). In first principles questions, it is the students’ responsibility to present multiple steps to allow the marker to recognise that the answer was not obtained through simple derivative laws
* made errors when finding the equation of the tangent in part (b). These included but were not limited to:
	+ never writing on the left-hand side of their equation
	+ never substituting in into , resulting in the equation of the tangent being incorrectly listed as
	+ using technology to obtain a decimal answer, rather than the exact answer requested
* incorrectly substituted the coordinate in part (c) instead of the required .

Question 4

This question involved the routine application of the Binomial Distribution for the purpose of computing probabilities and determining if a given claim could be supported. The question culminated in students finding a confidence interval for a proportion, then using it determine if they can support a given claim (with % confidence). Although students did well in this question, they found it challenging to achieve all marks on offer, often a result of not being careful and concise with their language in part (b)(iii) when discussing if they could support the websites claim.

The more successful responses commonly:

* used the formula in part (a)(i)(1)
* used technology to calculate answers to part (a) and (b)(ii). It’s worth noting that students generally were able to successfully manage answers which their calculator outputted as scientific notation
* clearly showed that the sample size was (and not ) in part (b)(i)
* used technology to find their confidence interval for the proportion in part (b)(ii) rather than substituting values into the formula manually. There is no requirement to write the formula down with evidence of substitution
* made a clear statement of ‘No’, followed by a brief justification in part (b)(iii). By being brief in their justification, it reduced the possibility of a student contradicting themselves or making an incorrect statement.

The less successful responses commonly:

* did not take note of the discrete nature of a binomial random variable, incorrectly calculating in part (a)(i)(3)
* did not take careful note of number of zeros in the given value of (i.e. instead of ) resulting in an incorrect statement for in part (b)(i) and often incorrect bounds of their confidence interval in part (b)(ii)
* used incorrect notation in their confidence interval in part (b)(ii) such as , , or , rather than the required *p*
* incorrectly interpreted their confidence in relation to the websites claim part (b)(iii). Some examples of this are:
	+ responses with a ‘yes’ thought to be based on the value stated in the claim being inside the confidence interval. The recuring problem lies in the misconception among many students that if the given value is ‘inside’ the confidence interval, it implies the claim is supported, whereas ‘outside’ the confidence interval implies that the claim is not supported. This is not correct.
	+ responses with a vague statement that was not a clear ‘yes’ or ‘no’
	+ responses which followed a correct response of ‘no’ with a statement that was too definitive. For example, “No, the proportion of platinum cards produced is more than 0.00327”.

Question 5

Early parts of this question contained routine knowledge of Calculus, followed by a more challenging conjecture question requiring students to carefully find a derivative in terms of a constant, . While the initial sections of the question were commonly navigated successfully by students, as with past years, many students continue to find it challenging to implement an appropriate process to successfully solve conjecture style problems. The diverse responses of students in the final part of the question were pleasantly surprising due to the multiple successful approaches to solving the resulting equation and proving the conjecture. The data showed that a large number of students achieved full marks (23%). However, surprisingly the distribution of results was reasonably uniform from 0 marks achieved to 9 marks achieved (with each category having between approximately 4% and 7% of results).

The more successful responses commonly:

* showed clear algebraic evidence in part (a)(i) to reach the given answer. Some important steps required (when using the chain rule) was evidence of a common denominator, and evidence of the expansion of the numerator
* used technology to successfully complete the table in part (b)(i)
* implemented clever and concise algebraic processes to quickly solve in part (b)(iii). These often included the use of the difference of two squares.

The less successful responses commonly:

* placed on a common denominator before finding in part (a)(i), resulting in a more complex process (i.e. the quotient rule). This was also true with in part (b)(iii) when proving their conjecture. Although this was a mathematically correct process, it was considerably more complex than using the chain rule
* did not show clear evidence of setting and factorisation in part (a)(ii)
* struggled with the demands of the algebraic complexity in part (b)(iii), perhaps a result of the presence of throughout the question. Some points of the question where students seemingly experienced difficulties were
	+ finding
	+ putting on a common denominator and expanding the numerator without making any minor algebraic mistakes
	+ solving the fraction and resulting quadratic function to successfully isolate .

Question 6

This question asked students to use attributes of the given graph to construct a sign diagram and a graph of its derivative. Additionally, students were required to answer multiple choice questions relating to the values of , and for given values of . The unfamiliar aspect of this question was the oblique asymptote, resulting in some unfamiliar complexities when drawing the graph of the derivative. The data suggest that students found this question challenging, particularly part (d) with markers commenting that only a small number of students achieved all marks for their graph. This question had the lowest average percentage of marks achieved in Booklet 1 (56%), which was approximately the same average as Question 8 and Question 9 in Booklet 2.

The more successful responses commonly:

* carefully considered the sign of and when selecting their response to part (b)(i)
* took careful note of the inclusion of the equality (i.e. as opposed to ) in part (c), and hence included them in their answer when stating the appropriate interval for
* considered the inclusion of on the -axis in Figure 5 when drawing their graph in part (d).
A correct possible graph of should not have passed below a horizontal line at

The less successful responses commonly:

* included additional information in their sign diagram in part (a). The most common mistake was to include all given letters (i.e. , , , and ) on their sign diagram. Also, many students incorrectly added to their sign diagram
* responded with the sign diagram for in part (a) instead of the requested
* drew a graph of in part (d) with missing critical points or incorrect aspects. Some common mistakes were:
	+ not extending the graph for or
	+ the inclusion of additional -intercepts (commonly the origin). The two -intercepts needed to be at and
	+ the inclusion of extra turning points in addition to the two required:
	+ the local minimum needed to have an -coordinate of and be below the -axis
	+ the local maximum needed have an -coordinate of and be above the -axis
	+ not showing evidence that they had considered the properties of when . This led to many graphs passing below when

It is worth noting that in this year’s exam, students were not penalised if:

* + the minimum at was below
	+ the absolute value of the -coordinate of the maximum at was smaller than the absolute value of the -coordinate of the minimum at

Question 7

This rather routine question contained mainly familiar statistical computations utilising technology. Students needed to show understanding of the distribution of sample sums and confidence intervals. Students generally demonstrated good skills in applying their knowledge of statistical concepts within the context given. However, many were unsuccessful in using their calculated confidence interval to answer questions with appropriate justification. Because of the many routine parts of the question, it had a very high average marks achieved (75% - similar to Question 3 in Booklet 1); however, a low proportion of students achieved full marks. Many of the less successful responses related to determining and using their calculated confidence intervals mirror those of Question 4 and hence I have not discussed the equivalent errors here. Please refer to the notes on Question 4 for more advice on questions involving confidence intervals.

The more successful responses commonly:

* used technology to successfully calculate the requested probabilities throughout the question
* made clear refence to ‘’ and ‘’ in their answer to part (e) and part (f)(ii). Due to the fact that there were many values in this question, the burden of proof was on students to demonstrate they were comparing the bounds of their confidence interval to the correct value (as students often compare their confidence intervals to the calculated sample mean by mistake).

The less successful responses commonly:

* found a value of such that instead of in part (a)(ii)
* referred to the sample size being large enough for normality, or just quoted the central limit theorem in part (b). Students needed to specifically reference that the original distribution was normally distributed
* used incorrect notation in their confidence interval, such as , or instead of the required in part (d)
* mentioned ‘supporting a claim’ or ‘not supporting a claim’ in part (e) and part (f)(ii) despite no claim being mentioned in the question at all.

Question 8

This question, which involved trigonometric functions in a contextual setting had many different aspects. It necessitated students to use technology, carefully interpret their results, and perform challenging derivatives. The question took on the guise of something different while fundamentally being a kinematics question, something that students should be rather familiar with from past exams. The average percentage of marks obtained for this question was approximately 56%. The distribution of all marks was negatively skewed with a mode of 5 marks gained. The marker’s feedback suggests that most marks were lost in the interpretation of in part (a)(iii), and when algebraically finding all values of such that in part (b)(ii).

The more successful responses commonly:

* used technology to successfully calculate the requested values in part (a)
* provided a comprehensive interpretation of the value of in part (a)(iii). Students who achieved well in this question addressed the following in their answer:
* represents ‘1.3 seconds after release’
* the value of relates to the fact that the ‘angle of lean is changing at a rate of …’, or that the ‘angle of lean is increasing at …’
* the units of are radians per second
* acknowledged the word ‘left’ in part (a)(iv) in their selection of stationary points
* successfully took out a factor of in part (b)(i) or in part (b)(ii) to allow its removal when solving in part (b)(ii).

The less successful responses commonly:

* incorrectly found instead of in part (a)(ii), often resulting in marks also being lost in part (a)(iii) due to the simplified interpretation of the value of in comparison to
* did not use the product rule in part (b)(i), instead differentiating each term separately and then incorrectly multiplying them together
* contained algebraic errors, a lack of consideration of the given domain, or did not take note of requirements of the question in part (b)(ii). Some common errors by students were a result of:
* only relying on the visual cues of the graph or a lack of understanding of the unit circle when finding all solutions. With the ‘amplitude’ of the graph rapidly decreasing, the number of stationary points cannot be determined by using the graph in Figure 8 alone. Possibly as a result of this, or a lack of understanding of the unit circle, many students only listed one stationary point
* ignoring the request for ‘exact values’
* making careless errors with negatives leading to the equation (or equivalent) instead of . Often these errors were already present in a student’s derivative in part (b)(i)
* lacking the knowledge to solve . This can be done either by thinking of the values on the unit circle that satisfy , or by dividing each term by , eventually resulting in the equation .

Question 9

This question introduced some less familiar concepts embedded within the familiar framework of calculus and discrete random variables. For a discrete random variable with infinitely many outcomes, we cannot easily find the values of the mean and standard deviation using the traditional methods of the course. Hence, the use of a moment generating function was introduced here as an alternative method. Students were tasked with finding derivatives, performing substitutions, and demonstrating an understanding of the discrete nature of the value of . Although I was pleasantly surprised by the responses of students in this question as a whole, part (d) was not well completed by most students. This highlighted that many students have a lack of understanding, as they often used the normal distribution to approximate the probability. The challenges experienced in this question was reflected in the data, as only 7% of students achieved full marks for this question.

The more successful responses commonly:

* understood how to use the probability mass function in part (a) to calculate the requested probabilities. Some students who were confused by factorials found using the given , then used the property that all probabilities sum to to determine without ever needing to find
* acknowledged the discrete nature of the random variable and successfully rounded the values of ‘in’, when finding their answer to part (d)
* used the table of values given in part (a) to find in part (d).

The less successful responses commonly:

* overlooked the statement “State your answers correct to four decimal places” given in part (a)
* incorrectly let in part (a), then solved the equation
* incorrectly differentiated in part (b). Many students incorrectly listed . Some incorrect responses did not allow the final mark to be gained when did not equal to (unless a student commented on this inconsistency)
* did not show clear process in the ‘show’ questions involving substitution (i.e. part (b) and
part (c) (ii))
* expanded the given equation in part (c)(i), rather than differentiating to find the requested
* gave an answer of in part (d). The error was disappointingly very common and results from incorrectly using a normal distribution to find .

Question 10

The final question in this exam required the mastering of very challenging algebraic processes and understanding of how transformations (in this case translations) impact the area under a function. Although there were several routine parts to this question involving the approximation of areas using rectangles, the more complex sections of the question (part (b)(ii) and part (d)) were not generally well completed by students. As anticipated, this question exhibited the lowest average percentage of marks gained; approximately 40%, with fewer than 4% of students who completed the exam achieving full marks in this question. The diverse approaches employed by students in demonstrating the requested relationship in part (b)(ii) were again pleasantly surprising.

The more successful responses commonly:

* successfully used the product rule in part (b)(i)
* implemented a complex algebraic process to demonstrate the requested relationship in part (b)(ii). The two most common approaches to successfully solve this problem were as follows:
* Approach 1: simplification using the laws of logarithms, then integrating using knowledge of basic integration rules and the given result in part (b)(i).

The rough steps were as follows:

* + rearranging the results of part (b)(i) to state that
	+ using the laws of logarithms to simplify to (often over many lines)
	+ integrating each term of this function by appreciating that is a constant, and hence . Additionally, using the rearranged answer to part (b)(i) to find (the first dot point above in this section)
	+ removing a common factor of before using the laws of logarithms to place the equation into the prescribed form
* Approach 2: Use of the given result in part (b)(i), then an ‘additive method’ in which students ‘built up’ in conjunction with knowledge of basic integration rules.

The rough steps were as follows:

* + rearranging the result of part (b)(i) to state that
	+ doubling both sides of the equations (or adding to both sides of the equation) eventually resulting in
	+ adding to both sides of the equation resulting in
	+ removing a common factor of before using the laws of logarithms to place the equation into the prescribed form
* gave a clear statement describing the transformation of required to obtain (or vice versa) in part (c). In this case a horizontal translation one unit to the right
* used the given result in part (b)(ii) and the transformation in part (c) to replace the with in part (d). This relied on students understanding that the value of these expressions is equal due to the nature of horizontal translations
* successfully expanded the brackets when finding the value of in
part (d). Many students did not multiply the by when attempting to express their answer in the prescribed form.

The less successful responses commonly:

* drew and calculated the underestimates in part (a)(i) and part (a)(ii)
* used an incorrect width of rectangles when calculating the area in part (a)(ii) (despite drawing their rectangles correctly in part (a)(i)). It was not uncommon to see a value resulting from students using a width of or
* stated an incorrect transformation (i.e. a dilation, vertical translation), or listed additional transformation in addition to the correct translation (stated above) in part (c)
* made errors in the replacement of in part (d). Some common errors were:
* replacing with without any change in the bounds of the integral
* translating in the wrong direction to replace with .

Students should have realised these approaches were not correct when the answer could not be expressed in the prescribed form (i.e. )

* did not consider the guidance stated to help direct their approaches i.e.
* in part (b)(ii), to “use part (b)(i)”
* in part (d) “to use part (b)(ii) and part (c)”.

The responses which did not take note of the given guidance led to a notable oversimplification of the required algebraic processes and understanding.