



South Australian  
Certificate of Education

# General Mathematics

## 2019

### Question booklet

- Questions 1 to 9
- Answer **all** questions
- Write your answers in this question booklet
- You may write on page 23 if you need more space

### Examination information

#### Materials

- Question booklet
- SACE registration number label

#### Instructions

- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used — complete the box below

**Total time:** 130 minutes

**Total marks:** 90

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<p>Attach your SACE registration number label here</p>	<p><b>Graphics calculator</b></p> <p>1. Brand _____</p> <p>Model _____</p> <p>2. Brand _____</p> <p>Model _____</p>
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Government  
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## **Question 1** (4 marks)

A council is constructing a new playground. This project involves completing tasks A to H. The various tasks and their prerequisites are shown in the precedence table below.

<i>Task</i>	<i>Time (weeks)</i>	<i>Prerequisites</i>	<i>Earliest start time</i>	<i>Latest start time</i>
A	2	None	0	0
B	1	None	0	3
C	1	A	2	2
D	5	A and B	2	4
E	3	C	3	4
F	2	C	3	3
G	2	E	6	7
H	4	F	5	5

Use the information in the table above to answer parts (a) to (d).

- (a) State the task that takes the longest time to complete.

(1 mark)

- (b) List all of the tasks that must be completed on time in order to cause no delay.

(1 mark)

- (c) Calculate the minimum completion time for the construction of the playground.

(1 mark)

- (d) If a network diagram were to be drawn for this project, which task would require a dummy link leading into it? Tick the appropriate box to indicate your answer.

### Task A

### Task D

**Task E**

### Task G

(1 mark)

**Question 2** (10 marks)

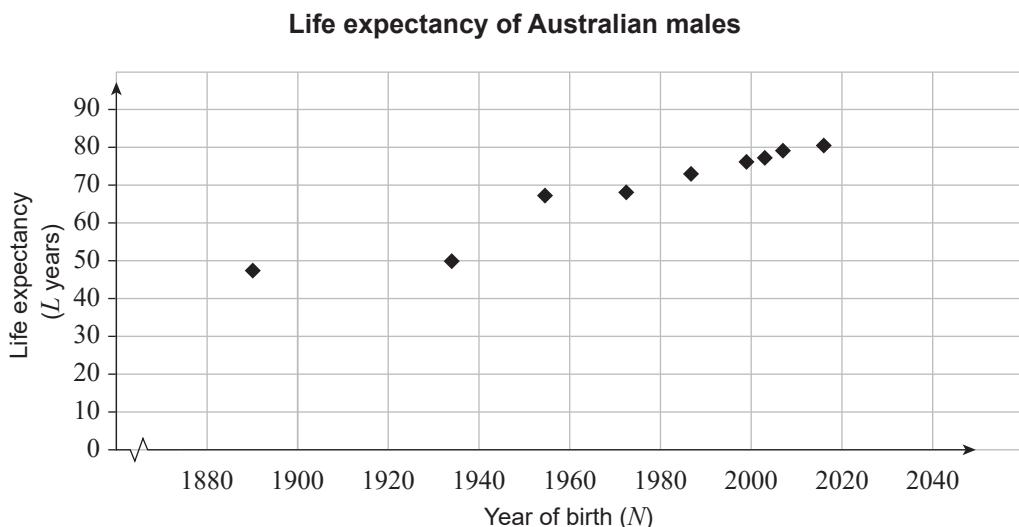
'Life expectancy' measures how long, on average, a person is expected to live from the time they are born.

The following table shows the life expectancy of Australian males.

Year of birth ( <i>N</i> )	1890	1934	1955	1972	1987	1999	2003	2007	2016
Life expectancy ( <i>L</i> years)	47.2	49.0	67.1	67.8	72.7	76.2	77.8	79.0	80.4

Source: based on Australian Institute of Health and Welfare; Registries of Births, Deaths and Marriages; National Coronial Information System; and Australian Bureau of Statistics material

These data have been plotted on the graph below.



- (a) Using a linear model, state the coefficient of determination ( $r^2$ ) between the variables. Give your answer correct to three decimal places.

(1 mark)

- (b) Describe the strength and nature of the relationship between the variables.

(2 marks)

(c) For the linear regression equation for these data:

- (i) state the values of ' $a$ ' and ' $b$ ', correct to three decimal places.

(1 mark)

- (ii) write the equation using the appropriate variables.

(1 mark)

- (d) Using the equation from your answer to part (c)(ii), predict the year of birth when the life expectancy of Australian males will first reach 100 years.

(1 mark)

- (e) One of the values for life expectancy in the table on page 4 has been recorded incorrectly, and appears as an outlier in the data.

- (i) Identify which *one* of the points below is most likely to be the outlier. Tick the appropriate box to indicate your answer.

(1890, 47.2)

(1934, 49.0) [ ]

(2016, 80.4) □

(1 mark)

- (ii) Remove the point that you identified in part (e)(i) from the data, and find the new linear regression equation.

(1 mark)

- (iii) Hence calculate and state the effect that removing the outlier will have on the prediction that you made in part (d).

(2 marks)

### **Question 3** (7 marks)

The residents' association of a retirement village needs \$275 000 to upgrade the village's facilities. The association obtains a 5-year, interest-only loan for \$275 000 that charges a flat interest rate of 5.2% per annum.

- (a) Calculate the quarterly interest payment for the interest-only loan.

(1 mark)

In order to repay the loan, the association sets up a sinking fund and plans to make monthly payments of \$4116.44 over the 5 years. The sinking fund earns interest at a rate of 4.3% per annum, compounded monthly.

- (b) Calculate the total cost of the interest-only loan and the sinking fund.

(2 marks)

- (c) (i) Calculate the value of the sinking fund at the end of the first year.

(2 marks)

- (ii) Calculate how much interest the sinking fund will earn in the first year.

(1 mark)

- (iii) State why the association could expect the sinking fund to earn more interest in the second year than in the first year.

(1 mark)

**Question 4** (9 marks)

In tennis tournaments, 'service time' is the time taken by a player to serve the ball at the beginning of a point.

Over several rural tournaments, the service times were recorded and found to be normally distributed with a mean of 20.3 seconds and a standard deviation of 2.4 seconds.

- (a) Calculate the percentage of service times expected to be between 18 seconds and 23 seconds.

(1 mark)

- (b) Calculate the longest service time that would be within the fastest 2% of service times.

(2 marks)

Tournament rules require that a player be penalised if their service time exceeds 25 seconds. Out of 12 500 serves, umpires issued service-time penalties 53 times.

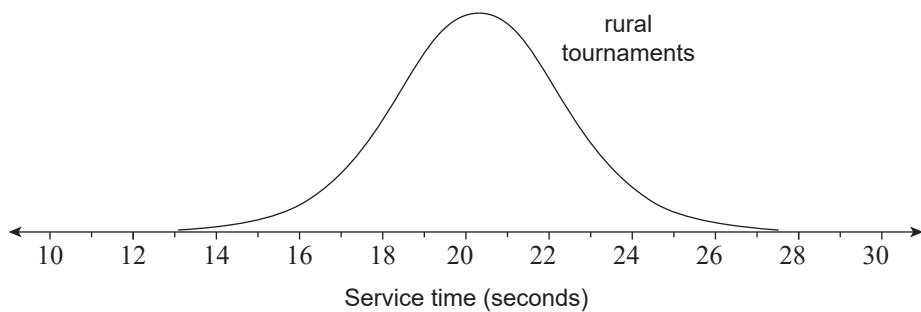
- (c) (i) Calculate the proportion of service times that would be expected to exceed 25 seconds.

(1 mark)

- (ii) Using your answer to part (c)(i), discuss whether or not the umpires are applying the service-time penalty according to the rules.

(2 marks)

The graph below shows the probability distribution of service times recorded at rural tournaments.



- (d) At the state tennis championship, the service time was found to be normally distributed with a mean of 18.5 seconds and a standard deviation of 1.5 seconds.

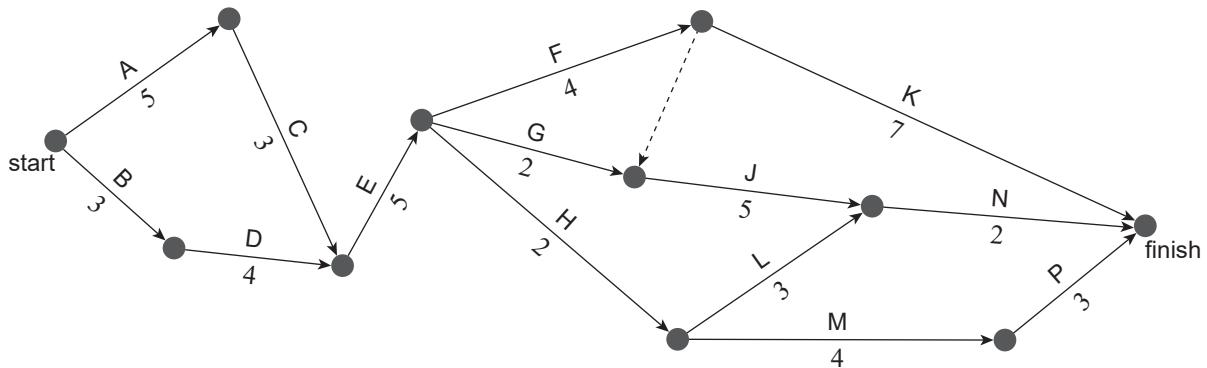
*On the graph above, sketch the probability distribution graph for this state tennis championship.*

(3 marks)

## Question 5

(10 marks)

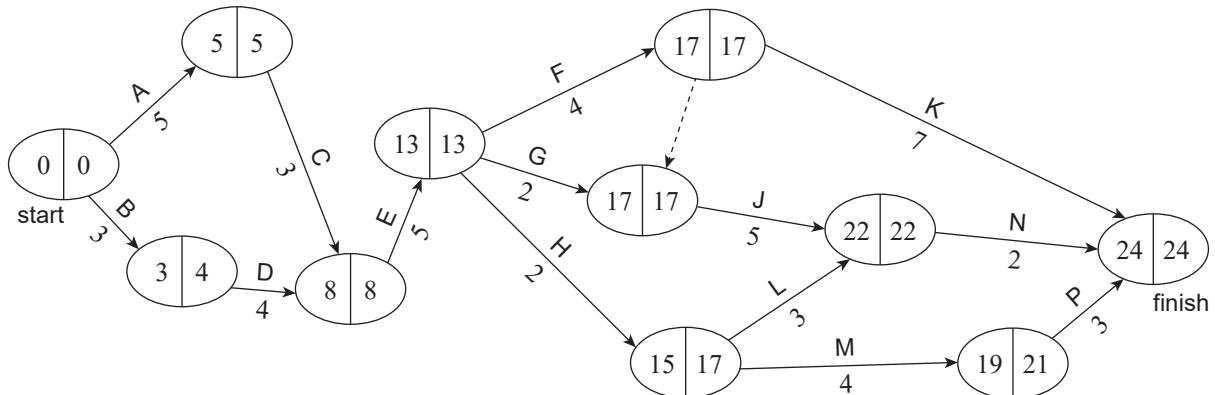
Activities A to P are part of the set-up of a music festival. The network diagram below shows these activities and their completion time in days.



- (a) Which activity *must* be on the critical path?

(1 mark)

A forward and backward scan has been completed on the network diagram below.



- (b) State the critical path(s) for the set-up of the music festival.

(2 marks)

(c) Calculate the amount of slack time available for activity L.

(1 mark)

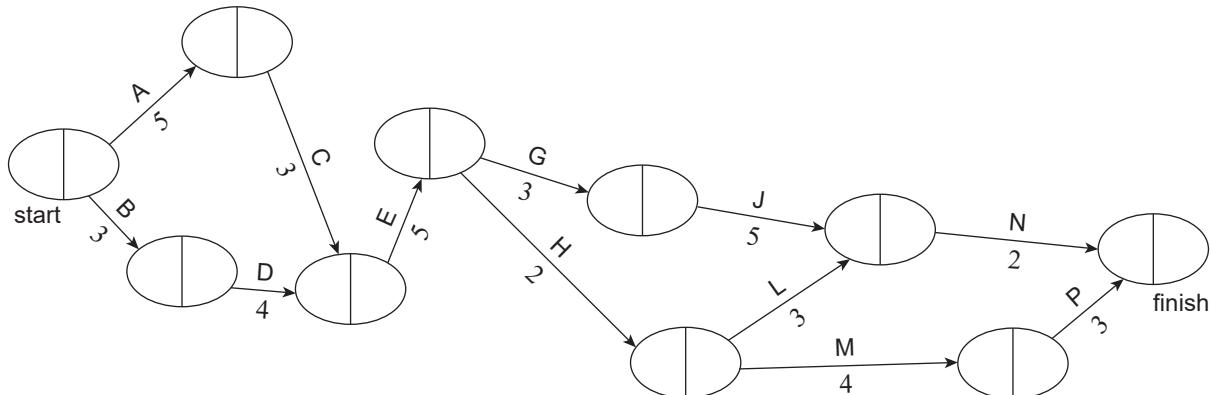
The set-up of the music festival has been changed by a new manager. The precedence table for the new set-up of the music festival is given below.

<i>Activity</i>	A	B	C	D	E	G	H	J	L	M	N	P	Q
<i>Prerequisites</i>	–	–	A	B	C, D	E	E	G	H	H	Q, J, L	M	A
<i>Time (days)</i>	5	3	3	4	5	3	2	5	3	4	2	3	12

(d) (i) Activity Q has been omitted from the network diagram below.

Using the information in the table above, add activity Q to the network diagram below.

(2 marks)



(ii) On the network diagram above, complete a forward and backward scan.

(2 marks)

(iii) Discuss how the change to the set-up of the music festival has affected the optimal solution.

(2 marks)

**Question 6** (9 marks)

Lewis wants to install an in-ground pool and has been quoted a price of \$32 000. He can afford to save \$500 per fortnight and has decided to invest this in an account that pays 3.2% per annum, compounded fortnightly.

- (a) Show that it will take Lewis approximately 62 fortnights to save \$32 000.

(2 marks)

- (b) Lewis wants to install the pool in 2 years' time.

Calculate how much Lewis would have to pay into the account per fortnight if he is to achieve this.

(1 mark)

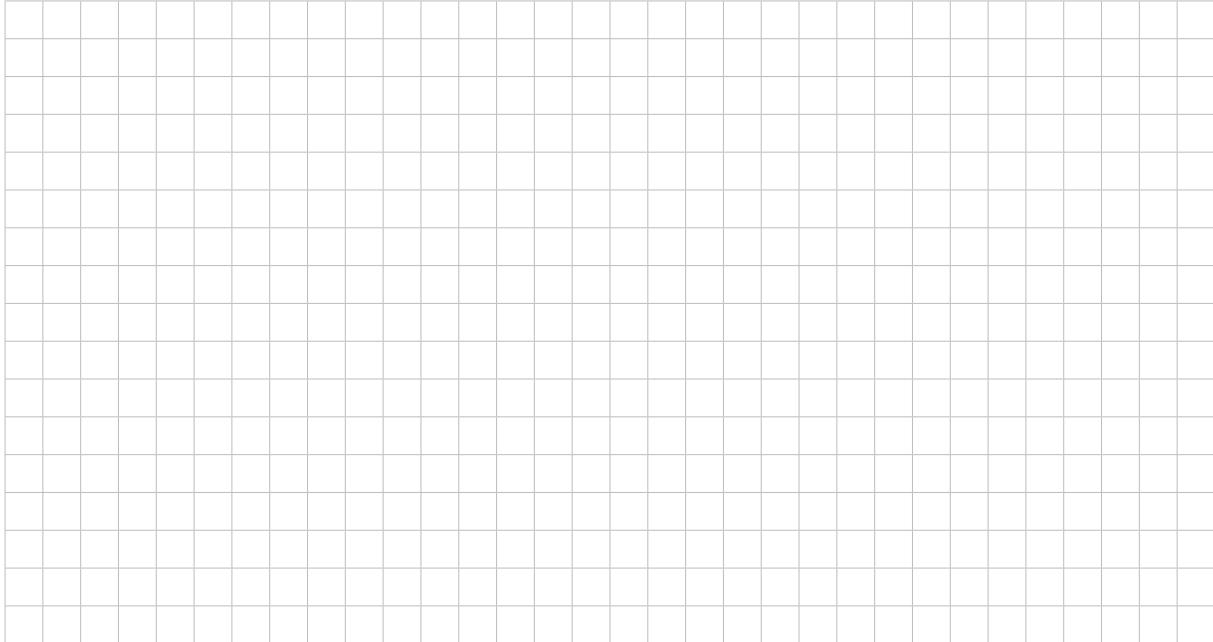
- (c) Assuming that inflation averages 1.7% per annum, calculate the expected cost of installing the pool in 2 years' time.

(1 mark)

(d) Lewis decides that he needs the pool installed as soon as possible and will take out a 3-year loan for the \$32 000. He considers the following two options:

- Option A charges 7.2% per annum, compounded weekly, with no establishment fees and no ongoing weekly charge.
- Option B charges 5.39% per annum, compounded weekly, with a \$495 establishment fee and an ongoing charge of \$5 per week.

(i) Find the comparison rate for Option A and for Option B.



(4 marks)

(ii) Hence complete the following statement:

Lewis should select Option \_\_\_\_\_ because \_\_\_\_\_

\_\_\_\_\_ (1 mark)

**Question 7** (12 marks)

The following table shows the number of subscribers (in millions) to a worldwide video streaming service, Netflix, since it started at the beginning of 2007.

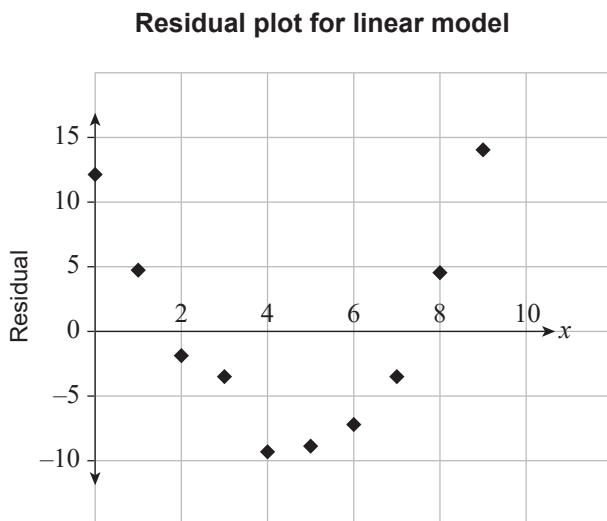
Number of years since the beginning of 2007 ( $x$ )	0	1	2	3	4	5	6	7	8	9
Millions of subscribers ( $y$ )	7.5	9.4	12.3	20.0	23.5	33.3	44.4	57.4	74.8	93.8

Source: adapted from a diagram in Richter, F 2017, 'Netflix's subscriber growth', Statista, viewed 4 July 2019, www.statista.com; data: Netflix

The linear regression equation for the Netflix data in the table is  $y = 9.35x - 4.44$ .

- (a) Complete the table of residuals below for this linear model. Give your answer correct to two decimal places. (1 mark)

$x$	Residual
0	11.94
1	4.49
2	-1.96
3	-3.61
4	-9.46
5	-9.02
6	-7.27
7	
8	4.43
9	14.08

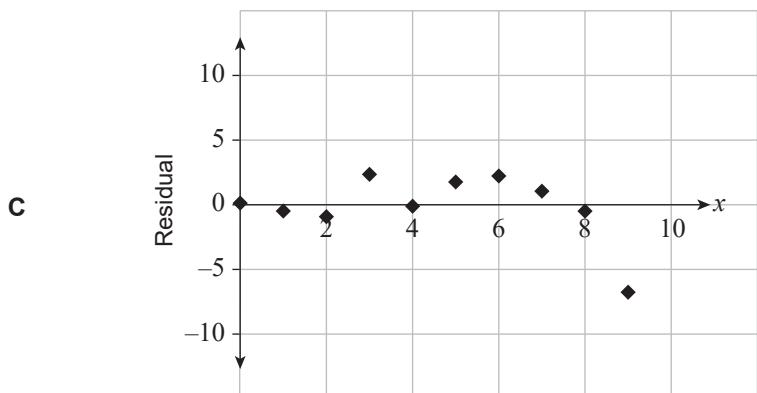
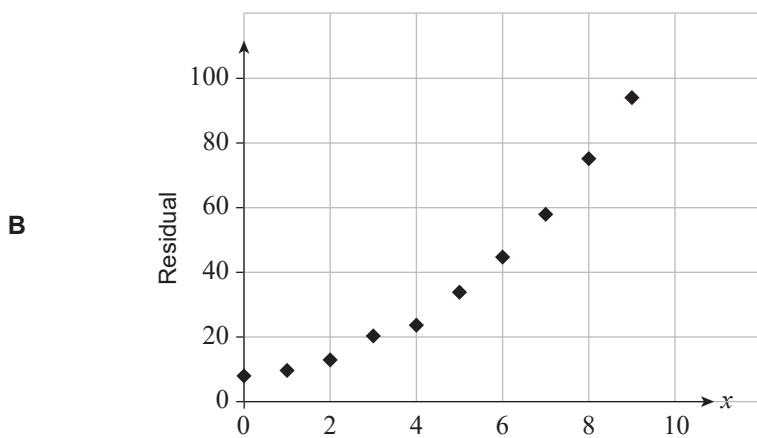
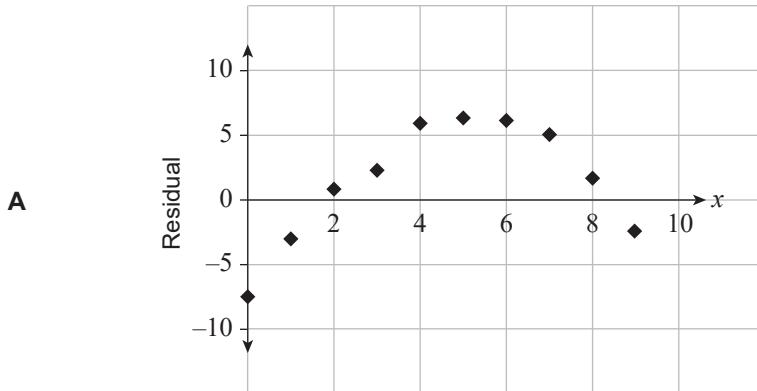


- (b) (i) Complete the table below for the Netflix data.

Model	Regression equation	$r^2$
Linear ( $y = ax + b$ )	$y = 9.35x - 4.44$	0.919
Exponential ( $y = a \times b^x$ )		

(2 marks)

- (ii) Circle the letter (A, B, or C) next to the plot that best represents the *residuals* for the exponential regression. (1 mark)



- (iii) Using evidence from parts (a), (b)(i), and (b)(ii), discuss which model best fits the Netflix data.

(2 marks)

A competing streaming service, Company Z, also started at the beginning of 2007. The number of subscribers to Company Z's service can be modelled by the exponential equation

$$y = 3.86 \times 1.409^x,$$

where  $x$  is the number of years since the beginning of 2007 and  $y$  is the number of subscribers (in millions).

- (c) (i) Explain the meaning of 3.86 in this context.

(1 mark)

- (ii) Explain the meaning of 1.409 in this context.

(1 mark)

- (iii) Using this exponential equation, estimate the number of subscribers that Company Z could expect to have by the beginning of 2023.

(1 mark)

- (iv) According to this exponential model, in what year could Company Z expect to have 350 million subscribers?

(2 marks)

- (v) State a reason why the answer to part (c)(iv) might be more reliable than the answer to part (c)(iii).

(1 mark)

## **Question 8** (16 marks)

A city operates four train lines: the Airport, Bayside, Circle and District lines. The number of passengers (in thousands) who use these train lines from Thursday to Sunday is as follows:

## Scenario 1

<i>Train line</i>	<i>Thursday</i>	<i>Friday</i>	<i>Saturday</i>	<i>Sunday</i>
Airport	11	16	15	16
Bayside	14	13	8	7
Circle	15	19	6	8
District	21	20	18	15

- (a) State the number of passengers who use the Airport line on a Saturday.

(1 mark)

From Thursday to Sunday, each train line will be closed for 1 day for maintenance.

- (b) (i) Apply the Hungarian algorithm to the array above, in order to minimise the total number of passengers affected.

Draw lines on your final array to show that the optimal solution has been reached, and state the day on which each train line should be closed.

(4 marks)

- (ii) State the minimum number of passengers who would be inconvenienced by the maintenance closures.

(1 mark)

The city decides to explore the option of doing the maintenance over 5 days, Wednesday to Sunday, during the quieter school holiday period. For this scenario, the number of passengers (in thousands) who use these train lines from Wednesday to Sunday is as follows:

## Scenario 2

<i>Train line</i>	<i>Wednesday</i>	<i>Thursday</i>	<i>Friday</i>	<i>Saturday</i>	<i>Sunday</i>
Airport	11	11	18	14	17
Bayside	13	13	12	7	6
Circle	14	14	18	5	7
District	19	20	18	17	13

- (c) State the modification that must be made to this array before the Hungarian algorithm can be applied.

(1 mark)

Some of the steps of the Hungarian algorithm have been applied to the array for Scenario 2, and the result is shown below.

0	0	7	3	7
6	6	5	0	0
9	9	13	0	3
5	6	4	3	0
0	0	0	0	1

- (d) (i) Draw lines on the reduced array above to show that the optimal solution has not yet been reached. (1 mark)

- (ii) Use the reduced array above to complete the application of the Hungarian algorithm.



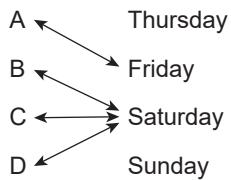
(2 marks)

- (iii) Give a complete interpretation of the optimal solution(s) in the context of the question.



(3 marks)

One assumption made when using the Hungarian algorithm model to make assignments between two groups is that all the elements of both groups must be assigned. This means that assignments such as the one in the diagram below are not considered.



- (e) (i) Referring to the diagram above, discuss the reasonableness of this assumption in the context of the question.

(2 marks)

- (ii) Referring to the information in the table below, show that a solution can be found that affects fewer passengers than the solution given by the Hungarian algorithm in Scenario 1.

## Scenario 1

<i>Train line</i>	<i>Thursday</i>	<i>Friday</i>	<i>Saturday</i>	<i>Sunday</i>
Airport	11	16	15	16
Bayside	14	13	8	7
Circle	15	19	6	8
District	21	20	18	15

(1 mark)

## **Question 9** (13 marks)

Dakota applies for a 25-year home loan of \$350 000. The loan has an interest rate of 4.18% per annum, compounded monthly.

- (a) (i) Show that the minimum monthly repayment is approximately \$1880.

(2 marks)

- (ii) Calculate the total cost of the loan.

(1 mark)

- (iii) State one assumption that may affect the reasonableness of your answer to part (a)(ii).

(1 mark)

Prior to taking out the loan, Dakota inherits \$10 000. The bank offers her an offset account.

The 25-year home loan with an offset account has an interest rate of 4.38% per annum, compounded monthly, with a minimum monthly repayment of \$1921.65.

- (b) (i) Calculate the balance on which interest will be charged at the beginning of this loan if Dakota deposits the \$10 000 in the offset account.

(1 mark)

- (ii) (1) Calculate the total cost of this loan if Dakota leaves the \$10 000 in the offset account for the entire term of the loan.

(3 marks)

- (2) Hence state one reason why Dakota may choose the loan with an offset account.

(1 mark)

- (3) State one reason (other than having the choice of where to invest the \$10 000) why Dakota may choose the original loan (without the offset account).

(1 mark)

Dakota is unsure about using an offset account. Instead, she considers investing the \$10 000 in an investment account that offers 2.3% per annum, compounded quarterly.

- (c) (i) Calculate the value of the \$10 000 investment at the end of 1 year.

(2 marks)

- (ii) Calculate the tax that Dakota will need to pay on the investment earnings, assuming that her marginal tax rate is 37.5%.

(1 mark)

*You may write on this page if you need more space to finish your answers. Make sure to label each answer carefully (e.g. 6(c)(i) continued).*

A large grid of 20 columns and 25 rows, intended for writing additional answers. The grid is composed of thin, light gray lines forming small squares.



## **GENERAL MATHEMATICS 2019**

### **ACKNOWLEDGMENT**

Question 2: table: Based on AIHW 2018, Deaths web report: supplementary tables, Australian Institute of Health & Welfare; Registries of Births, Deaths and Marriages; National Coronial Information System; and Australian Bureau of Statistics, Canberra, Table S6.1

Question 7: table: Adapted from a diagram in Richter, F 2017, 'Netflix's subscriber growth: Netflix on the brink of a major milestone', *Statista*, viewed 4 July 2019, [www.statista.com](http://www.statista.com); Data: Netflix

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