South Australian
Certificate of Education

Essential Mathematics
2019

Question booklet

Topic 2: Measurement (Questions 1 to 3) 30 marks

Topic 4: Statistics (Questions 4 to 6) 30 marks

Topic 5: Investments and loans (Questions 7 to 9) 30 marks

- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on pages 10, 17, and 24 if you need more space
- Allow approximately 40 minutes for ***each*** topic

Examination information

Materials
- Question booklet
- SACE registration number label

Instructions
- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used — complete the box below

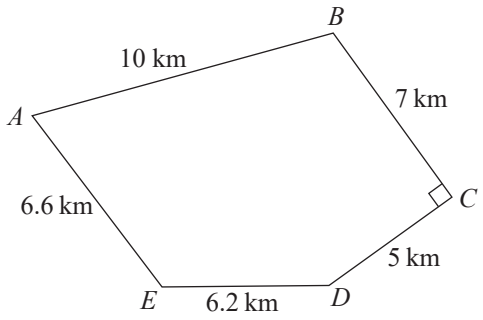
Total time: 130 minutes
Total marks: 90

Attach your SACE registration number label here	**Graphics calculator** 1. Brand _____ Model _____ 2. Brand _____ Model _____

Government of South Australia

Question 1 (9 marks)

A group of hikers planned a trail through a national park. The trail is shown below.



The hikers must start and finish at A, and follow the trail in alphabetical order (A to B to C to D to E, and back to A).

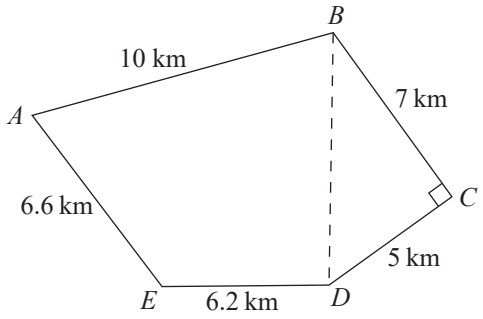
(a) (i) Calculate the distance that the hikers will travel in order to complete the trail.

(1 mark)

(ii) Discuss *one* assumption that could affect the reasonableness of your answer to part (a)(i).

(2 marks)

The hikers consider shortening the trail by following a path directly between B and D, as shown below.



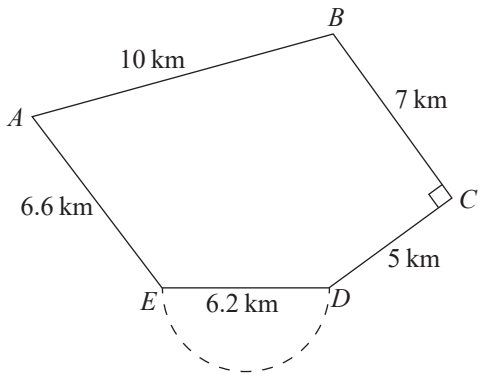
(b) (i) Calculate the distance of the direct path between B and D.

(1 mark)

(ii) Calculate by how many kilometres the original trail will be shortened when following the direct path between B and D.

(1 mark)

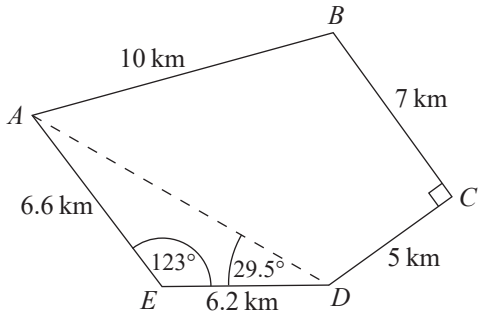
When the hikers arrive at D, they find that the straight path between D and E cannot be used. They decide to follow a semicircular path between D and E, as shown below.



(c) Calculate the distance from D to E via the semicircular path.

(1 mark)

One of the hikers sprains their ankle at D, and needs to take the direct path to A, as shown below.



(d) Calculate the distance of the direct path between D and A.

(3 marks)

Question 2 begins on page 6.

Question 2 (11 marks)

One animal welfare group recommends that a maximum of 1500 free-range hens be kept in an area of 1 hectare.

One hectare is equivalent to an area of 100 metres by 100 metres.

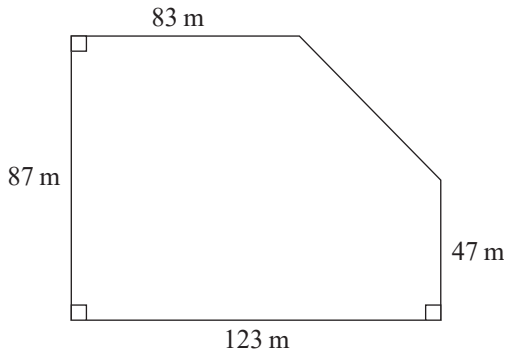
(a) Convert 1 hectare to square metres (m^2).

(1 mark)

(b) Calculate the area (in m^2) that the animal welfare group recommends each hen have.

(1 mark)

Farmer Wilkins has one paddock that has the dimensions shown in the diagram below.



(c) (i) Calculate the area of the paddock, in m^2.

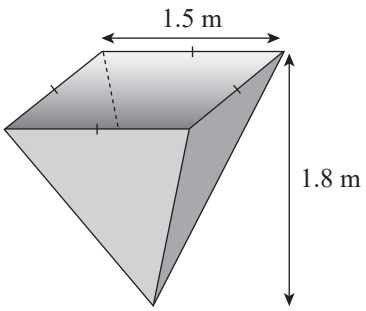
(3 marks)

Farmer Wilkins wants each hen to have an area of $8\ \text{m}^2$.

(ii) Calculate the maximum number of hens that Farmer Wilkins could keep in the paddock.

(2 marks)

Farmer Wilkins has a silo for storing grain to feed the hens. The silo is an inverted square-based pyramid with a height of $1.8\ \text{m}$, as shown in the diagram.



(d) (i) Show that the volume, V, of the silo is 1.35 cubic metres (m^3).

Note that $V_{(\text{pyramid})} = \frac{1}{3} \times \text{area of base} \times \text{height}$.

(1 mark)

(ii) The mass of $1\ \text{m}^3$ of grain is 790 kilograms (kg).

Calculate how much grain (in kg) the silo can hold.

(1 mark)

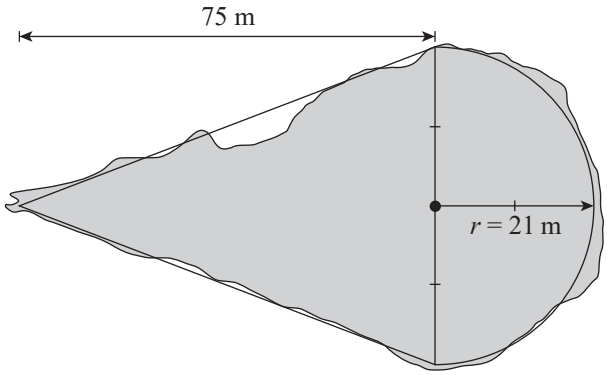
(iii) It is expected that each hen will eat approximately 120 grams (g) of grain per day.

Using calculations, show whether or not the grain stored in this full silo could feed 1200 hens for 1 week.

(2 marks)

Question 3 (10 marks)

An oil spill has been found at sea. The diagram below shows an aerial view of the oil spill and the shapes used to approximate the area of the spill.



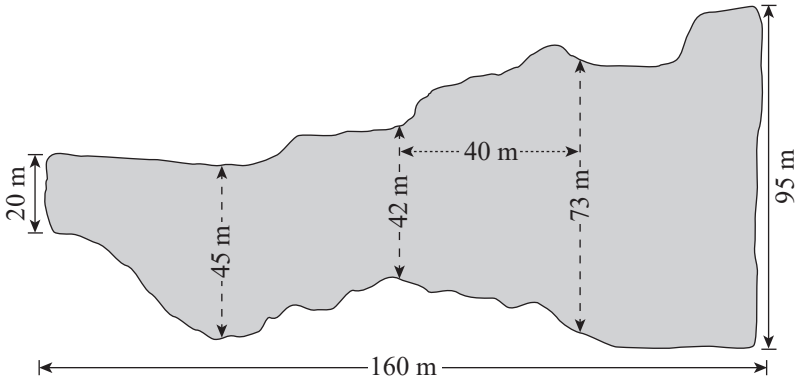
(a) (i) Using the information in the diagram above, calculate the approximate area of the oil spill.

(3 marks)

(ii) Assuming that the measurements provided on the diagram are accurate, discuss whether or not the area that you calculated in part (a)(i) is a reasonable approximation of the actual area of the spill.

(2 marks)

(b) A storm has spread the oil further. The diagram below shows an aerial view of the oil spill after the storm and the measurements taken at intervals of $40\ \text{m}$ along the length of the oil spill.



[*This diagram is not drawn to scale.*]

Using Simpson's rule, calculate the approximate area of the oil spill after the storm.

(3 marks)

A plane will spray a chemical that will help clean up the oil spill. The maximum mass of chemical that the plane can carry is 2500 kilograms (kg).

The density of the chemical is $1100\ \text{kg/m}^3$.

(c) Calculate the volume of $2500\ \text{kg}$ of the chemical, in litres (L).

(2 marks)

You may write on this page if you need more space to finish your answers to any questions in Topic 2. Make sure to label each answer carefully (e.g. 2(c)(ii) continued).

Question 4 (4 marks)

Amos is researching how many text messages the students at one school send each day, on average.

There are 1200 students in this school. The principal suggests to Amos that he email a survey to all students in the school.

(a) State the type of survey method that the principal has suggested.

(1 mark)

(b) State *one* limitation of the survey method that the principal has suggested.

(1 mark)

Amos decides to use a stratified sample of 150 students.

(c) There are 265 Year 12 students at the school.

Calculate how many Year 12 students should be included in the survey.

(2 marks)

Question 5 (12 marks)

The daily water consumption (in litres, L) of one household was recorded for 10 days before and 10 days after a water-saving strategy was introduced by a local authority.

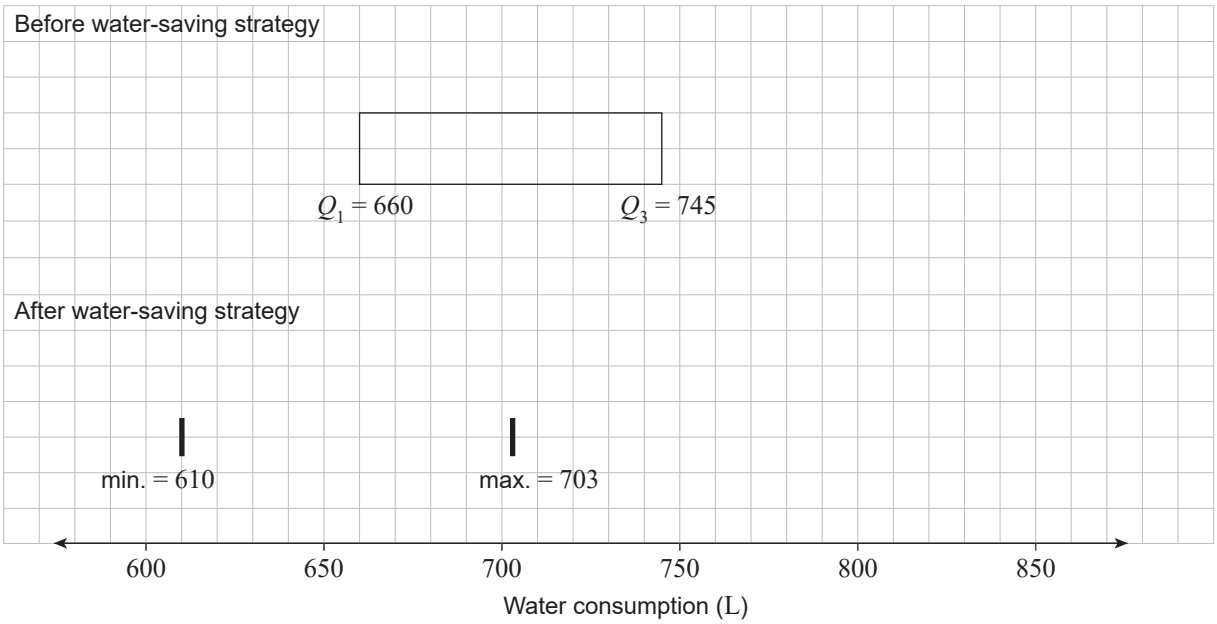
The following results were recorded.

Table 1: Daily water consumption (L)

Before water-saving strategy	*After water-saving strategy*
756	645
660	672
702	650
644	620
822	640
698	660
720	610
745	660
650	703
705	630

(a) Complete and label the box and whisker diagrams below.

Daily water consumption



(4 marks)

(b) State the impact that the water-saving strategy appears to have had on the daily water consumption of the household.

(1 mark)

(c) Complete Table 2 below (correct to one decimal place).

Table 2: Daily water consumption (L)

Statistical measure	*Before water-saving strategy*	*After water-saving strategy*
standard deviation		
range	178.0	93.0
interquartile range (IQR)	85.0	
mean		649.0

(3 marks)

(d) Using the statistical measures in Table 2, compare the consistency of this household's daily water consumption before and after the water-saving strategy was introduced.

(2 marks)

(e) This local authority claims that the average daily water consumption was reduced by 15% after the water-saving strategy was introduced.

State whether or not this claim is supported by the statistical measures in Table 2. Justify your answer with a calculation.

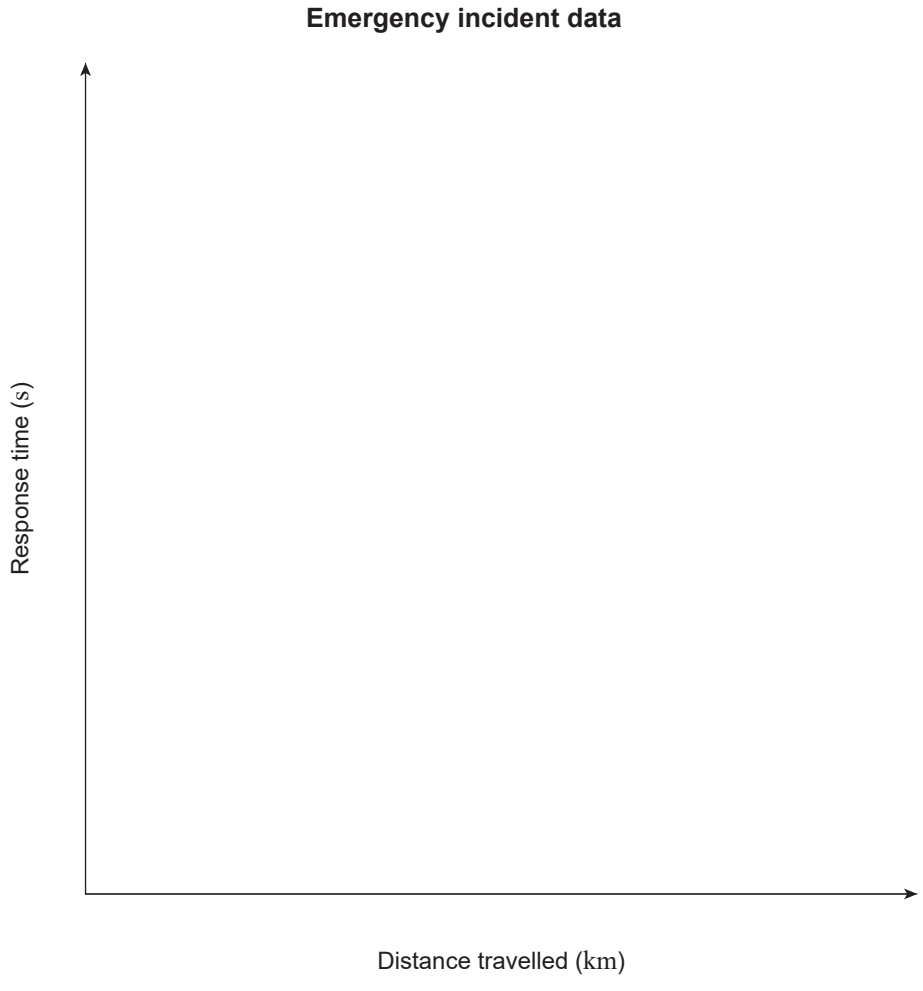
(2 marks)

Question 6 (14 marks)

One particular ambulance travels to emergency incidents in a suburban area. A study was made of the relationship between response time (in seconds, s) and the distance travelled to an incident (in kilometres, km) by this ambulance. The following results were recorded:

Incident number	*Distance travelled (km)*	*Response time (s)*
1	9.3	630
2	10.2	765
3	6.3	435
4	7.5	900
5	9.9	720
6	10.5	750
7	8.7	695
8	8.3	477
9	8.7	588

(a) On the axes below, sketch a scatter plot of the data from the table above. Include an indication of the scale on each axis.



(3 marks)

(b) (i) Calculate the coefficient of determination (r^2), and state the strength of the relationship between response time and distance travelled.

(2 marks)

(ii) Discuss the reliability of any interpolated predictions that could be made using the equation of the least squares regression line (line of best fit) for these data.

(2 marks)

(c) There is one outlier in the data.

(i) State the coordinates of the outlier.

(1 mark)

Question 6 continues on page 16.

(ii) The outlier was found to be a recording error.

Remove the outlier from the data and calculate the equation of the least squares regression line (line of best fit).

Fill in the boxes below to complete the equation.

$$\underset{\left[\begin{array}{c}\text{response}\\ \text{time}\\ (\text{s})\end{array}\right]}{y} = \square \times \underset{\left[\begin{array}{c}\text{distance}\\ \text{travelled}\\ (\text{km})\end{array}\right]}{x} - \square$$

(1 mark)

(d) Using the equation that you completed in part (c)(ii), predict:

(i) the response time (s), if the distance travelled was $7.7\ \text{km}$.

(1 mark)

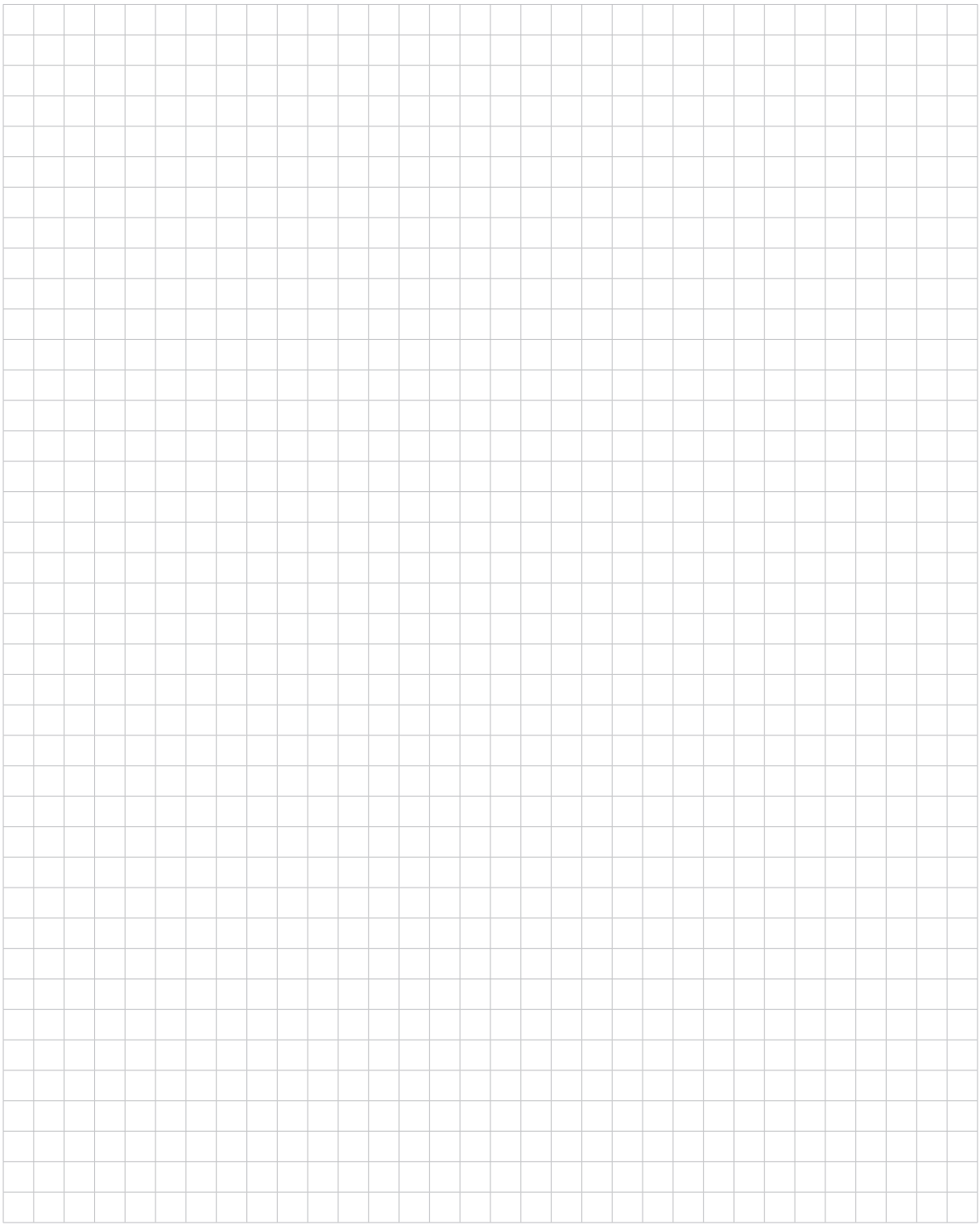
(ii) the distance travelled (km), if the response time was 13 minutes.

(3 marks)

(e) State *one* factor that may affect the reliability of the response times calculated when using the least squares regression line to make predictions.

(1 mark)

You may write on this page if you need more space to finish your answers to any questions in Topic 4. Make sure to label each answer carefully (e.g. 6(b)(i) continued).



Topic 5: Investments and loans begins on page 18.

Question 7 (7 marks)

Tianying has $\$7500$ that she wants to invest for 15 months. She has investigated two different accounts, as shown below.

Account A: savings account (interest compounded monthly)

Term of investment	*Rate*
Up to 2 years	2.67% per annum

Account B: fixed-term investment (interest paid at maturity)

Term of investment	*Rate*
Up to 3 months	1.75% per annum
More than 3 months, up to 4 months	2.20% per annum
More than 4 months, up to 5 months	2.30% per annum
More than 5 months, up to 6 months	2.40% per annum
More than 6 months, up to 10 months	2.65% per annum
More than 10 months, up to 12 months	2.60% per annum

Note: in Account B, money cannot be withdrawn before the end of the term of the investment.

(a) Calculate the amount of interest that Tianying will earn if she invests her money in Account A for 15 months.

(2 marks)

Tianying also considers investing her money in Account B over the same period of time.

(b) (i) If Tianying initially invests her money for 5 months, show that the interest earned will be $\$71.88$.

(1 mark)

(ii) Calculate the total amount that Tianying will have after investing her money in Account B for 5 months.

(1 mark)

(iii) For the remaining 10 months, Tianying will reinvest the amount that you calculated in part (b)(ii) in Account B.

Calculate the total interest that Tianying will earn from Account B over the 15 months.

(2 marks)

(c) State which account — Account A or Account B — Tianying should invest her money in if she wants to maximise the interest earned.

(1 mark)

Question 8 (12 marks)

Paisley started working for her employer at 21 years of age, with a starting salary of $\$45\,000$. The employer contributed 9.5% of Paisley's salary into a superannuation account.

(a) Show that the employer contributed approximately $\$165$ each fortnight into Paisley's superannuation account.

(1 mark)

(b) (i) Calculate the balance of Paisley's superannuation account after 9 years if the account earns 7% per annum, compounded fortnightly.

(2 marks)

(ii) State and explain *one* factor that could affect the reasonableness of the superannuation balance that you calculated in part (b)(i).

(2 marks)

(c) Paisley is now 40 years of age, and her superannuation account has a balance of $\$170\,000$.

Paisley aims to retire at 65 years of age with a balance of $\$1\,500\,000$ in her superannuation account.

(i) Calculate the total fortnightly contribution that will need to be made into Paisley's account after age 40 years if she is to achieve this aim. Assume that the superannuation account conditions remain the same.

(2 marks)

(ii) If the total fortnightly contribution into Paisley's account is $\$475$ after age 40 years, at what age will she be able to retire with a balance of $\$1\,500\,000$ in her superannuation account?

(3 marks)

(d) At retirement, Paisley will invest the $\$1\,500\,000$ in an account paying 3.5% per annum, compounded weekly.

Calculate the weekly withdrawal that Paisley can make from this account, if she wants this money to last for 25 years.

(2 marks)

Question 9 (11 marks)

Jordan has taken out a home loan, which he will repay over 25 years. The loan has an interest rate of 4.6% per annum, compounded monthly. The bank advised Jordan that his monthly loan repayment will be $\$4214.80$.

(a) Show that Jordan's initial loan amount is approximately $\$750\,000$.

(2 marks)

(b) Jordan makes monthly repayments of $\$4214.80$ for 1 year.

(i) Calculate how much money he still owes the bank at the end of the first year.

(2 marks)

(ii) Calculate by how much the loan balance has reduced at the end of the first year.

(1 mark)

(iii) Calculate how much Jordan has paid to the bank over the first year.

(1 mark)

(iv) State why the loan balance has not reduced by the full amount that Jordan has paid.

(1 mark)

(c) Jordan's loan balance after the first 5 years is $\$660\,500$. At this time he receives an inheritance of $\$30\,000$, which he decides to pay into his loan to reduce the loan balance.

(i) If Jordan continues to make the same monthly repayments of $\$4214.80$, calculate how many months it will now take him to pay off his loan.

(2 marks)

(ii) Calculate the interest that Jordan has saved by paying the inheritance into the loan.

(2 marks)

You may write on this page if you need more space to finish your answers to any questions in Topic 5. Make sure to label each answer carefully (e.g. 9(a) continued).