South Australian
Certificate of Education

Essential Mathematics
2018

Question booklet

- **Topic 2: Measurement** (Questions 1 to 3) 30 marks
- **Topic 4: Statistics** (Questions 4 to 6) 30 marks
- **Topic 5: Investments and loans** (Questions 7 to 10) 30 marks
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on pages 9 and 22 if you need more space
- Allow approximately 40 minutes for ***each*** topic
- Approved calculators may be used — complete the box below

Examination information

Materials

- Question booklet
- SACE registration number label

Reading time

- 10 minutes
- You may begin writing during this time
- You may begin using an approved calculator during this time

Writing time

- 2 hours
- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams

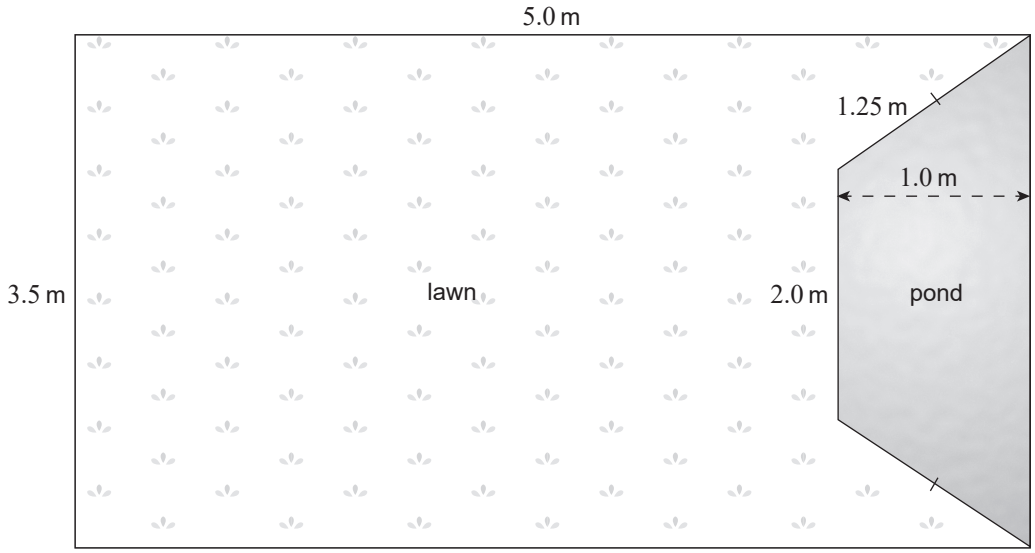
Total marks 90

Government of South Australia

Attach your SACE registration number label here	**Graphics calculator** 1. Brand _____ Model _____ 2. Brand _____ Model _____

Question 1 (9 marks)

Jessie has a rectangular garden with a pond, as shown in the diagram below.



(a) (i) Calculate the perimeter of the pond in metres (m).

(1 mark)

(ii) Calculate the cost of fencing the pond, if fencing costs $\$94.50$ per metre.

(1 mark)

(b) Calculate the area of the pond in square metres (m^2).

(2 marks)

(c) Jessie wants to keep fish in the pond.

(i) The water in the pond is 75 cm deep.

Calculate the volume of water in the pond in cubic metres (m^3).

(2 marks)

(ii) Calculate the number of litres (L) of water in the pond.

Note that $1\text{ L} = 1000\text{ cm}^3$.

(1 mark)

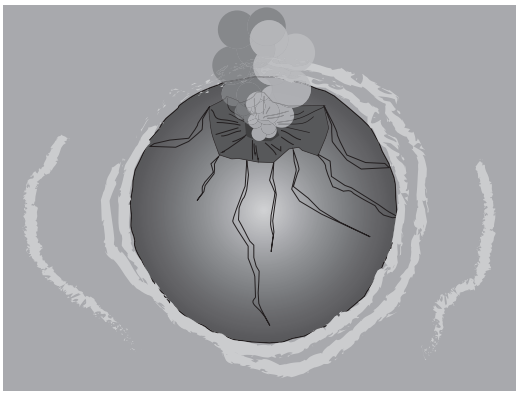
(iii) Each fish needs 110 litres of water.

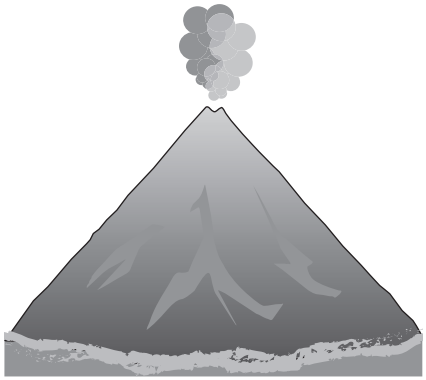
Calculate the maximum number of fish that Jessie can keep in the pond.

(2 marks)

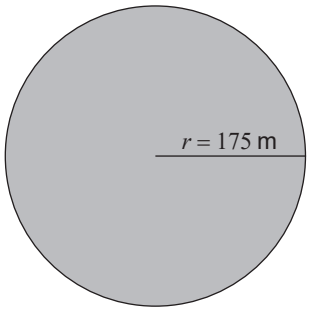
Question 2 (10 marks)

The following illustrations show a volcanic island from above and from the side:





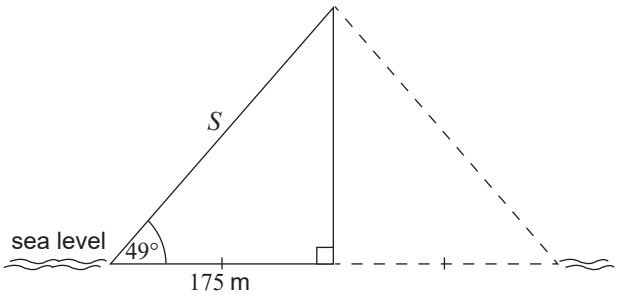
Refer to the following diagram, which shows the base of the island at sea level:



(a) Calculate the circumference of the base of the island at sea level in kilometres (km).

(2 marks)

Refer to the following diagram, which shows a vertical cross-section of the island:



(b) Show that the length (S) of the sloped side of the island is approximately 266 metres.

(2 marks)

(c) (i) Calculate the surface area of the island above sea level in square metres (m^2), using the formula $A = \pi rS$.

(1 mark)

(ii) Discuss *one* assumption that could affect the reasonableness of your answer to part (c)(i).

(2 marks)

(d) The Department for the Environment wants to conduct an aerial photographic survey of the island above sea level.

(i) Calculate the number of photographs required to complete the survey if each photograph captures an area of 75 square metres (m^2).

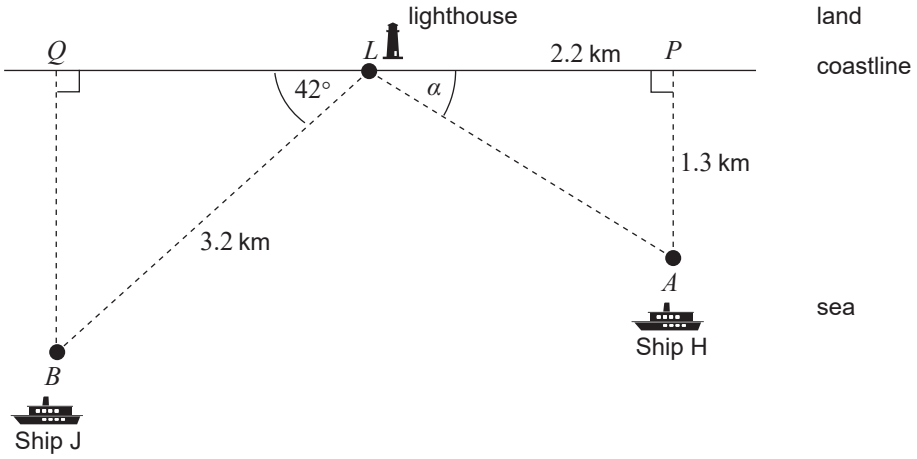
(1 mark)

(ii) Calculate the number of flights required to photograph the entire island, if the aeroplane's camera can take 450 photographs per flight.

(2 marks)

Question 3 (11 marks)

Ship H and Ship J were at sea off a straight section of coastline. Ship H was at point A, and Ship J was at point B, which is 3.2 km from a lighthouse at point L, as shown in the diagram below.



(a) Calculate the distance (AL) from Ship H to the lighthouse.

(2 marks)

(b) Find angle α ($\angle PLA$) to two decimal places.

(2 marks)

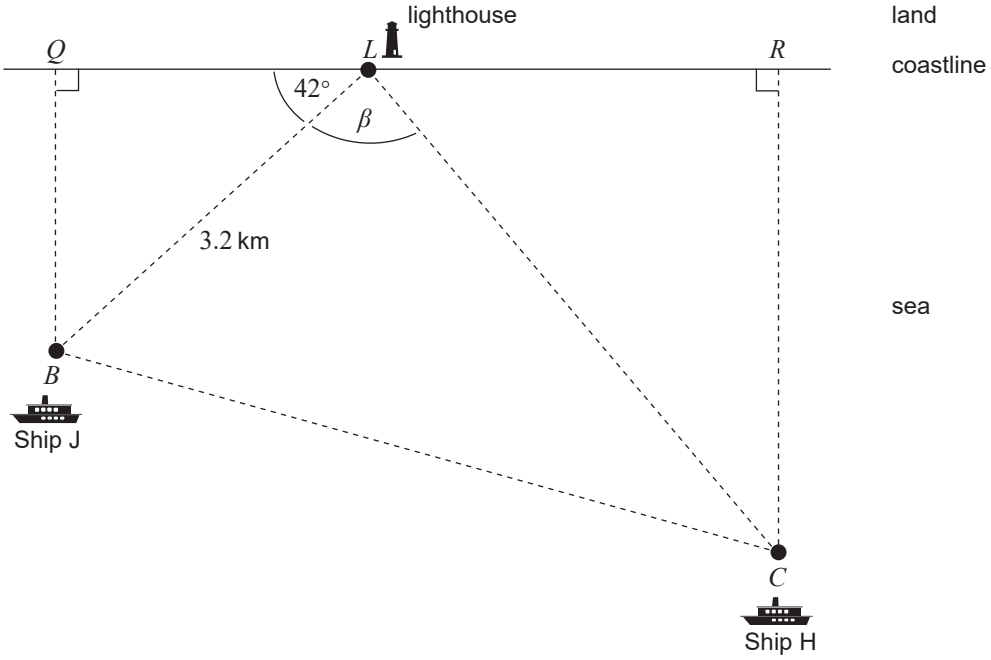
(c) Calculate the distance (BQ) from Ship J to the coastline at point Q.

(2 marks)

An hour later, Ship H received a distress call from Ship J.

At that time, Ship H was at point C, which is 5.5 km from the lighthouse at an angle of $50°$ from the coastline ($\angle RLC$).

(d) Write this distance and angle on the diagram below.



(1 mark)

(e) Show that angle β ($\angle BLC$) is $88°$.

(1 mark)

(f) Find the distance (CB) that Ship H had to travel to reach Ship J.

(3 marks)

You may write on this page if you need more space to finish your answers to any questions.
Make sure to label each answer carefully (e.g. 2(c)(ii) continued).

Question 4 (9 marks)

A travel association was investigating the number of nights away for group travellers and single travellers holidaying in Australia.

Table 1 shows the data (number of nights away) for 16 single travellers.

Table 1

25	19	5	12	25	20	23	1
2	20	1	24	38	24	6	15

(a) Using the data in Table 1, complete the ordered stem plot for single travellers.

Number of nights away

Group travellers									*Stem*	*Single travellers*							
								2	**0**								
								6	**0***								
					2	1	1	0	**1**								
			8	7	6	6	5	5	**1***								
						3	3	2	**2**								
								8	**2***								
									3								
									3*								

Key

Stem **0** leaves 0 – 4

Stem **0*** leaves 5 – 9

(2 marks)

(b) Hence tick the appropriate box to indicate which *one* of the following statements is correct.

The data for single travellers *are not* symmetrical, and *do not* appear to have an outlier. ☐

The data for group travellers have a larger *range* of values than the data for single travellers do. ☐

The data for single travellers *are not* symmetrical, and *do* appear to have an outlier. ☐

The data for single travellers *are* approximately symmetrical, and *do* appear to have an outlier. ☐

(1 mark)

(c) (i) Complete Table 2 below (correct to one decimal place).

Table 2

Statistical measure	*Group travellers*	*Single travellers*
mean	15.3	
standard deviation	6.7	
median	15.5	

(2 marks)

(ii) Explain which statistical measure in Table 2 is the most reliable measure of centre of the data for single travellers.

(2 marks)

(iii) State whether group travellers or single travellers have a more consistent number of nights away when holidaying in Australia. Justify your answer using evidence from Table 2.

(2 marks)

Question 5 (11 marks)

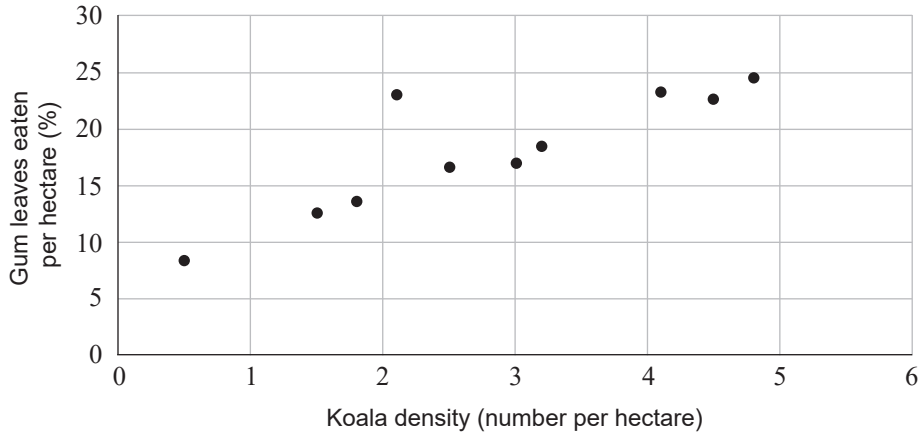
An environmental group wanted to assess the damage that koalas cause to the gum trees that they feed on.

The group collected data on koala density (number of koalas per hectare) and the percentage of gum leaves eaten per hectare, at 10 different sites. The data are shown in the table below.

Site	1	2	3	4	5	6	7	8	9	10
Koala density (number per hectare)	1.8	2.5	3.2	4.5	1.5	2.1	3.0	4.1	4.8	0.5
Gum leaves eaten per hectare (%)	13.5	16.5	18.4	22.6	12.5	23.0	16.9	23.2	24.5	8.5

A scatter plot of the data is given below.

Koala density versus percentage of gum leaves eaten



(a) State the dependent variable in this scenario.

(1 mark)

(b) Describe the relationship between the two variables shown in the scatter plot above.

(1 mark)

(c) (i) Circle a possible outlier in the scatter plot on page 12. (1 mark)

(ii) If the outlier is due to an error, suggest *one* possible reason for this error.

(1 mark)

(iii) The coefficient of determination (r^2) of the original data, including the outlier, is 0.75.

Tick the appropriate box to indicate which one of the following values is most likely to be the r^2 value if the outlier is removed from the data. Explain your answer.

-0.95 ☐ 0.75 ☐ 0.98 ☐

(2 marks)

(d) With the outlier removed, the equation of the least squares regression line (line of best fit) for the data is $y = 3.68x + 6.80$. Use this equation to answer parts (d)(i) and (d)(ii) below.

(i) Predict the percentage of gum leaves eaten per hectare if the koala density is 5.5 per hectare.

(1 mark)

(ii) Predict the koala density when 17.8% of the gum leaves per hectare have been eaten.

(2 marks)

(iii) Identify which one of the calculations — part (d)(i) or part (d)(ii) — is an extrapolation. Explain your answer.

(2 marks)

Question 6 (10 marks)

A department store in a town that has $10\,000$ residents wants to find out what mobile phone covers it should stock.

For 1 month, every tenth customer who entered the store was surveyed about the brand of mobile phone that they own.

(a) State the sampling method that the store used.

(1 mark)

(b) State *one* limitation of this sampling method.

(1 mark)

During the first day of data collection the store had 12 survey responses.

(c) Explain whether it is likely that these responses accurately represent the brands of mobile phones owned by the town's residents.

(2 marks)

The store recorded the brands of mobile phone owned by the customers surveyed. The data are shown in the table below.

Brand of mobile phone	*Number of customers*
Ascent	72
Buzz	125
Connect	24
Ring-a-Ding	
Texter	101
Total	**350**

(d) (i) Calculate the number of customers surveyed who own a Ring-a-Ding mobile phone.

(1 mark)

(ii) Calculate the percentage of customers surveyed who own an Ascent mobile phone. Give your answer correct to one decimal place.

(1 mark)

(e) (i) The store can only stock covers to fit four brands of mobile phone.

Based on the data collected, explain which one of the five brands of mobile phones in the table above you would recommend the store does ***not*** stock covers for.

(2 marks)

(ii) Based on the data collected, explain whether your answer to part (e)(i) is reasonable.

(2 marks)

Question 7 (4 marks)

Keisha is thinking of depositing $\$7800$ into an investment account with a flat interest rate of 4.5% per annum for 1 year.

(a) Calculate the amount of interest that Keisha will earn after 1 year.

(1 mark)

(b) Calculate the total amount that Keisha will have in her account after 1 year.

(1 mark)

Keisha can only invest her money for 5 months.

(c) Calculate the amount that Keisha would need to deposit into this investment account in order to earn, in 5 months, the amount of interest calculated in part (a).

(2 marks)

Question 8 (6 marks)

Jane's grandmother started a savings account for her when she was born. Every month her grandmother deposited $\$180$ into an account with an interest rate of 4.2% per annum, compounded monthly.

(a) Show that there will be approximately $\$58\,000$ in the account after 18 years.

(2 marks)

When Jane turned 18 years old, her grandmother gave her $\$58\,000$. Jane decided to deposit the money into an investment account that compounds weekly in order to provide an income while she is studying for the next 4 years.

(b) Jane estimated that she would need $\$300$ per week to cover her living expenses.

Calculate the interest rate (compounded weekly) that Jane's investment account will need to earn.

(2 marks)

(c) Jane realises that she will need $\$375$ per week over the 4 years.

Calculate the amount that Jane will need to invest at the beginning of the 4 years if the bank offers her an interest rate of 2.3% per annum, compounded weekly.

(2 marks)

Question 9 (12 marks)

Josiah borrows $\$355\,000$ in order to buy a house. The term of the loan is 30 years, and the interest rate for the loan is 3.8% per annum, compounded monthly.

(a) (i) Show that the minimum monthly repayment for the loan is approximately $\$1650$.

(2 marks)

(ii) State *one* assumption that you made when calculating the minimum monthly repayment.

(1 mark)

(b) (i) Calculate the total amount that Josiah will repay over the term of the loan.

(1 mark)

(ii) Calculate the total amount of interest that Josiah will pay over the term of the loan.

(1 mark)

(c) Josiah wants to pay more than the minimum monthly repayment. He decides to pay $\$2000$ per month from the beginning of the loan.

(i) Calculate how long (in years) it will take Josiah to repay the loan in full.

(3 marks)

(ii) Calculate the approximate amount of interest that Josiah will save by making these larger repayments.

(2 marks)

(iii) Explain one *other* strategy that would help Josiah to pay off his loan sooner.

(2 marks)

Question 10 (8 marks)

Lin Hao wants to travel after finishing Year 12. To fund his travel, he has a savings goal of $\$20\,000$ over 3 years.

Lin Hao knows that inflation will affect the value of his savings.

(a) If the inflation rate averages 1.9% per annum over the 3 years, show that Lin Hao's savings goal should be adjusted to approximately $\$21\,160$.

(2 marks)

Lin Hao increases his savings goal to $\$22\,000$.

He currently has $\$10\,000$ in a savings account with an interest rate of 2.5% per annum, compounded fortnightly. He will deposit fortnightly payments into this account over 3 years.

(b) Calculate how much Lin Hao needs to deposit into this account each fortnight in order to reach his savings goal.

(2 marks)

Lin Hao can only afford to deposit $\$80$ per fortnight.

(c) Calculate how long (in fortnights) it will take Lin Hao to reach his savings goal.

(1 mark)

(d) Calculate the interest rate that Lin Hao's investment account would need to earn in order to achieve his savings goal within the 3 years, if he deposits $\$80$ per fortnight.

(1 mark)

(e) Discuss whether it is likely that Lin Hao will achieve his savings goal of $\$22\,000$ within 3 years.

(2 marks)

You may write on this page if you need more space to finish your answers to any questions.
Make sure to label each answer carefully (e.g. 9(c)(i) continued).