Government of South Australia

SACE Board of SA | South Australian Certificate of Education

Essential Mathematics
2017

Question Booklet

- **Topic 2: Measurement** (Questions 1 to 3) 30 marks
- **Topic 4: Statistics** (Questions 4 to 6) 30 marks
- **Topic 5: Investments and Loans** (Questions 7 to 10) 30 marks
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on pages 10, 18, and 25 if you need more space
- Allow approximately 40 minutes for ***each*** topic
- Approved calculators may be used — complete the box below

Examination information

Materials

- Question Booklet
- SACE registration number label

Reading time

- 10 minutes
- You may make notes on scribbling paper

Writing time

- 2 hours
- Show all working in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams

Total marks 90

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Attach your SACE registration number label here

Graphics calculator

1. Brand _____

   Model _____

2. Brand _____

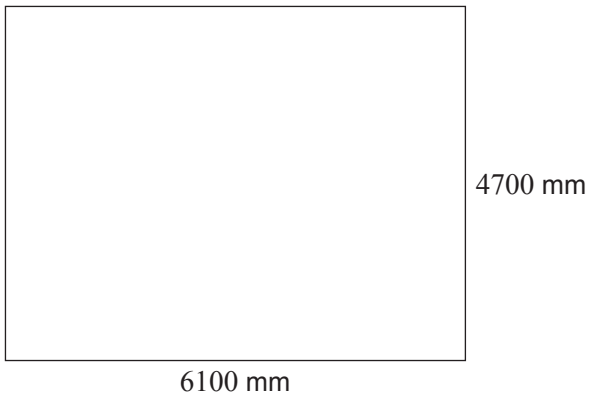
   Model _____

For office use only

Supervisor check	Re-marked

Question 1 (8 marks)

A diagram of a rectangular garage floor is shown below.



(a) Convert each of the dimensions of the floor to centimetres (cm).

(1 mark)

(b) Calculate the area of the floor in square centimetres (cm^2).

(1 mark)

(c) Varnish was applied to the floor to a thickness of 0.5 millimetres (mm).

(i) Show that the volume of varnish used is $14\,335$ cm^3.

(2 marks)

(ii) A can of varnish contains 2 litres (L) of varnish.

Calculate the number of cans of varnish needed to varnish the floor.

Note that 1 L $= 1000$ cm^3.

(3 marks)

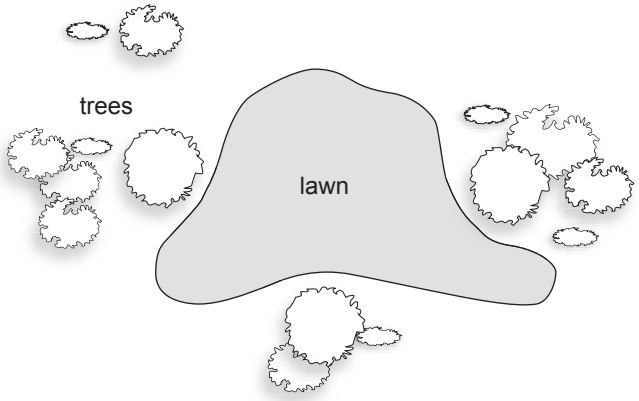
(iii) A 2 L can of varnish costs \$$32$.

Calculate the total cost of the cans of varnish needed.

(1 mark)

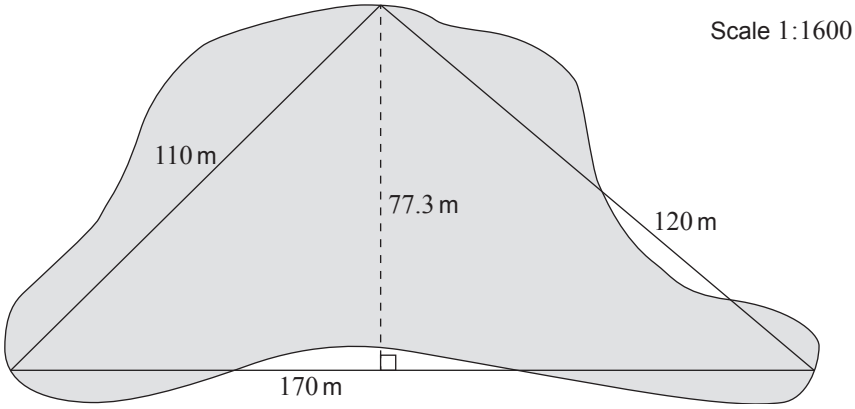
Question 2 (10 marks)

A council is redeveloping a lawn at a local park by laying roll-out lawn. To work out costs, the council needs an estimate of the area of the lawn, which is an irregular shape, as shown below.



[This diagram is not drawn to scale.]

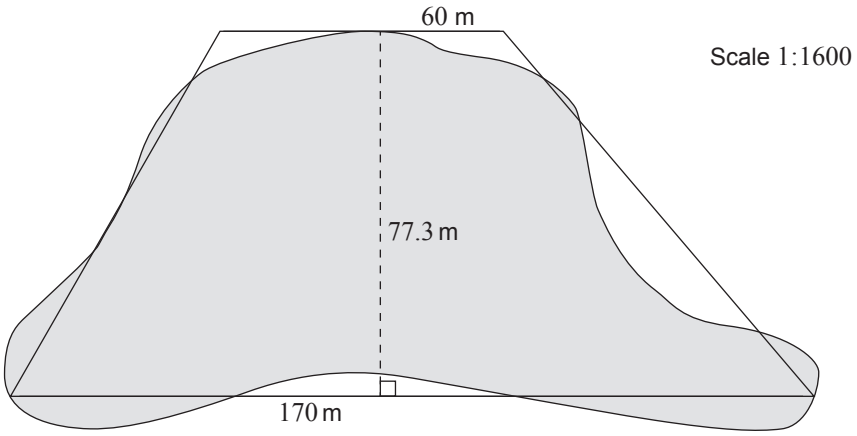
(a) The gardener thinks that the shape of the lawn looks roughly like a triangle.



Using the information in the diagram above, estimate the area of the lawn in square metres (m^2).

(2 marks)

(b) The assistant gardener thinks that the shape of the lawn looks more like a trapezium.



Using the information in the diagram above, find a second estimate of the area of the lawn in square metres (m^2).

(2 marks)

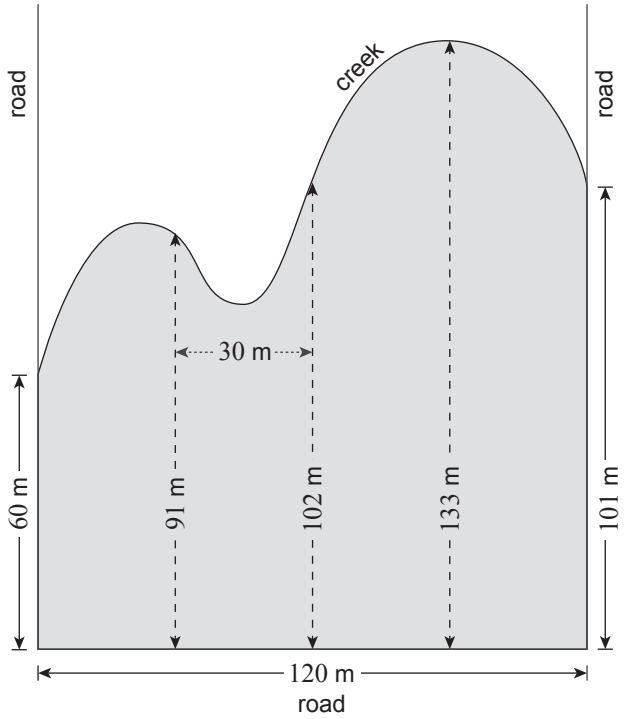
(c) (i) State which shape — the triangle in part (a) or the trapezium in part (b) — provides the more accurate estimate of the area of the lawn.

(1 mark)

(ii) Give a reason for your answer in part (c)(i).

(1 mark)

(d) The council has also decided to replant a section of the park that is shaded in the diagram below. The diagram shows measurements of the road boundaries and measurements taken at 30-metre intervals along the length of the section.



[*This diagram is not drawn to scale.*]

(i) Using Simpson's rule, estimate the area in square metres (m^2) to be replanted.

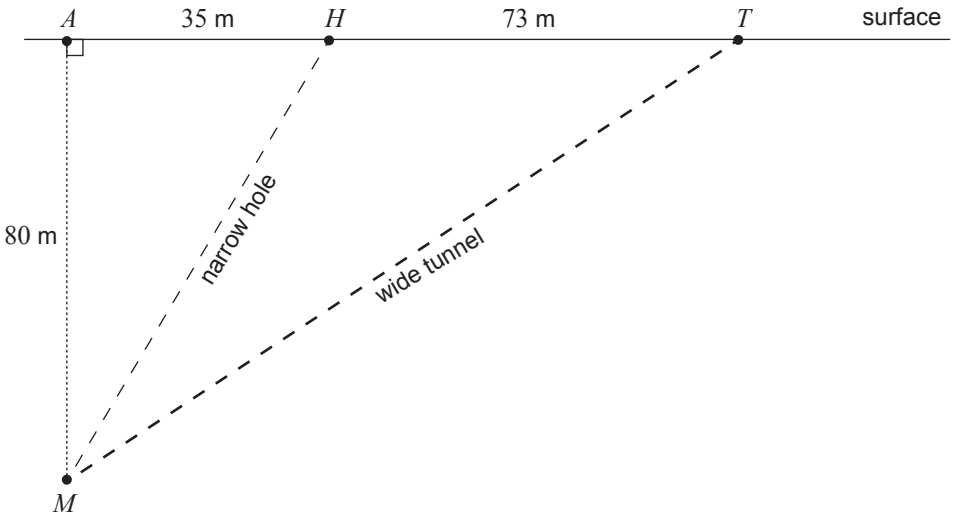
(3 marks)

(ii) Suggest *one* way to improve the accuracy of this estimate of the area.

(1 mark)

Question 3 (12 marks)

An underground rockfall has trapped a miner at point M. The rescue team will first drill a narrow hole (starting at point H) to supply water and food to the uninjured miner during the rescue operation. The team will then drill a wide tunnel (starting at point T) to allow the miner to return safely to the surface.



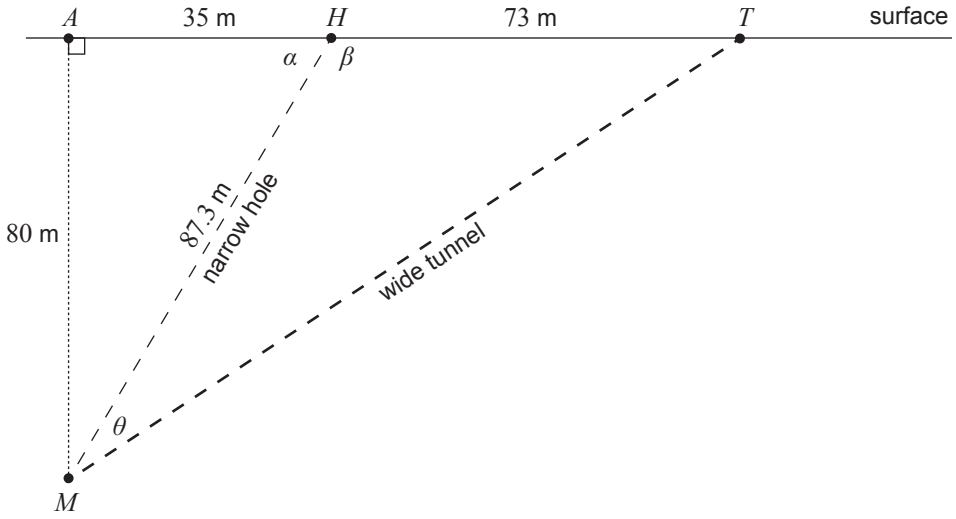
[This diagram is not drawn to scale.]

(a) Show that the narrow hole (HM) will be 87.3 metres long, rounded to one decimal place.

(2 marks)

Question continues on page 8.

(b) The drilling engineer needs to determine the size of angle β ($\angle THM$).



[*This diagram is not drawn to scale.*]

(i) Show that angle α ($\angle AHM$) is approximately $66°$.

(2 marks)

(ii) Find angle β ($\angle THM$).

(1 mark)

(c) Show that the wide tunnel (TM) will be approximately 134 metres long.

(3 marks)

(d) (i) Calculate the size of angle θ ($\angle HMT$).

You may use any of the information in part (b) and part (c).

(3 marks)

(ii) A safety requirement is that the angle (θ) formed by the narrow hole and the wide tunnel meeting at point M is *greater* than 15º.

Based on your calculation in part (d)(i), state whether or not angle θ ($\angle HMT$) meets the safety requirement.

(1 mark)

You may write on this page if you need more space to finish your answers to Topic 2. Make sure to label each answer carefully (e.g. Question 2(d)(i) continued).

Question 4 begins on page 12.

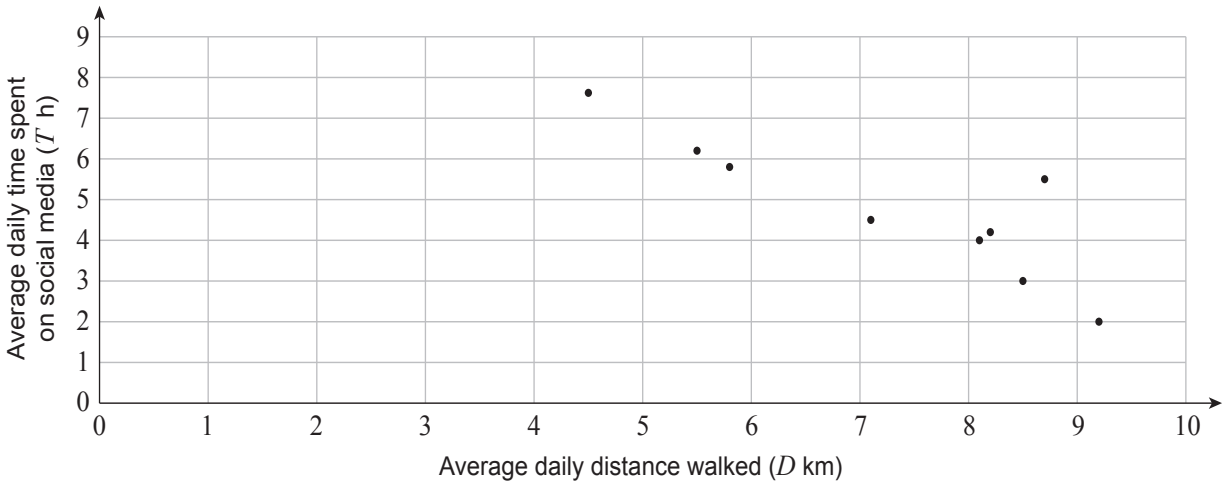
Question 4 (11 marks)

Each day for 1 week, 10 students recorded the distance that they walked and the time that they spent on social media. The recorded information was used to calculate the average daily distance walked and the average daily time spent on social media for each student. The calculated data are given in the table below.

Student	1	2	3	4	5	6	7	8	9	10
Average daily distance walked (D kilometres)	5.5	8.1	8.2	9.2	5.8	7.1	8.7	8.5	4.5	7.5
Average daily time spent on social media (T hours)	6.2	4.0	4.2	2.0	5.8	4.5	5.5	3.0	7.6	4.0

A scatter plot of the data for students 1 to 9 is given below.

Average daily time spent on social media versus average daily distance walked



(a) On the scatter plot, use an **X** to plot the data for student 10. (1 mark)

(b) Calculate the coefficient of determination (r^2), and comment on the strength of the relationship between the variables.

(2 marks)

(c) There is an outlier in the data, as a result of a recording error by a student.

(i) Circle the outlier on the scatter plot. (1 mark)

(ii) State which student is represented by the outlier that you have circled.

(1 mark)

(iii) State how the coefficient of determination (r^2) would be affected if the outlier was removed from the data.

(1 mark)

With the outlier removed, the equation of the least squares regression line (line of best fit) is:

$$T = -1.06D + 12.17$$

(d) Using the equation $T = -1.06D + 12.17$, calculate:

(i) the average time a student could be expected to spend on social media each day if the student walked an average of 7 kilometres each day.

(1 mark)

(ii) the average distance a student could be expected to walk each day if the student spent an average of 10 hours on social media each day.

(2 marks)

(e) Comment on the reasonableness of your calculation in part (d)(i). Justify your answer.

(2 marks)

Question 5 (9 marks)

A small group of residents has been lobbying their local council to change the speed limits in the council's streets. The council decides to survey a larger group of residents before considering any changes.

(a) One council member suggests conducting a survey of visitors to the council library on a weekday.

State *two* possible reasons why this sampling method could be biased.

(2 marks)

(b) Another council member suggests inviting users of the council's website to participate in an online survey.

(i) State *one* advantage of this sampling method.

(1 mark)

(ii) State *one* disadvantage of this sampling method.

(1 mark)

The council decides to conduct a survey of the residents who live in the council area, based on the suburbs in which the residents live.

(c) The council area is made up of four suburbs.

Fill in the *two* blank cells to complete the table below.

Suburb populations

Suburb	*Number of residents*	*Percentage of total residents*
Acacia Park	8 460	27.3%
Banksia Valley	6 358	20.5%
Jacaranda Grove	7 192	23.2%
Wattle Heights		
Total	**31 000**	

(2 marks)

(d) The council considers surveying 2000 residents using stratified sampling.

Using information in the table above, calculate the number of residents that should be surveyed in the suburb of Jacaranda Grove.

(1 mark)

(e) The council decides that the sample size is too large, and reduces the survey to 900 residents.

Calculate the number of residents that should now be surveyed in the suburb of Jacaranda Grove.

(2 marks)

Question 6 (10 marks)

Shufeng grows sweetcorn plants and wanted to investigate how a recommended fertiliser would affect the heights of his plants.

To test the fertiliser, Shufeng planted 40 sweetcorn seeds where they had the same soil, water, and sunlight conditions. He applied equal amounts of fertiliser to 20 of the plants once a week. He did not apply any fertiliser to the other 20 plants.

After 6 weeks, Shufeng measured the heights of all 40 sweetcorn plants to the nearest centimetre. The data for the 20 fertilised plants are in Table 1 below.

Table 1

Heights of fertilised plants (cm)				
38	42	39	45	39
41	35	27	36	42
40	32	31	47	26
35	38	32	36	31

(a) Using the information in Table 1 and Table 2, fill in the *six* blank cells to complete Table 2 below.

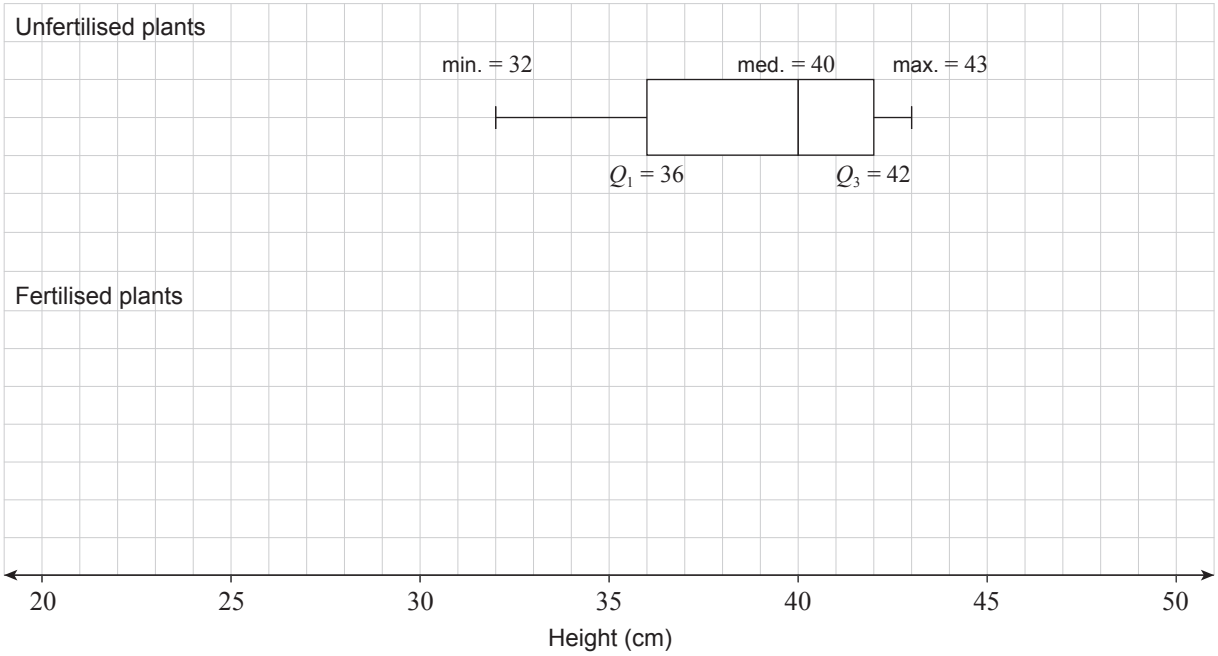
Table 2

Statistical measure	*Fertilised plants*	*Unfertilised plants*
mean	36.6	39
minimum		32
Q_1		36
median		40
Q_3		42
maximum	47	43
range	21	
interquartile range (IQR)	8.5	

(2 marks)

(b) Using the axis below, and the information in Table 2, draw and label a box-and-whisker diagram for the fertilised plants.

Height of sweetcorn plants



(3 marks)

(c) Complete the following sentences by circling the correct capitalised word:

(i) The mean and median heights for the fertilised plants are HIGHER / LOWER than the mean and median heights of the unfertilised plants. (1 mark)

(ii) On average, the fertilised plants have grown MORE / LESS than the unfertilised plants over the 6 weeks. (1 mark)

(iii) The range of heights of the fertilised plants is greater than the range of heights of the unfertilised plants. This indicates that plant growth is MORE / LESS consistent *without* fertiliser. (1 mark)

(d) (i) State whether or not Shufeng should apply this fertiliser to other sweetcorn plants in the future.

(1 mark)

(ii) Give a reason for your answer in part (d)(i).

(1 mark)

You may write on this page if you need more space to finish your answers to Topic 4. Make sure to label each answer carefully (e.g. Question 4(d)(ii) continued).

Question 7 (4 marks)

For 10 years, Diego has deposited $\$40$ each week into an investment account that has an interest rate of 2.75% per annum, compounded weekly.

(a) Calculate how much Diego has in the account after 10 years.

(2 marks)

(b) In the final year of his investment, Diego earned $\$620$ in interest from the investment account.

(i) Calculate how much tax Diego has to pay on the $\$620$ if his marginal tax rate is 37%.

(1 mark)

(ii) Calculate Diego's after-tax return from his investment for the final year.

(1 mark)

Question 8 (9 marks)

Mary has a lump sum of \$5300 to invest for 3 years. Her bank offers two options for investing the money:

- Option A is an account with a flat interest rate of 4.3% per annum.
- Option B is an account with an interest rate of 4.2% per annum, compounded monthly.

(a) (i) Calculate the total amount of interest that Mary would earn over the 3 years if she invested her lump sum in Option A.

(2 marks)

(ii) Calculate the total amount that Mary would have after 3 years if she invested her lump sum in Option A.

(1 mark)

(b) (i) Calculate the total amount that Mary would have after 3 years if she invested her lump sum in Option B.

(2 marks)

(ii) Calculate the total amount of interest that Mary would earn over the 3 years if she invested her lump sum in Option B.

(1 mark)

(c) Based on your calculations, state which option (A or B) Mary should choose if she wants to maximise her investment.

(1 mark)

(d) Calculate the flat interest rate that Mary would need to be offered if she wanted to earn a total of $700 in interest by investing her $5300 lump sum for 3 years.

(2 marks)

Question 9 (11 marks)

Sharla wants to buy a car. She borrows $\$8000$ from the bank and will make monthly repayments over 4 years, with an interest rate of 7% per annum, compounded monthly.

(a) Calculate how many monthly repayments Sharla will have to make over the 4 years.

(1 mark)

(b) Show that Sharla's repayments will be approximately $\$192$ per month.

(2 marks)

(c) Calculate how much Sharla will owe the bank after making repayments for 2 years.

(2 marks)

(d) (i) Calculate the total amount that Sharla will pay the bank over the 4 years.

(1 mark)

(ii) Calculate the total amount of interest that Sharla will pay the bank over the 4 years.

(1 mark)

Sharla's friend, Zoe, also wants to borrow $\$8000$ to buy a car. She goes to the same bank as Sharla and takes out a loan with the same interest rate of 7% per annum, but compounded fortnightly instead of monthly. Zoe halves the approximate monthly repayment of $\$192$ and agrees to pay $\$96$ per fortnight.

(e) (i) Calculate how long (in fortnights) Zoe will need to pay off her loan.

(2 marks)

(ii) Calculate the total amount that Zoe will pay the bank over the period of her loan.

(1 mark)

(f) State *one* reason why Zoe will pay less to the bank than Sharla will over the period of their loans.

(1 mark)

Question 10 (6 marks)

Kaleb has started his first full-time job and wants to make sure that his superannuation will provide him with a comfortable retirement. He has heard that the *current* superannuation goal for a single person to retire comfortably is $\$500\,000$.

(a) Kaleb and his employer will make a combined contribution of $\$267$ per fortnight into Kaleb's superannuation account, which returns 5.2% per annum, compounded fortnightly.

Show that it will take Kaleb approximately 30 years to accumulate $\$500\,000$ in his superannuation account.

(3 marks)

(b) (i) If the *current* superannuation goal for comfortable retirement is $\$500\,000$, calculate the *expected* superannuation goal in 30 years' time if inflation averages 2.1% per annum.

(2 marks)

(ii) State *one* recommendation that will allow Kaleb to achieve the amount calculated in part (b)(i) in his superannuation account.

(1 mark)

You may write on this page if you need more space to finish your answers to Topic 5. Make sure to label each answer carefully (e.g. Question 9(e)(i) continued).