DRAFT FOR ONLINE CONSULTATION

Specialist Mathematics

Subject Outline

Stage 1 and Stage 2

Version 1.0 – Draft for Online Consultation
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Government of South Australia
SACE Board of SA

CONTENTS

INTRODUCTION

SUBJECT DESCRIPTION

Specialist Mathematics is a 10-credit subject or a 20-credit subject at Stage 1, and a 20-credit subject at Stage 2.

Specialist Mathematics draws on and deepens students' mathematical knowledge, skills, and understanding and provides opportunities for students to develop their skills in using rigorous mathematical arguments and proofs, and using mathematical models. It includes the study of functions and calculus.

The subject leads to study in a range of tertiary courses such as mathematical sciences, engineering, computer science, and physical sciences. Students envisaging careers in related fields will benefit from studying this subject.

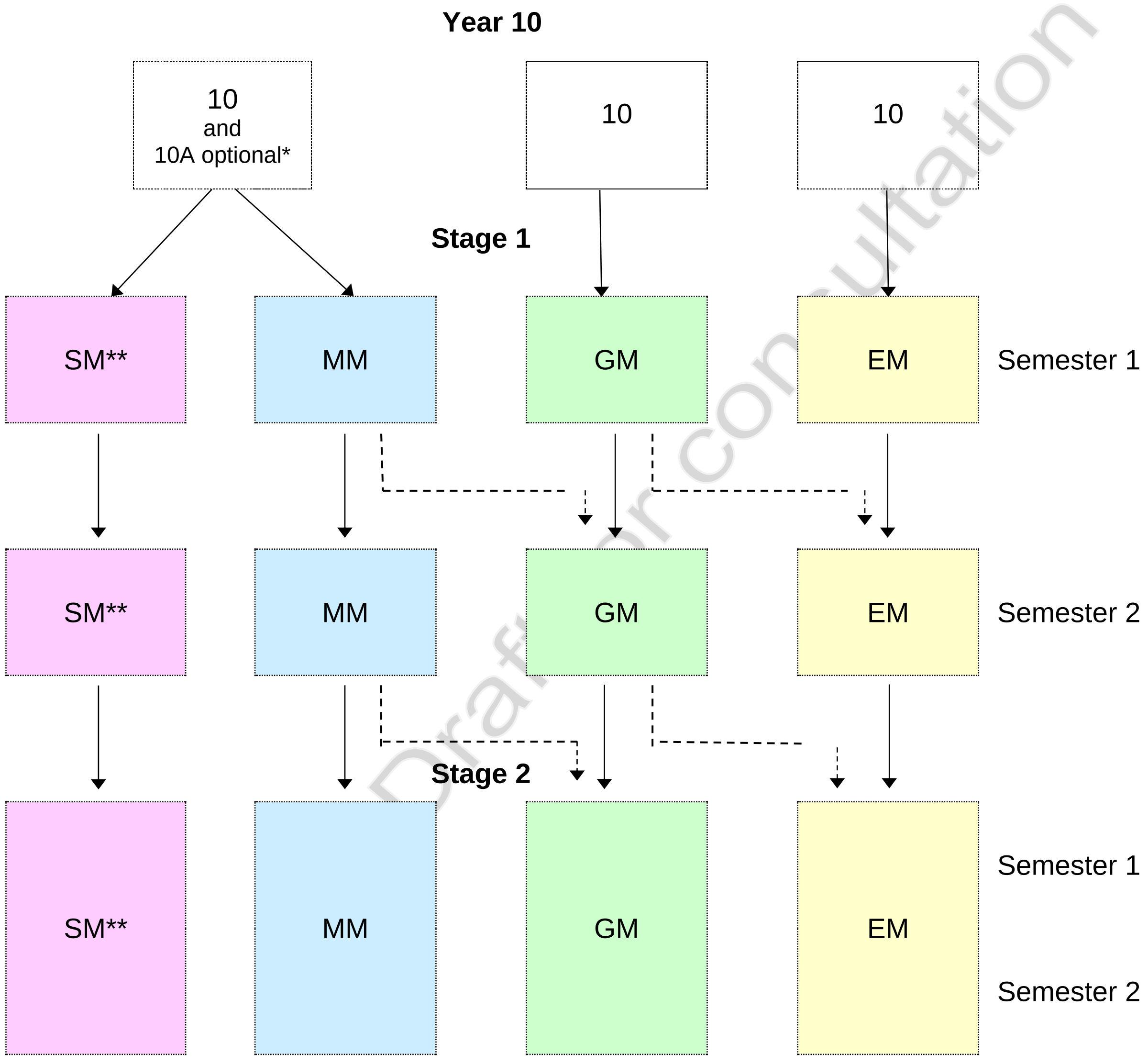
Specialist Mathematics is designed to be studied in conjunction with Mathematical Methods.

MATHEMATICAL OPTIONS

The diagram below represents the possible mathematical options that students might select as Stage 1 and Stage 2 subjects.

Solid arrows indicate the mathematical options that lead to completion of each subject at Stage 2. Dotted arrows indicate a pathway that may provide sufficient preparation for an alternative Stage 2 mathematics subject.

SM	Specialist Mathematics
MM	Mathematical Methods
GM	General Mathematics
EM	Essential Mathematics



Notes: * Although it is advantageous for students to study Australian Curriculum 10 and 10A in Year 10, the 10A curriculum *per se* is not a prerequisite for the study of Specialist Mathematics and Mathematical Methods. The essential aspects of 10A are included into the curriculum for Specialist Mathematics and Mathematical Methods.

** Mathematical Methods can be studied as a single subject; however, Specialist Mathematics is designed to be studied together with Mathematical Methods.

CAPABILITIES

The capabilities connect student learning within and across subjects in a range of contexts. They include essential knowledge and skills that enable people to act in effective and successful ways.

The SACE identifies seven capabilities. They are:
- literacy
- numeracy
- information and communication technology capability
- critical and creative thinking
- personal and social capability
- ethical understanding
- intercultural understanding.

Literacy

In this subject students develop their literacy capability by, for example:
- communicating mathematical reasoning and ideas for different purposes, using appropriate language and representations, such as symbols, equations, tables, and graphs
- interpreting and responding to appropriate mathematical language and representations
- analysing information and explaining mathematical results.

Mathematics provides a specialised language to describe and analyse phenomena. It provides a rich context for students to extend their ability to read, write, visualise, and talk about situations that involve investigating and solving problems.

Students apply and extend their literacy skills and strategies by using verbal, graphic, numerical, and symbolic forms of representing problems and displaying statistical information. Students learn to communicate their findings in different ways, using different systems of representation.

Numeracy

Being numerate is essential for participating in contemporary society. Students need to reason, calculate, and communicate to solve problems. Through the study of mathematics, they understand and use mathematical skills, concepts, and technologies in a range of contexts that can be applied to:
- using measurement in the physical world
- gathering, representing, interpreting, and analysing data
- using spatial sense and geometric reasoning
- investigating chance processes
- using number, number patterns, and relationships between numbers
- working with graphical, statistical and algebraic representations, and other mathematical models.

Information and communication technology capability

In this subject students develop their information and communication technology (ICT) capability by, for example:
- understanding the role of electronic technology in the study of mathematics
- making informed decisions about the use of electronic technology
- understanding the mathematics involved in computations carried out using technologies, so that reasonable interpretations can be made of the results.

Students extend their skills in using technology effectively and processing large amounts of quantitative information.

Students use ICT to extend their theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software relevant for study and/or workplace contexts. This may include tools for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, address problems, and operate systems in particular situations.

Critical and creative thinking

In this subject students develop critical and creative thinking by, for example:

- building confidence in applying knowledge and problem-solving skills in a range of mathematical contexts
- developing mathematical reasoning skills to think logically and make sense of the world
- understanding how to make and test projections from mathematical models
- interpreting results and drawing appropriate conclusions
- reflecting on the effectiveness of mathematical models, including the recognition of assumptions, strengths, and limitations
- using mathematics to solve practical problems and as a tool for learning
- making connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas
- thinking abstractly, making and testing conjectures, and explaining processes.

Problem-solving in mathematics builds students' depth of conceptual understanding and supports development of critical and creative thinking. Learning through problem-solving helps students when they encounter new situations. They develop their creative and critical thinking capability by listening, discussing, conjecturing, and testing different strategies. They learn the importance of self-correction in building their conceptual understanding and mathematical skills.

Personal and social capability

In this subject students develop their personal and social capability by, for example:

- arriving at a sense of self as a capable and confident user of mathematics through expressing and presenting ideas in a variety of ways
- appreciating the usefulness of mathematical skills for life and career opportunities and achievements
- understanding the contribution of mathematics and mathematicians to society.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for informed decision-making, active citizenship, and effective self-management. Students build their personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, and building adaptability, communication, and teamwork.

Students use mathematics as a tool to solve problems they encounter in their personal and working lives. They acquire a repertoire of strategies and build the confidence needed to:

- meet the challenges and innovations of a rapidly changing world
- be the designers and innovators of the future, and leaders in their fields.

Ethical understanding

In this subject students develop their ethical understanding by, for example:

- gaining knowledge and understanding of ways in which mathematics can be used to support an argument or point of view
- examining critically ways in which the media present particular perspectives

- sharing their learning and valuing the skills of others
- considering the social consequences of making decisions based on mathematical results
- acknowledging and learning from errors rather than denying findings and/or evidence.

Areas of ethical understanding relevant to mathematics include issues associated with ethical decision-making and working collaboratively as part of students' mathematically related explorations. They develop ethical understanding in mathematics through considering social responsibility in ethical dilemmas that may arise when solving problems in personal, social, community, and/or workplace contexts.

Intercultural understanding

In this subject students develop their intercultural understanding by, for example:

- understanding mathematics as a body of knowledge that uses universal symbols which have their origins in many cultures
- understanding how mathematics assists individuals, groups, and societies to operate successfully across cultures in the global, knowledge-based economy.

Mathematics is a shared language that crosses borders and cultures, and is understood and used globally.

Students read about, represent, view, listen to, and discuss mathematical ideas. They become aware of the historical threads from different cultures that have led to the current bodies of mathematical knowledge. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

ABORIGINAL AND TORRES STRAIT ISLANDER KNOWLEDGE, CULTURES, AND PERSPECTIVES

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of high-quality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

- providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
- recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society
- drawing students' attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
- promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

SACE NUMERACY REQUIREMENT

Completion of 10 or 20 credits of Stage 1 Specialist Mathematics with a C grade or better, or 20 credits of Stage 2 Specialist Mathematics with a C- grade or better, will meet the numeracy requirement of the SACE.

Stage 1
Specialist Mathematics

LEARNING SCOPE AND REQUIREMENTS

LEARNING REQUIREMENTS

The learning requirements summarise the knowledge, skills, and understanding that students are expected to develop and demonstrate through their learning in Stage 1 Specialist Mathematics.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. think mathematically by posing questions and solving problems, including making and testing conjectures
4. interpret results, draw conclusions, and determine the reasonableness of solutions in context
5. make discerning use of electronic technology
6. communicate mathematically and present mathematical information in a variety of ways.

CONTENT

Stage 1 Specialist Mathematics may be studied as a 10-credit subject or a 20-credit subject.

At Stage 1 students broaden their mathematical experience and increase their mathematical flexibility and versatility by developing mathematical arguments, proof, and problem solving in a variety of contexts.

Topics studied provide a blending of algebraic and geometric thinking. At Stage 1 there is a progression of content, applications, level of sophistication, and abstraction leading to Stage 2. For example, vectors in two dimensions are introduced in Stage 1 then studied for three-dimensional space in Stage 2.

Key concepts from Australian Curriculum 10A Mathematics have been incorporated into the Mathematical Methods and Specialist Mathematics subject outlines.

Stage 1 Specialist Mathematics consists of the following list of six topics:

Topic 1: Arithmetic and Geometric Sequences and Series
Topic 2: Geometry
Topic 3: Vectors in the Plane
Topic 4: Trigonometry
Topic 5: Matrices
Topic 6: Real and Complex Numbers.

Programming

For a 10-credit subject students study three of the topics.

For a 20-credit subject students study all six topics.

The topics selected can be sequenced and structured to suit individual cohorts of students. The suggested order of the topics in the list is a guide only. However, as Specialist Mathematics is designed to be studied in conjunction with Mathematical Methods, consideration should be given to appropriate sequencing of the topics across the two subjects.

Each topic consists of a number of subtopics. These are presented in the subject outline in two columns, as a series of ‘key questions and key concepts’ side by side with ‘considerations for developing teaching and learning strategies’.

The ‘key questions and key concepts’ cover the prescribed content for teaching, learning, and assessment in this subject. The ‘considerations for developing teaching and learning strategies’ are provided as a guide only.

A problem-based approach is integral to the development of the mathematical models and associated key concepts in each topic. Through key questions students deepen their understanding of concepts and processes that relate to the mathematical models required to address the problems posed.

The ‘considerations for developing teaching and learning strategies’ present problems and guidelines for sequencing the development of the concepts. They also give an indication of the depth of treatment and emphases required.

Students use electronic technology, where appropriate, to enable complex problems to be solved efficiently.

Topic 1: Arithmetic and Geometric Sequences and Series

Arithmetic and geometric sequences and series and their applications are introduced and their recursive definitions applied. They provide examples of applications such as growth and decay.

Sub-topic 1.1: Arithmetic Sequences and Series

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Are there examples of sequences of numbers where there is a constant amount of increase (or decrease) in the values?	Students are presented with a variety of situations in which arithmetic sequences occur (e.g. simple interest).
• Find the generative rule for a sequence, both recursive and explicit, using $t_{n+1} = t_n + d$ and $t_n = t_1 + (n-1)d$	The structure of a sequence (as a starting value continually augmented by a constant adder) can be clearly seen in the simple programs used to generate the terms on a calculator or computer.
• Determine the value of a term or the position of a term in a sequence	Questions can be framed in the context of the growth situation being investigated.
• Describe the nature of the growth observed	Graphs are used here, and links between the algebraic rule and the shape of the graph are emphasised.
• $S_n = \frac{n}{2}(t_1 + t_n) = \frac{n}{2}(2t_1 + (n-1)d)$	The sum of series is useful to find; for example, how many seats in a section of a stadium.

Sub-topic 1.2: Geometric Sequences and Series

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Are there examples of sequences of numbers where there is a constant ratio of increase (or decrease) in the values?	Some examples are compound interest, depreciation, half-lives of radioactive material, simple population models (bacteria, locusts, etc.).
• Recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_1$	
• Use the formula $t_n = r^{n-1}t_1$ for the general form of a geometric sequence and recognise its exponential nature.	
• Understand the limiting behaviour as $n \to \infty$ of the terms t_n and its dependence on the value of the common ratio r.	Graphs are used and links between the algebraic rule and the shape of the graph are emphasised.
• Establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence	Finding the total value of the investment after a given number of periods requires the summing of this sequence. At this point the general formula for the sum of a geometric series can be derived and used to answer questions such as 'How much will I have after…?' or 'How much do I need to put away each month?' or 'How long will it take me to save…?'
• If |r|< 1 then $S_n \to \frac{t_1}{1-r}$ as $n \to \infty$	Investigate the consequence of $\|r\| < 1$ $1 + x + x^2 + \ldots + x^n = \frac{1 - x^{n+1}}{1 - x}$ And if $\|x\| < 1$ then the right hand side tends to $\frac{1}{1-x}$ as $n \to \infty$

Topic 2: Geometry

The context of this topic is the geometry of planar figures. The focus is on forming and testing hypotheses about their properties, which if proved to be true become theorems. Students form ideas about which properties of a figure might be universal, and test enough examples to be convinced that their idea is correct before they attempt a formal proof. For testing to be effective and efficient, electronic technology is used whenever possible.

Sub-topic 2.1: Circle Properties

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What properties are found when angles, lines, and polygons are drawn on and within circles?	Questions can be posed to guide the investigation of the properties of a circle; for example:
• Chord and tangent properties – Radius and tangent property – Angle between tangent and chord (alternate segment theorem) – Length of the two tangents from an external point • Properties of angles within circles – Angle subtended at the centre is twice the angle subtended at the circumference by the same arc – Angles at the circumference subtended by the same arc are equal – Opposite angles in a cyclic quadrilateral are supplementary – Angle in a semicircle is a right angle – Chords of equal length subtend equal angles at the centre – Converses of the above properties – Intersecting chords theorem, including internal and external intersections, and the special case of a tangent and chord through an external point	• How can a right angle be marked out with no measuring equipment other than some pegs and lengths of string? • In a penalty shot or conversion attempt in rugby union, where is the best place to stand on a circular arc to get the widest angle of attack at the goal? Or, where is the best place to sit in a row at the theatre? • Can a circle always be inscribed round a rectangle? How? Can this be done with any other kinds of parallelograms? Why not? What kinds of other quadrilaterals do have an inscribing circle? Do they have special properties?

Sub-topic 2.2: The Nature of Proof

Key Questions and Key Concepts

How can the properties discovered by investigation be proved?

- Justification of properties of circles

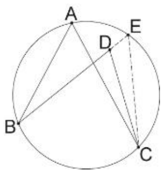
Considerations for Developing Teaching and Learning Strategies

The hypotheses developed when answering questions about circle properties could be tested first by trial and error, preferably using electronic technology.

Once this testing process has been completed for each property, students are led through a set of logical steps that justify the property for all cases.

The nature of proof:

- use of similarity and congruence is required in some proofs
- use implication, converse, equivalence, negation, contra positive
- use examples and counter-examples
- use proof by contradiction, for example, to prove that 'Angles at the circumference subtended by the same arc are equal'. That is, consider 3 concyclic points *A*, *B*, *C* with a fourth, *E*, on the circle as shown



Given $\angle BAC = \angle BDC$, prove *D* must be at position *E*.

Topic 3: Vectors in the Plane

The study of vectors in the plane provides new perspectives for working with two-dimensional space. Vectors are used to specify quantities that have size (magnitude) and direction. These quantities include velocity, force, acceleration, displacement, and are used in fields such as physics and engineering. The topic includes vector operations, their applications, and their use in proving results in geometry.

Sub-topic 3.1: Vector Operations

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Representing vectors in the plane by directed line segments. What are the rules for vector operations?	Define magnitude and direction of a vector. Examples include displacement and velocity.
• Vector addition and subtraction • Scalar multiples of a vector • Applications of scalar multiples: parallel vectors, ratio of division	Drawing software provides an excellent environment for manipulating vectors and appreciating the triangular nature of vector addition. The problem of moving a robot from A to B, using only the repetition of a small number of possible movements (in vector form), reinforces the concept that the order of addition changes the path followed but not the resultant vector. In the same context, the concept of scalar multiples as several steps using the same vector is made clear. Similarly, negative multiples simply involve moving backwards along an arrow.

Sub-topic 3.2: Component and Unit Vector Forms

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can a vector be described in the Cartesian plane?	In sub-topic 3.1 students have operated with vectors without reference to a grid system that requires the vectors to be defined in component form.
• Use ordered pair notation and column vector notation • Convert a vector into component and unit vector forms	Students practise vector addition and scalar multiplication. They explore the concept of component vectors, column vectors, combinations of unit vectors, and position vectors.
• Determine length and direction of a vector from its components	Concepts from physics can be explored, for example, the effects on horizontal motion of forces from different directions and the analysis of static systems (in conjunction with force table experiments if the equipment is available).

Sub-topic 3.3: Projections

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How much of one vector is operating in the direction of another?	In sub-topic 3.2, students concentrated on the *x* and *y* components of a vector, which are projections of the vector onto the axes. In this sub-topic, students work out the projection of one vector onto another. For instance, how much help does an aeroplane travelling north gain from a south-westerly wind, and what are the consequences for fuel consumption? How fast can a person sail on a particular course in a specified wind and with the tide running? Shadows also provide a practical context for this work.
• The dot (scalar) product • The angle between two vectors • Perpendicular vectors • Parallel vectors	Calculating the projection algebraically leads to the definition of the dot product and the formula for the cosine of the angle between the two vectors. From here the implications of a zero dot product and other properties can be investigated.

Sub-topic 3.4: Geometric Proofs using Vectors

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Can vector concepts be used to prove results in geometry?	Understanding of scalar multiples and the dot product provides a powerful tool for proving results about parallelism and perpendicularity.
Geometric proofs using vectors in the plane include: • The diagonals of a parallelogram meet at right angles if and only if it is a rhombus • Midpoints of the sides of a quadrilateral join to form a parallelogram • The sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides.	

Topic 4: Trigonometry

In this topic students extend their understanding of trigonometric functions developed in Stage 1 Mathematical Methods. Students model circular motion in the familiar contexts of, for example, ferris wheels, merry-go-rounds, and bicycle wheels. These functions are fundamental to understanding many natural oscillatory phenomena such as lunar illumination, tidal variation, and wave propagation.

Sub-topic 4.1: Trigonometric Functions

Key Questions and Key Concepts	**Considerations for Developing Teaching and Learning Strategies**
How can periodic phenomena be modelled mathematically?	Note that Stage 1 Mathematical Methods Topic 2, sub-topic 2.3 also considers trigonometric functions and must precede this topic.
• The general function $y = A\sin B(x - C) + D$ • Extend to: $y = A\cos B(x - C) + D$ $y = A\tan B(x - C) + D$	Using graphing technology, students can explore the effects of the four control numbers (individually and in combination) in the general sinusoidal model $y = A\sin B(x - C) + D$ on transforming the graph of $y = \sin x$. Students explore fitting functions of this form to their data from 'circle' examples such as the Ferris wheel. They relate the values they find for *A*, *B*, *C*, and *D* back to the parameters given in the real situation.
• Sketch graphs of sinusoidal functions	Having explored the effects of the four parameters, students sketch the graphs of sinusoidal functions without using technology (except to check their answers).
• Solve trigonometric equations of the form $y = k$ (where y is one of the functions above), finding all solutions.	Students use the resulting equations to answer questions such as 'How high off the ground will you be if the wheel stops after 3 minutes?' or 'For how long in each revolution is a person able to see over the 3-metre fence round the Ferris wheel?' Students solve these equations both graphically (using technology) and algebraically.

Sub-topic 4.2: Trigonometric Identities

Key Questions and Key Concepts

What special relationships can be observed by examining the sine and cosine functions and their behaviour in the unit circle?

- $\sin(-x), \cos(-x)$, $\sin^2 x + \cos^2 x, \sin 2x$, $\cos 2x, \sin \frac{1}{2}x, \cos \frac{1}{2}x$

  In terms of sin x, cos x

- $\cos(A \pm B)$, and hence $\sin(A \pm B)$

In terms of sin *A*, cos *A,* sin *B*, cos *B*

- Conversion of $A\sin x + B\cos x$ into the form $k\sin(x + \alpha)$

- The reciprocal trigonometric functions:

  $\sec\theta$, $\operatorname{cosec}\theta$, $\cot\theta$

Considerations for Developing Teaching and Learning Strategies

Students are guided through the deduction of many of these useful identities by looking at the unit circle. They discover others by comparing their graphs. The formula for $\cos(A - B)$ can be derived from the unit circle, using the cosine rule. The other angle sum formulae follow from it, using the identities already learnt.

Students prove these basic identities and apply them algebraically, to establish such results as

$$\cos 3x = 4\cos^3 x - 3\cos x$$

Students derive the results:

$$\text{for } k > 0,\ k = \sqrt{A^2 + B^2},\ \cos\alpha = \frac{A}{k},\ \sin\alpha = \frac{B}{k}$$

Working from their definitions, students sketch graphs and simple transformations of these new reciprocal functions.

Topic 5: Matrices

Matrices provide new perspectives for working with two-dimensional space. The study of matrices includes extension of matrix arithmetic to applications such as linear transformations of the plane and cryptography.

Sub-topic 5.1: Matrix Arithmetic

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is a matrix? • Order of matrices	Matrices can provide a useful representation of information in a wide range of contexts. From a given context students put numerical information in tabular form with the rows and/or columns labelled to identify what the elements represent.
What operations can be applied to matrices? • Addition and subtraction • Scalar multiplication • Matrix multiplication	Matrix addition is both commutative and associative. Matrix multiplication is associative but not commutative. Matrices can only be multiplied under certain conditions.
The identity matrix for matrix multiplication	The identity matrix I should be introduced via its properties: $AI = IA = A$.
What is the inverse of a square matrix A?	The matrix A^{-1} is defined by the property $$A^{-1}A = AA^{-1} = I$$ Not all square matrices have an inverse; those which do not are called singular. The inverse is not defined for non-square matrices.
• Calculating the inverse of a 2 x 2 matrix when it exists.	The formula $$A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$ for a matrix $$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$
• The determinant of a 2 x 2 matrix and its significance.	The number ad-bc is called the determinant of A and is denoted detA. If detA = 0, the inverse does not exist.
How can matrix inverses be used? • Find the unique solution to matrix equations of the form $$AX = B \text{ or } XA = B,$$ if it exists	For the system $AX = B$, if A^{-1} does exist, then $X = A^{-1}B$.

Sub-topic 5.2: Transformations in the Plane

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What are some applications of matrices? Transformations in the plane and their description in terms of matrices. • Translations and their representation as column vectors, that is 2x1 matrices • Define and use basic linear transformations • Consider dilations of the form $(x, y) \to (\lambda_1 x, \lambda_2 y)$, rotations about the origin where $\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ and reflection in a line which passes through the origin where $\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} \cos(2\theta) & \sin(2\theta) \\ \sin(2\theta) & -\cos(2\theta) \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$ • Apply transformations to points in the plane and geometric objects • Define and use composition of linear transformations and the corresponding matrix products • Establish geometric results by matrix multiplications • Show that the combined effect of two reflections in lines through the origin is a rotation.	The representation of points in the plane as a coordinate pair can be considered as an example of matrix notation (1x2 for a row vector and 2x1 for a column vector).
Can transformations be 'undone'? • Define and use inverses of linear transformations and the relationship with the matrix inverse • Examine the relationship between the determinant and the effect of a linear transformation on area • Note that if the determinant of a matrix is zero, then the corresponding transformation has no inverse	An example is that of matrix codes: arithmetic via a matrix and decrypting via its inverse, using modulo 26. Not every 2x2 matrix will be suitable. The value of the determinant needs to be considered to ensure that decryption is possible. An example with no inverse: Projection on an axis e.g. $\begin{bmatrix} x \\ y \end{bmatrix} \to \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$

Topic 6: Real and Complex Numbers

This topic is a continuation of students' study of numbers. Mathematical induction is introduced as a way of proving a given statement for all integers. Complex numbers extend the concept of the number line to the two-dimensional complex plane. This topic introduces operations with complex numbers, their geometric representation, and their use in solving problems that cannot be solved with real numbers alone.

Sub-topic 6.1: The Number Line

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
The number line represents all real numbers. What are some properties of special subsets of the reals? • Rational and irrational numbers	Consider surds and their operations • express rational numbers as terminating or eventually recurring decimals and vice versa • prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $\log_2 5$.
• Prove simple results involving numbers	Some examples include: • sum of two odd numbers is even • product of two odd numbers is odd • the sum of two rational numbers is rational.
Is there a convenient notation for 'pieces' of the number line? • Interval notation.	Use of square brackets and parentheses to denote intervals of the number line that include or exclude the endpoints. For example, the set of numbers x such that $a < x \leq b$ is denoted $(a, b]$.

Sub-topic 6.2: Principle of Mathematical Induction

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can a statement concerning all positive integers be proved?	
An introduction to proof by mathematical induction: • understand the nature of inductive proof including the 'initial statement' and inductive step	Formal proofs are expected, i.e. Let there be associated with each positive integer *n*, a proposition *P*(*n*). If *P*(1) is true, and for all *k*, *P*(*k*) is true implies *P*(*k* + 1) is true, then *P*(*n*) is true for all positive integers *n*. • prove results for sums, such as 1 + 4 + 9 … + n^2 = $\frac{n(n+1)(2n+1)}{6}$ for any positive integer *n* • prove divisibility results, such as 3^{2n+4} - 2^{2n} is divisible by 5 for any positive integer *n*

Sub-topic 6.3: Complex Numbers

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Why were complex numbers 'invented'? Define the imaginary number i as a solution to the quadratic equation $x^2+1=0$.	Analogy can be drawn with the extension of the natural numbers to the integers, integers to rationals, rationals to reals — in each case with a view to ensuring that certain kinds of equations have solutions. A parallel is drawn with the arithmetic of surds and how surds arise — for example, solving the quadratic equation $x^2+2x-1=0$ leads to numbers of the form $\{a+b\sqrt{2} : a, b \text{ rational}\}$.
Operations with complex numbers • Real and imaginary parts • Complex conjugates • Arithmetic with complex numbers • $i^2=-1$	• define the imaginary number as a root of the equation $x^2 = -1$ • use complex numbers in the form $a+bi$ where a and b are the real and imaginary parts • determine and use complex conjugates • perform complex-number arithmetic: addition, subtraction, multiplication and division Students can add and subtract complex numbers using the usual rules of arithmetic and algebra. Multiplication calls for the same approach with the additional need to simplify i^2 using the fact that $i^2=-1$. Division of complex numbers can be introduced by presenting a complex product such as $(2-i)(1+i)=3+i$, inferring the result for $\dfrac{3+i}{2-i}$ and then asking: How can this result be obtained through calculation?

Sub-topic 6.4: The Complex (Argand) Plane

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can complex numbers be represented geometrically?	The Cartesian plane as extension of the real number line to two dimensions.
• Cartesian form on the Argand Diagram	Correspondence between the complex number $a+bi$, the coordinates (a,b), and the vector $[a,b]$.
• Vector addition in the complex plane	Complex number addition corresponds to vector addition via the parallelogram rule. Complex numbers can be regarded as two-dimensional vectors with rules for multiplication and division to supplement addition and subtraction.
• Locating complex conjugates in the complex plane (Argand Diagram)	Relative positions of $z = a + bi$ and its conjugate; their sum is real and difference purely imaginary.
• Modulus	Recognising that $\lvert z\rvert = \sqrt{a^2+b^2}$ represents the length of a complex number when represented as a vector.

Sub-topic 6.5: Roots of Equations

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What has been the advantage of introducing complex numbers? • The introduction of i enables the solution of all real quadratic equations and the factorisation of all quadratic polynomials into linear factors.	Consider the formula for solution of quadratic equations, with emphasis on arithmetic involving i. When the solutions of a real quadratic equation are complex they are conjugates.

ASSESSMENT SCOPE AND REQUIREMENTS

Assessment at Stage 1 is school based.

EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 1 Specialist Mathematics:

Assessment Type 1: Skills and Applications Tasks
Assessment Type 2: Mathematical Investigation.

For a 10-credit subject, students should provide evidence of their learning through four assessments. Each assessment type should have a weighting of at least 20%.

Students complete:

- at least two skills and applications tasks
- at least one mathematical investigation.

For a 20-credit subject, students should provide evidence of their learning through eight assessments. Each assessment type should have a weighting of at least 20%.

Students complete:

- at least four skills and applications tasks
- at least two mathematical investigations.

It is anticipated that from 2018 all assessments will be submitted electronically.

ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by teachers to:

- clarify for the student what he or she needs to learn
- design opportunities for the student to provide evidence of his or her learning at the highest level of achievement.

The assessment design criteria consist of specific features that:

- students need to demonstrate in their evidence of learning
- teachers look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships

CT2 Selection and application of techniques and algorithms to solve problems in a variety of contexts

CT3 Development and application of mathematical models

CT4 Use of electronic technology to find solutions to mathematical problems.

Reasoning and Communication

The specific features are as follows:

RC1 Interpretation and evaluation of mathematical results, with an understanding of their reasonableness and limitations

RC2 Knowledge and use of appropriate mathematical notation, representations, and terminology

RC3 Communication of mathematical ideas and reasoning, to develop logical arguments

RC4 Development and testing of valid conjectures.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks

For a 10-credit subject, students complete at least two skills and applications tasks.

For a 20-credit subject, students complete at least four skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher.

Students find solutions to mathematical problems that may:

- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine, analytical, and/or interpretative problems.

Students provide explanations and arguments, and use correct mathematical notation, terminology, and representation throughout the task. They may be required to use electronic technology appropriately to aid and enhance the solution of some problems.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

Assessment Type 2: Mathematical Investigation

For a 10-credit subject, students complete at least one mathematical investigation.

For a 20-credit subject, students complete at least two mathematical investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers may give students a clear, detailed, and sequential set of instructions for part of the investigation or to initiate the investigation, or may provide guidelines for students to develop contexts, themes, or aspects of their own choice. Teachers should give some direction about the appropriateness of each student's choice and guide and support students' progress in a mathematical investigation.

A mathematical investigation may provide an opportunity for students to work collaboratively to achieve the learning requirements. If an investigation is undertaken by a group, students explore the problem and gather data together then develop a model or solution individually. Each student must submit an individual report.

Teachers may need to provide support and clear directions for the first mathematical investigation. Where students undertake more than one investigation, subsequent investigations could be less directed and set within more open-ended contexts.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. Computer Algebra Systems, spreadsheets, statistical packages) to enhance their investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may provide an important focus. From these, students may recognise different patterns or structures. Notation, terminology, forms of representation of information gathered or produced, calculations, technological skills, and results are important considerations.

Students complete a report on the mathematical investigation.

In the report, they formulate and test conjectures, interpret and justify results, draw conclusions, and give appropriate explanations and arguments.

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem
- a bibliography and appendices, as appropriate.

The format of an investigation report may be written or multimodal.

Each investigation report should be a maximum of 8 pages if written, or the equivalent in multimodal form.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E. Each level of achievement describes the knowledge, skills and understanding that teachers refer to in deciding how well a student has demonstrated his or her learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student’s completion of study of a subject, the teacher makes a decision about the quality of the student’s learning by:

- referring to the performance standards
- taking into account the weighting given to each assessment type
- assigning a subject grade between A and E.

Performance Standards for Stage 1 Specialist Mathematics

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation and critical evaluation of mathematical results, with an in-depth understanding of their reasonableness and possible limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Effective development and testing of valid conjectures.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and use of mathematical techniques and algorithms to find accurate solutions to routine and some complex problems in a variety of contexts. Mostly successful development and application of mathematical models to find accurate solutions. Mostly appropriate and effective use of electronic technology to find accurate solutions to routine and some complex problems.	Mostly appropriate interpretation and some critical evaluation of mathematical results, with some depth of understanding of their reasonableness and possible limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop logical arguments. Mostly effective development and testing of valid conjectures.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and use of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Some development and successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation and evaluation of mathematical results, with some understanding of their reasonableness and possible limitations. Generally appropriate use of mathematical notation, representations, and terminology, with some inaccuracies. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures.
D	Basic knowledge and some understanding of concepts and relationships. Some effective selection and use of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation and attempted evaluation of mathematical results, with some awareness of their reasonableness and possible limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted development or testing of a reasonable conjecture.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited use of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology with limited accuracy in solving routine problems.	Limited interpretation or evaluation of mathematical results, with limited awareness of their reasonableness and possible limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to develop or test a conjecture.

ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (wwww.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement in the school assessment are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 1 are available on the SACE website (www.sace.sa.edu.au).

SUPPORT MATERIALS

SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au) Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website. (www.sace.sa.edu.au).

Stage 2
Specialist Mathematics

LEARNING SCOPE AND REQUIREMENTS

LEARNING REQUIREMENTS

The learning requirements summarise the key skills, knowledge and understanding that students are expected to develop and demonstrate through learning in Stage 2 Specialist Mathematics.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. think mathematically by posing questions and solving problems; developing and evaluating models; making, testing, and proving conjectures
4. interpret results, draw conclusions, and determine the reasonableness of solutions in context
5. make discerning use of electronic technology to solve problems and refine and extend mathematical knowledge
6. communicate mathematically and present mathematical information in a variety of ways.

CONTENT

Stage 2 Specialist Mathematics is a 20-credit subject.

The topics in Stage 2 extend students' mathematical experience and their mathematical flexibility and versatility, in particular, in the areas of complex numbers and vectors. The general theory of functions, differential equations, and dynamic systems provide opportunities to analyse the consequences of more complex laws of interaction.

Specialist Mathematics topics provide different scenarios for incorporating mathematical arguments, proofs, and problem solving.

Stage 2 Specialist Mathematics consists of the following list of five topics:

Topic 1: Complex Numbers

Topic 2: Functions and Sketching Graphs

Topic 3: Vectors in Three Dimensions

Topic 4: Integration Techniques and Applications

Topic 5: Rates of Change and Differential Equations.

The suggested order of the topics in the list is a guide only; however, students study all five topics.

Each topic consists of a number of subtopics. These are presented in the subject outline in two columns, as a series of 'key questions and key concepts' side by side with 'considerations for developing teaching and learning strategies'.

The 'key questions and key concepts' cover the prescribed content for teaching, learning, and assessment in this subject. The 'considerations for developing teaching and learning strategies' are provided as a guide only.

A problem-based approach is integral to the development of the mathematical models and associated key concepts in each topic. Through key questions teachers can develop the key concepts and processes that relate to the mathematical models required to address the problems posed.

The ‘considerations for developing teaching and learning strategies’ present problems for consideration, and guidelines for sequencing the development of the concepts. They also give an indication of the depth of treatment and emphases required.

Although the material for the external examination will be based on the ‘key questions and key concepts’ outlined in the five topics, the ‘considerations for developing teaching and learning strategies’ may provide useful contexts for examination questions.

Students us electronic technology, where appropriate, to enable complex problems to be solved efficiently.

Topic 1: Complex Numbers

The Cartesian form of complex numbers was introduced in Stage 1, and the study of complex numbers is now extended to the polar form. The arithmetic of complex numbers is developed and their geometric interpretation as an expansion of the number line into a number plane is emphasised. Their fundamental feature – that every polynomial equation has a solution over the complex numbers – is reinforced and de Moivre's theorem is used to find nth roots.

Sub-topic 1.1: Cartesian and Polar Forms

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Understanding complex numbers involves: • real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number • Cartesian form • arithmetic using Cartesian forms	
Is the Cartesian form always the most convenient representation for complex numbers? Is it the most convenient form for multiplying complex numbers?	Consider describing sets of points in the complex plane, such as circular regions or regions bounded by rays from the origin. For example $\lvert z-i\rvert \leq 2,\ \arg(z)=\frac{\pi}{4}$
How to do arithmetic using polar form • Conversion between Cartesian form and polar form	The conversions $x=r\cos\theta, y=r\sin\theta,$ where $r=\sqrt{x^2+y^2}$ and $\tan\theta=\left(\frac{y}{x}\right)$, and $-\pi < \theta \leq \pi$ along with $\cos\theta=\left(\frac{x}{\sqrt{x^2+y^2}}\right)$ and $\sin\theta=\left(\frac{y}{\sqrt{x^2+y^2}}\right)$, and their use in converting between Cartesian form and polar form. Calculators can be used, both to check calculations and enable students to consider examples that are not feasible by hand.
• The properties $\lvert z_1z_2\rvert=\lvert z_1\rvert\lvert z_2\rvert$ $\arg(z_1z_2)=\arg(z_1)+\arg(z_2)$ $cis\theta_1.cis\theta_2=cis(\theta_1+\theta_2)$ $\frac{\text{cis}\,\theta_1}{\text{cis}\,\theta_2}=\text{cis}(\theta_1-\theta_2)$ They are the basis on which multiplication by $r\,\text{cis}\,\theta$ is interpreted as dilation by r and rotation by θ	These properties make polar form the most powerful representation for dealing with multiplication. Students observe that the real and imaginary parts of the identity $\text{cis}(\theta_1+\theta_2)=\text{cis}\,\theta_1.\text{cis}\,\theta_2$ are the addition of angles formulae for cosine and sine. Thus complex multiplication encodes these trigonometric identities in a remarkable and simple way.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
• The utility of the polar form in calculating multiplication, division, and powers of complex numbers, and the geometrical interpretation of these	The properties $\lvert z_1 z_2 \dots z_n \rvert = \lvert z_1 \rvert \lvert z_2 \rvert \dots \lvert z_n \rvert$ and $\operatorname{cis}(\theta_1 + \theta_2 + ... + \theta_n) = \operatorname{cis}\theta_1 . \operatorname{cis}\theta_2 \dots \operatorname{cis}\theta_n$ could be argued by mathematical induction. The geometric significance of multiplication and division as dilation and rotation is emphasised, along with the geometric interpretation of modulus as distance from the origin. Geometry from Stage 1 Specialist Mathematics can be used — for example, using properties of a rhombus to determine the polar form of $z + 1$ from that of z when z is on the unit circle.
• Prove and use de Moivre's Theorem for integral powers using mathematical induction	Note also the special case $\lvert z^n \rvert = \lvert z \rvert^n$ when all the zs are equal, and $\operatorname{cis}(n\theta) = (\operatorname{cis}\theta)^n$ when all the θs are equal, which is de Moivre's theorem for positive integral n. Extension to negative powers via
• Extension to negative integral powers and fractional powers	$\operatorname{cis}(-n\theta) = \frac{1}{\operatorname{cis}(n\theta)} = \frac{1}{(\operatorname{cis}\theta)^n} = (\operatorname{cis}\theta)^{-n}$

Sub-topic 1.2: The Complex (Argand) Plane

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What connections are there with previous work on vectors and linear transformations in the plane?	
• Examine and use addition of complex numbers as vector addition in the complex plane	Correspondence between the complex number $a+bi$, the coordinates (a,b), and the vector $[a,b]$. Complex number addition corresponds to vector addition via the parallelogram rule. Complex numbers can be regarded as two-dimensional vectors with rules for multiplication and division to supplement addition and subtraction.
• Examine and use multiplication as a linear transformation in the complex plane	Multiplying a complex number w by $z = r\text{cis}\,\theta$ dilates w by a factor of r and rotates w through an angle of θ.
• Multiplication by i as anticlockwise rotation through a right angle	Although this follows from the addition formula for $\text{cis}\,\theta$, it can be demonstrated directly that: $i(x+iy) = -y+ix$ shows that multiplication by i sends (x,y) to $(-y,x)$, which can be shown to be the effect on the coordinates of rotating a point about the origin anticlockwise through a right angle. Note the connection between matrices and vectors in Stage 1 Topic 5.
• Geometric notion of $\lvert z-w \rvert$ as the distance between points in the plane representing them	
• Triangle inequality	The triangle inequality may be used when considering the distance between points in the complex plane. Extension to the sum of several complex numbers can be argued by mathematical induction.
• Geometrical interpretation and solution of equations describing circles, lines, rays, and inequalities describing associated regions; obtaining equivalent Cartesian equations and inequalities where appropriate	Graphical solution of equations and inequalities such as $\lvert z\rvert < 2, \lvert z-3i\rvert = 4, \lvert z+i\rvert = \lvert z-1\rvert, \lvert z-1\rvert = 2\lvert z-i\rvert$ $\theta = \frac{\pi}{4}, \frac{\pi}{4} \le \arg z \le \frac{3\pi}{4}, Re(z) > Im(z), 0 < Im(z) < 1$ strengthen the geometric interpretation. An important aim is the conversion of such equations and inequalities into Cartesian form in the case of circles and lines, through geometric understanding of the descriptions used above directly in terms of modulus and argument. Dynamic geometry software can be used as an aid. Students can investigate various polar graphs, using graphing technology.

Sub-topic 1.3: Roots of Complex Numbers

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Can polar form be used to efficiently find all solutions of the simplest n^{th} degree polynomial?	
• Solution of $z^n = c$ for complex c but in particular the case $c = 1$	
• Finding solution of nth roots of complex numbers and their location in the complex plane	As a particular example of the use of de Moivre's theorem, recognise the symmetric disposition of the n^{th} roots of unity in the complex plane. The fact that their sum is zero can be linked in the vector section of the subject outline to the construction of a regular n-gon.

Sub-topic 1.4: Factorisation of Polynomials

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
Operations on real polynomials: polynomials can be added and multiplied. What can be said about division of polynomials?	Briefly consider the multiplication process. The division algorithm using long division or synthetic division or the multiplication process with inspection. The use of undetermined coefficients and equating coefficients in factoring when one factor is given.
• Roots, zeros, factors • Prove and apply the Factor and Remainder theorem; its use in verifying zeros	Explore and understand the correspondence between the roots of a polynomial equation, the zero of a polynomial, and the linear factor of a polynomial.
• Consider conjugate roots in factorisation of cubics and quartics with real coefficients (given a zero)	Use of conjugate pairs to create real quadratic factor of a given real polynomial. (The existence of a formula for the roots of a cubic and a quartic, but not polynomials of higher degree, can be mentioned as background.)
• Solve simple real polynomial equations	Real polynomials can be factored into real linear and quadratic factors, and into linear factors with the use of complex numbers. Explore the connection between the zeros and the shape and position of the graph of a polynomial. There are many opportunities to make use of graphing technology. Special examples: $x^n = 1$ or -1 as solved above by de Moivre's theorem; the factorisation of $x^3 \pm a^3, x^4 \pm a^4,$ and so on, as an illustration of the use of the remainder theorem. The statement of the fundamental theorem can be considered to answer the question 'Why were complex numbers "invented"?' Though not every real polynomial of degree n has n real zeros, in the field of complex numbers every real or complex polynomial of degree n has exactly n zeros (counting multiplicity).

Topic 2: Functions and Sketching Graphs

The study of functions and techniques of graph sketching, introduced in Stage 1 Mathematical Methods, is extended and applied in the exploration of inverse functions and the sketching of graphs of composite functions involving absolute value, reciprocal, and rational functions.

Sub-topic 2.1: Composition of Functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How and when can new functions be built from other functions?	
• Determine when the composition of two functions is defined	The composition $(f \circ g)(x) = f\big(g(x)\big)$ is defined for those values of x for which $g(x)$ is in the domain of f. Equivalently $f \circ g$ is defined when the range of g is contained in the domain of f.
• Finding compositions	Using examples of known functions, obtaining such compositions as $\sqrt{(\sin x)}$, $\frac{1}{x-3}$ with appropriate domains.
Can information be found about the derivative of a function even when there is no explicit formula for it in the form $y = f(x)$?	
• Implicit differentiation	Following on from the rules of differentiation studied in Mathematical Methods, implicit differentiation is used to find the gradient of curves in implicit form. Knowledge of implicit differentiation is required to justify the derivative of the logarithm function. That is: If $y = \ln x$ then $e^y = x$. Consider $e^y = x$ and differentiate both sides with respect to x to show that $\frac{dy}{dx} = \frac{1}{x}$.

Sub-topic 2.2: One-to-one Functions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
When is it possible for one function to 'undo' another function? Consider functions that are one-to-one:	
• Determine if a function is one-to-one	Function f is one-to-one if $f(a) = f(b)$ only when $a = b$. As the 'vertical line test' (for a relation to be a function) is mentioned in Mathematical Methods, it would be useful to consider that a function passes the 'horizontal line test' if and only if it is one-to-one.
• The inverse $f^{(-1)}$ of a one-to-one function • Determine the inverse of a one-to-one function	Appreciate that an inverse function for f can only be defined when there is a unique value of its domain corresponding to each element of its range. For example, if $y = f(x) = 2x + 3$ then $x = \frac{(y-3)}{2}$ so $f^{(-1)}(x) = \frac{(x-3)}{2}$.
• Relationship between the graph of a function and the graph of its inverse • Symmetry about $y = x$	Examples such as the pair $2x + 3, \frac{x-3}{2}$ and (for non-negative x) the pair $\sqrt{x}, x^2$. Also $\sin x$ for $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ and its inverse $\arcsin x$ for $-1 \leq x \leq 1$ Dynamic geometry software and graphics calculators can be used to investigate these relationships. (Note that the exponential and logarithmic functions and their inverse relationship are studied in Mathematical Methods.)

Sub-topic 2.3: Sketching Graphs

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can available information be put together to deduce the behaviour of various functions that are composite functions?	
• Absolute value function and its properties	Use and apply the notation $\lvert x\rvert$; the graph of $y = \lvert x\rvert$
• Compositions involving absolute values and reciprocals	If $f(x)$ is some given function, the relationship between the graphs of $y = f(x)$ and the graphs of these compositions $y = \frac{1}{f(x)}, y = \lvert f(x)\rvert, y = f(\lvert x\rvert)$ is investigated. Note: when referring to 1/f(x), f(x) is linear, quadratic or trigonometric.
• Graphs of rational functions	Sketch the graphs of rational functions where the numerator and denominator are polynomials of up to degree 2 with real zeros. Students use technology to determine the behaviour of the function near the asymptotes.

Topic 3: Vectors in Three Dimensions

The study of vectors was introduced in Stage 1 with a focus on vectors in two-dimensional space. Three-dimensional vectors are introduced enabling the study of lines and planes in three dimensions, their intersections, and the angles they form. Further development of vector methods of proof enables students to solve geometric problems in three dimensions.

Students gain an understanding of the interrelationships of Euclidean, vector, and coordinate geometry, and appreciate that the proof of a geometric result can be approached in different ways.

Sub-topic 3.1 The Algebra of Vectors in Three Dimensions

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is a vector?	Briefly consider vectors as directed line segments (with arrows) in space, generalising from the two-dimensional treatment in Stage 1 Topic 3, including unit vectors ***i***, ***j*** and ***k***.

Sub-topic 3.2 Vector and Cartesian Equations

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How are points represented in three-dimensional space? • Introduce Cartesian coordinates including plotting points and the equations of spheres	Teaching and learning in this topic can be enhanced with the use of current, free, 3D-dynamic geometry software.
How is the equation of a line in two and three dimensions written? • Vector, parametric, and Cartesian forms	Geometric considerations lead to the vector equation of a line; from this can be derived the parametric form and (less importantly) the Cartesian form. Exercises highlight the construction of parallel lines, perpendicular lines, and the phenomenon of skewness. Computation of the point of a given line that is closest to a given point; distance between skew lines; and angle between lines.
Can it be determined whether the paths of two particles cross?	Examine the position of two particles each described as a vector function of time, and determine if their paths cross or if the particles meet.
Can vectors be multiplied together? What is the meaning of the product? • Scalar (dot) product and vector (cross) product: their properties and their interpretation in context.	These two operations on pairs of vectors provide important geometric information. The scalar product (extended into three dimensions) and the vector product are treated in terms of coordinates and of length and angle. Conditions for perpendicularity and parallelism, and construction of perpendiculars.
• Cross-product calculation using	

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
the determinant to determine a vector normal to a given plane	2x2 Determinants are treated in Stage 1 Topic 5 of Specialist Mathematics.
• $\lvert \mathbf{a} \times \mathbf{b} \rvert$ is the area of the parallelogram with sides $\mathbf{a}$ and $\mathbf{b}$	The cross-product of two vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ in three dimensions is a vector, mutually perpendicular to $\boldsymbol{a}$ and to $\boldsymbol{b}$ whose length is the area of the parallelogram determined by $\boldsymbol{a}$ and $\boldsymbol{b}$. The right-hand rule determines its sense. The components and the vector itself may be expressed using determinants.
How can a plane be described by an equation in both vector and Cartesian form $ax + by + cz = d$?	The equation of a plane in Cartesian form $ax + by + cz = d$ is derived. This form is developed using the same geometric concepts as those used in developing vector equations.
What relationships between lines and planes can be described?	Intersection of a line and a plane, angle between a line and a plane, lines parallel to or coincident with planes. Computation of the point of a given plane that is closest to a given point in space.
Prove geometric results in the plane and construct simple proofs in three dimensions. • Equality of vectors • Coordinate systems and position vectors; components • The triangle inequality	Vectors are equal if they form opposite sides of a parallelogram. Applications (e.g. navigation and force) as encountered in Stage 1 Topic 3 can readily be extended to three dimensions.
• The use of vector methods of proof, particularly in establishing parallelism, perpendicularity, and properties of intersections	Students consider some examples of proof by vector methods, and appreciate their power. Students note the previous application of this inequality in sub-topic 1.2. Suitable examples include: • the angle in a semicircle is a right angle • medians of a triangle intersect at the centroid. The result: If $k_1\boldsymbol{a} + k_2\boldsymbol{b} = l_1\boldsymbol{a} + l_2\boldsymbol{b}$, where $\boldsymbol{a}$ and $\boldsymbol{b}$ are not parallel, then $k_1 = l_1$ and $k_2 = l_2$.

Sub-topic 3.3 Systems of Linear Equations

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can a system of linear equations be solved?	Recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of up to 3 X 3 linear equations.
For a system of equations with three variables, what solutions are possible and what is their geometric interpretation?	Intersections of planes: algebraic and geometric descriptions of unique solution, no solution and infinitely many solutions. Finding the intersection of a set of two or more planes amounts to solving a system of linear equations in three unknowns.

Topic 4: Integration Techniques and Applications

Integration techniques developed in Stage 2 Mathematical Methods are extended to a greater range of trigonometric functions and composite functions, using inverse trigonometric functions and integration by parts. These techniques are applied to finding the areas between curves and the volumes of solids of revolution.

Sub-topic 4.1: Integration Techniques

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What techniques can be used to enable integration of a wider class of functions?	
• Use identities to simplify integrals of squared trigonometric functions	$sin^2x = \frac{1}{2}(1 - cos2x)$, $cos^2x = \frac{1}{2}(1 + \cos 2x)$, $1 + tan^2x = sec^2x$.
• Use substitution $u = g(x)$ to integrate expressions of the form $f(g(x))g'(x)$	For example, $\int \sin^2 x \cos x \mathrm{d}x$.
• Establish and use the formula $\int \frac{1}{x}dx = \ln\lvert x\rvert + c$ for $x \neq 0$.	Compare the case $x > 0$
How can use of the inverse trigonometric functions enable integration of certain functions?	
• Find and use the inverse trig functions: arcsine, arccosine, arctangent	This will involve discussion of restriction of domain in order to obtain a one-to-one function. For example, the principal domain of $\sin x$ is the closed interval $\left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$. (See also Specialist Mathematics Topic 2.2 One-to-one functions.)
• Find and use the derivatives of these functions.	For example, if $y = \arcsin(x)$ then $x = \sin y$, and use implicit differentiation and trigonometric identities to obtain the results.
• Hence integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2-x^2}}$, $\frac{a}{a^2+x^2}$	
• Use partial fractions for integrating rational functions in simple cases.	For example: Verify that $\frac{1}{x+2} - \frac{1}{x-1} = \frac{-3}{(x+2)(x-1)}$ and hence find $\int \frac{1}{x^2+x-2}\mathrm{d}x$

• Use integration by parts.	For integrals which can be expressed in the form $\int f'(x)g(x)dx$, making use of the formula for differentiating a product: $$\int f'(x)g(x)dx = f(x)g(x) - \int f(x)g'(x)dx$$ For example, $\int xe^{x}\mathrm{d}x$

Sub-topic 4.2: Applications of Integral Calculus

Key Questions and Key Concepts	**Considerations for Developing Teaching and Learning Strategies**
What are some applications of the integration techniques?	
• Areas between curves determined by functions	If $f(x) \geq g(x)$ for $a \leq x \leq b$ then the area of the region bounded by the graphs of the functions between $x = a$ and $x = b$ is $\int_a^b \big(f(x) - g(x)\big)dx$.
• Volumes of solids of revolution about either axis	Let $y = f(x)$ be non-negative on the interval $[a, b]$. The volume of the solid obtained by rotating the region bounded by its graph on $[a, b]$ about the *x*-axis is $\int_a^b \pi(f(x))^2 dx$. The volume of the solid obtained by rotating the region under the curve $y = f(x)$ on $[a, b]$ (where $0 \leq a < b$) about the *y*-axis is $\int_a^b 2\pi x f(x) dx$. Derivation of these formulae can be motivated through graphical approaches.

Topic 5: Rates of Change and Differential Equations

A mathematical equation that relates a function to its derivatives is known as a differential equation. In applications, functions usually represent physical quantities, the derivatives represent their rates of change, and the equation defines a relationship between the two.

This topic continues the study of differentiation and integration of functions. Calculus techniques are applied to vectors and simple differential equations. The study of rates of change and differential equations demonstrates applications of learning throughout this subject, in a range of contexts.

This topic also highlights the fundamental theorem of calculus from the point of view of differential equations, intended to deepen students' perspective on indefinite integrals.

Sub-topic 5.1: Differential Equations

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What is the relationship between the rates of change of two related functions of time? • Related rates	Where two functions of time $x(t)$ and $y(t)$ are related by $y(t)=f(x(t))$, their rates of change are related by the chain rule $y'=\frac{\mathrm{d}f}{\mathrm{d}x}x'$ Examples of calculating y' from x' and the derivative of f. For instance, $x(t)$ might be the length of the side of a square that is changing over time, and $y(t)$ its area. Or, they might be the length of the side of a cube and its volume.
What is a differential equation?	A differential equation is an equation that expresses a relationship between a function and its rates of change.
• Solve differential equations of the form $\frac{dy}{dx}=f(x)$ • Solve differential equation of the form $\frac{dy}{dx}=f(x)g(y)$	The simplest differential equation is of the form $\frac{dy}{dx}=f(x)$ which may be solved by integration techniques. Differential equations of the form $\frac{dy}{dx}=g(y)$ and in general form $\frac{dy}{dx}=f(x)g(y)$ may be solved using separation of variables.
How can the information about the derivative of a function be described?	An equation $y'(x)=f(x)$ indicates the slope of the graph at each point x but not the value of y. A line of gradient $f(x_0)$ can be drawn at each point on each vertical line $x=x_0$, and one of these is the tangent line to the graph. This family of lines, one through each point in the plane, is called the 'slope field'.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
• Examine slope fields of first order differential equations • Reconstruct a graph from a slope field both manually and using graphics software	Given an initial value, say, y_0, for y at $x = x_0$, the slope field indicates the direction in which to draw the graph. These concepts should be displayed using graphics calculators or software. Students consider how the computer or graphics calculator might have traced the curves — for example, by following each slope line for a small distance, then following the slope line at the new point. These graphical results are compared with known solutions from integration, such as the solutions $y = x^2 + c$ for the equation $y'(x) = 2x$. What role does c play in the geometric picture? What is the key feature of a family of curves of the form $F(x)+c$?
How can differential equations be used in modelling? • Formulate differential equations in contexts where rates are involved.	
• Separable differential equations	The exponential equation $y'(x) = ky(x)$ is the simplest example of a separable differential equation. Other examples such as $y' = k(A - y)$ arise from Newton's law of cooling or as models of the spread of rumours. They are solved in terms of exponential functions. The family of solutions, and the use of initial conditions to determine which one describes a problem, is emphasised.
• The logistic differential equation	This logistic differential equation $y' = k(P - y)y$ is interpreted in terms of a population P (of molecules, of organisms, etc.) of which y are active or infected and $P - y$ are not. To carry out the method for solving the logistic differential equation students check the identity $\frac{1}{y} + \frac{1}{P-y} = \frac{P}{y(P-y)}$.

Sub-topic 5.2 Pairs of Uniformly Varying Quantities and Their Representation as a Moving Point

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How can a pair of time-varying quantities be modelled as a point moving in a plane?	Given two time-varying quantities, their values at any instant can be interpreted as the Cartesian coordinates $x(t)$ and $y(t)$ of a point moving in the plane.
When the quantities vary uniformly with time, what kind of curve do they trace out? • Coordinate representation (parametric)	Being uniform, the quantities are functions of the form $x(t)=x_0+at,\ y(t)=y_0+bt.$ These describe the parametric equation of a line in two dimensions. This is understood with reference to the vector and parametric equations of lines in three dimensions. The vector form of the two-dimensional equation is $[x_0+ta, y_0+tb]=[x_0, y_0]+t[a,b].$
• Vector interpretation	Emphasis is placed on the interpretation of the parameter t as time. It is noted that vector $[a,b]$ gives the change in position vector occurring in each unit of time, and that this is the fundamental notion of velocity.
Examples involving pairs of uniformly varying quantities.	Students consider examples such as production costs with two components, capital and labour, and a simplistic model of predator–prey relationships, defined by two uniformly varying populations.

Sub-topic 5.3 Pairs of Non-uniformly Varying Quantities – Polynomials of Degree 2 and 3

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
What curves are traced out by a moving point $\left(x(t), y(t)\right)$ in which the functions $x(t)$ and $y(t)$ are polynomials of degree 1 to 3?	
Consider examples of applications to: • objects in free flight • Bézier curves and equivalent examples.	The position of an object in free flight is given by the equations $x(t)=x_0+at,\ y(t)=y_0+bt-\frac{1}{2}gt^2$ where (x_0, y_0) is the initial position, $[a,b]$ is the initial velocity, and g is the acceleration due to gravity. Students observe the parabolic shape of the curve. This equation can be used to answer questions such as: ‘At what angle should a basketball be thrown to score a goal?’ or ‘At what angle should a cricket ball be struck to clear the fence?’ Degree 3 polynomials feature in computer-aided design. The use of Bézier curves attempts to mimic freehand drawing using cubic polynomials, and motion along Bézier curves is used to simulate motion in computer animations. They are constructed using four control points, two marking the beginning and end of the curve and two others controlling the shape. They can be constructed with interactive geometry software such as: (www.moshplant.com/direct-or/bezier/) Although the curves can be drawn interactively and intuitively by the designer, an accurate mathematical description is needed for: editing the curve; zooming in by the program; passing the design on to be printed or processed by specialised machining equipment. Bézier curves can be constructed in two or three dimensions. This relates to Topic 3.

Sub-topic 5.4 Related Rates, Velocity, and Tangents

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
How is the motion of a moving point described?	
• For a moving point $(x(t), y(t))$ the vector of derivatives $\mathbf{v} = [x'(t), y'(t)]$ is naturally interpreted as its instantaneous velocity.	For uniform motion the velocity $[a,\ b]$ has for its components the rates of change of the components of the position vector. The concept that $\mathbf{v} = [x'(t), y'(t)]$ is interpreted as the instantaneous velocity vector of the moving point $(x(t), y(t))$ is a logical extension of this concept, in which average rates of change are replaced by instantaneous rates of change, or derivatives.
• The Cartesian equation of the path of the moving point can be found by eliminating *t* and establishing the relationship between *y* and *x*.	If, given a function $f(x)$ and a function of time $x(t)$, you set $y(t) = f(x(t))$ then the moving point $(x(t), y(t))$ travels along the graph of $f(x)$.
• The velocity vector is always tangent to the curve traced out by a moving point.	The chain rule $y' = \frac{\mathrm{d}f}{\mathrm{d}x}x'$ shows that the ratio of y' to x' is $\frac{\mathrm{d}f}{\mathrm{d}x}$ and that the velocity vector is tangent to the graph. Since effectively every curve is the graph of a function (by rotation if necessary), the velocity vector is always tangent to the curve traced out by a moving point.
• Parametric equations of tangents to parametric curves.	At time $t = a$ the moving point passes through $(x(a), y(a))$ and the velocity vector $[x'(a), y'(a)]$ is tangent at this point to the curve traced out. The parametric line $[x(a)+tx'(a),\ y(a)+ty'(a)]$ passes through $(x(a), y(a))$ at time $t = 0$ and has velocity $[x'(a), y'(a)]$, so it is tangent to the curve at this point. Students can use this formula to calculate the parametric equations of tangents to parametric curves, for example, Bézier curves.

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
• Speed of the moving point as magnitude of velocity vector, that is $\sqrt{x'^2(t)+y'^2(t)} = \sqrt{\mathbf{v} \bullet \mathbf{v}}$	Calculation of the speed of projectiles or of points moving along Bézier curves.
• Arc lengths along parametric points	$\mathbf{v} = \frac{d}{dt}[x(t), y(t)]$ is the velocity vector of the moving point. If the point moves during the time from $t=a$ to $t=b$ the length of the path traced out is calculated using integration: $\int_a^b \sqrt{\mathbf{v} \bullet \mathbf{v}}\, dt$

Sub-topic 5.5 Trigonometric Parameterisations

Key Questions and Key Concepts	Considerations for Developing Teaching and Learning Strategies
• A point moving with unit speed around the unit circle can be described using the moving position vector $\mathbf{P}(t)=[\cos t, \sin t]$	Students can find the first and second derivatives of $\mathbf{P}(t)$ and establish the constant speed and centripetal acceleration of circular motion.
• Moving around other circles with other speeds.	The more general position vector $\mathbf{P}(t)=[R\cos\omega t, R\sin\omega t]$ allows for faster or slower motion around smaller or larger circles. Students can use the arc length formula to establish the well-known formula for the circumference of a circle.

ASSESSMENT SCOPE AND REQUIREMENTS

All Stage 2 subjects have a school assessment component and an external assessment component.

EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 2 Specialist Mathematics.

School Assessment (70%)

- Assessment Type 1: Skills and Applications Tasks (50%)
- Assessment Type 2: Mathematical Investigation (20%)

External Assessment (30%)

- Assessment Type 3: Examination (30%)

Students provide evidence of their learning through seven assessments, including the external assessment component. Students complete:

- five skills and applications tasks
- one mathematical investigation
- one examination.

It is anticipated that from 2018 all school assessments will be submitted electronically.

ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by:

- teachers to clarify for the student what he or she needs to learn
- teachers and assessors to design opportunities for the student to provide evidence of his or her learning at the highest possible level of achievement.

The assessment design criteria consist of specific features that:

- students should demonstrate in their learning
- teachers and assessors look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole, must give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships

CT2 Selection and application of techniques and algorithms to solve problems in a variety of contexts

CT3 Development and application of mathematical models

CT4 Use of electronic technology to find solutions to mathematical problems.

Reasoning and Communication

The specific features are as follows:

RC1 Interpretation and evaluation of mathematical results, with an understanding of their reasonableness and limitations

RC2 Knowledge and use of appropriate mathematical notation, representations, and terminology

RC3 Communication of mathematical ideas and reasoning, to develop logical arguments

RC4 Development, testing, and proof of valid conjectures.

SCHOOL ASSESSMENT

Assessment Type 1: Skills and Applications Tasks (50%)

Students complete five skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher.

Students find solutions to mathematical problems that may:
- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine, analytical, and/or interpretative problems. Some of these problems should be set in context; for example, social, scientific, economic, or historical.

Students provide explanations and arguments, and use correct mathematical notation, terminology, and representation throughout the task. They may be required to use electronic technology appropriately to aid and enhance the solution of some problems.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:
- concepts and techniques
- reasoning and communication.

Assessment Type 2: Mathematical Investigation (20%)

Students complete one mathematical investigation.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers should give some direction about the appropriateness of each student's choice and guide and support students' progress in an investigation. For this investigation there must be minimal teacher direction and teachers must allow the opportunity for students to extend the investigation in an open-ended context.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. Computer Algebra Systems, spreadsheets, statistical packages) to enhance their investigation. The generation of data and the exploration of patterns and structures, or changing parameters, may provide an important focus. From these, students may recognise different patterns or structures. Notation, terminology, forms of representation of information gathered or produced, calculations, evidence of technological skills, and results are important considerations.

Students complete a report for the mathematical investigation.

In the report, they formulate and test conjectures, interpret and justify results, draw conclusions, and give appropriate explanations and arguments.

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context

- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including
  - relevant data and/or information
  - mathematical calculations and results, using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem
- a bibliography and appendices, as appropriate.

The format of an investigation report may be written or multimodal.

The investigation report should be a maximum of 15 pages if written, or the equivalent in multimodal form.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

EXTERNAL ASSESSMENT

Assessment Type 3: Examination (30%)

Students undertake a 3-hour external examination.

The examination is based on the ‘key questions and key concepts’ in the five topics. The ‘considerations for developing teaching and learning strategies’ are provided as a guide only, although applications described under this heading may provide contexts for examination questions.

The examination consists of a range of problems, some focusing on knowledge and routine skills and applications, and others focusing on analysis and interpretation. Some problems may require students to interrelate their knowledge, skills, and understanding from more than one topic. Students provide explanations and arguments, and use correct mathematical notation, terminology, and representation throughout the examination.

The examination is divided into two parts:

- Part 1 (40%, 60 minutes): Calculations without electronic technology (graphics/scientific calculators).
- Part 2 (60%, 120 minutes): Calculations with access to approved electronic technology (graphics/scientific calculators).

Students have 10 minutes in which to read both Part 1 and Part 2 of the examination. At the end of the reading time, students begin their answers to Part 1. For this part, students do not have access to electronic technology (graphics or scientific calculators).

At the end of the specified time for Part 1, students stop writing. Students submit Part 1 to the invigilator.

Students have access to Board-approved calculators for Part 2 of the examination. The invigilator coordinates the distribution of calculators (graphics and scientific). Once all students have received their calculators, the time allocated for Part 2 begins, and students resume writing their answers.

The SACE Board will provide a list of approved graphics calculators for use in Assessment Type 3: Examination that meet the following criteria:

- have flash memory that does not exceed 5.0 MB (this is the memory that can be used to store add-in programs and other data)
- can calculate derivative and integral values numerically
- can calculate probabilities
- can calculate with matrices
- can draw a graph of a function and calculate the coordinates of critical points using numerical methods
- solve equations using numerical methods
- do not have a CAS (Computer Algebra System)
- do not have SD card facility (or similar external memory facility).

Graphics calculators that currently meet these criteria, and are approved for 2017, as follows:
Casio fx-9860G AU
Casio fx-9860G AU Plus
Hewlett Packard HP 39GS
Sharp EL-9900
Texas Instruments TI-83 Plus
Texas Instruments TI-84 Plus
Texas Instruments – TI 84 Plus C –silver edition
Texas Instruments – TI 84 Plus CE.

Other graphic calculators will be added to the approved calculator list as they become available.

Students may bring two graphics calculators or one scientific calculator and one graphics calculator into the examination room.

There is no list of Board-approved scientific calculators. Any scientific calculator, except those with an external memory source, may be used.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:
- concepts and techniques
- reasoning and communication.

PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers and assessors refer to in deciding how well a student has demonstrated his or her learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student's completion of study of each school assessment type, the teacher makes a decision about the quality of the student's learning by:

- referring to the performance standards
- assigning a grade between A+ and E- for the assessment type.

The student's school assessment and external assessment are combined for a final result, which is reported as a grade between A+ and E-.

Performance Standards for Stage 2 Specialist Mathematics

	Concepts and Techniques	Reasoning and Communication
A	Comprehensive knowledge and understanding of concepts and relationships. Highly effective selection and application of techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts. Successful development and application of mathematical models to find concise and accurate solutions. Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems.	Comprehensive interpretation and critical evaluation of mathematical results, with an in-depth understanding of their reasonableness and possible limitations. Proficient and accurate use of appropriate mathematical notation, representations, and terminology. Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. Effective development and testing of valid conjectures, with proof.
B	Some depth of knowledge and understanding of concepts and relationships. Mostly effective selection and use of mathematical techniques and algorithms to find accurate solutions to routine and some complex problems in a variety of contexts. Mostly successful development and application of mathematical models to find accurate solutions. Mostly appropriate and effective use of electronic technology to find accurate solutions to routine and some complex problems.	Mostly appropriate interpretation and some critical evaluation of mathematical results, with some depth of understanding of their reasonableness and possible limitations. Mostly accurate use of appropriate mathematical notation, representations, and terminology. Mostly effective communication of mathematical ideas and reasoning to develop logical arguments. Mostly effective development and testing of valid conjectures, with substantial attempt at proof.
C	Generally competent knowledge and understanding of concepts and relationships. Generally effective selection and use of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts. Some development and successful application of mathematical models to find generally accurate solutions. Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems.	Generally appropriate interpretation and evaluation of mathematical results, with some understanding of their reasonableness and possible limitations. Generally appropriate use of mathematical notation, representations, and terminology, with some inaccuracies. Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. Development and testing of generally valid conjectures, with some attempt at proof.
D	Basic knowledge and some understanding of concepts and relationships. Some effective selection and use of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts. Some application of mathematical models to find some accurate or partially accurate solutions. Some appropriate use of electronic technology to find some accurate solutions to routine problems.	Some interpretation and attempted evaluation of mathematical results, with some awareness of their reasonableness and possible limitations. Some appropriate use of mathematical notation, representations, and terminology, with some accuracy. Some communication of mathematical ideas, with attempted reasoning and/or arguments. Attempted development or testing of a reasonable conjecture.
E	Limited knowledge or understanding of concepts and relationships. Attempted selection and limited use of mathematical techniques or algorithms, with limited accuracy in solving routine problems. Attempted application of mathematical models, with limited accuracy. Attempted use of electronic technology, with limited accuracy in solving routine problems.	Limited interpretation or evaluation of mathematical results, with limited awareness of their reasonableness and possible limitations. Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy. Attempted communication of mathematical ideas, with limited reasoning. Limited attempt to develop or test a conjecture.

ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement, in both the school assessment and the external assessment, are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 2 are available on the SACE website (www.sace.sa.edu.au)

SUPPORT MATERIALS

SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).