DRAFT FOR ONLINE CONSULTATION

# General Mathematics

## Subject Outline

## Stage 1 and Stage 2



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# INTRODUCTION

## SUBJECT DESCRIPTION

General Mathematics is a 10-credit subject or a 20-credit subject at Stage 1, and a 20-credit subject at Stage 2.

General Mathematics extends students' mathematical skills in ways that apply to practical problem solving. A problems-based approach is integral to the development of mathematical models and the associated key ideas in the topics. These topics cover a diverse range of applications of mathematics, including personal financial management, measurement and trigonometry, the statistical investigation process, modelling using linear and non-linear functions, and discrete modelling using networks and matrices.

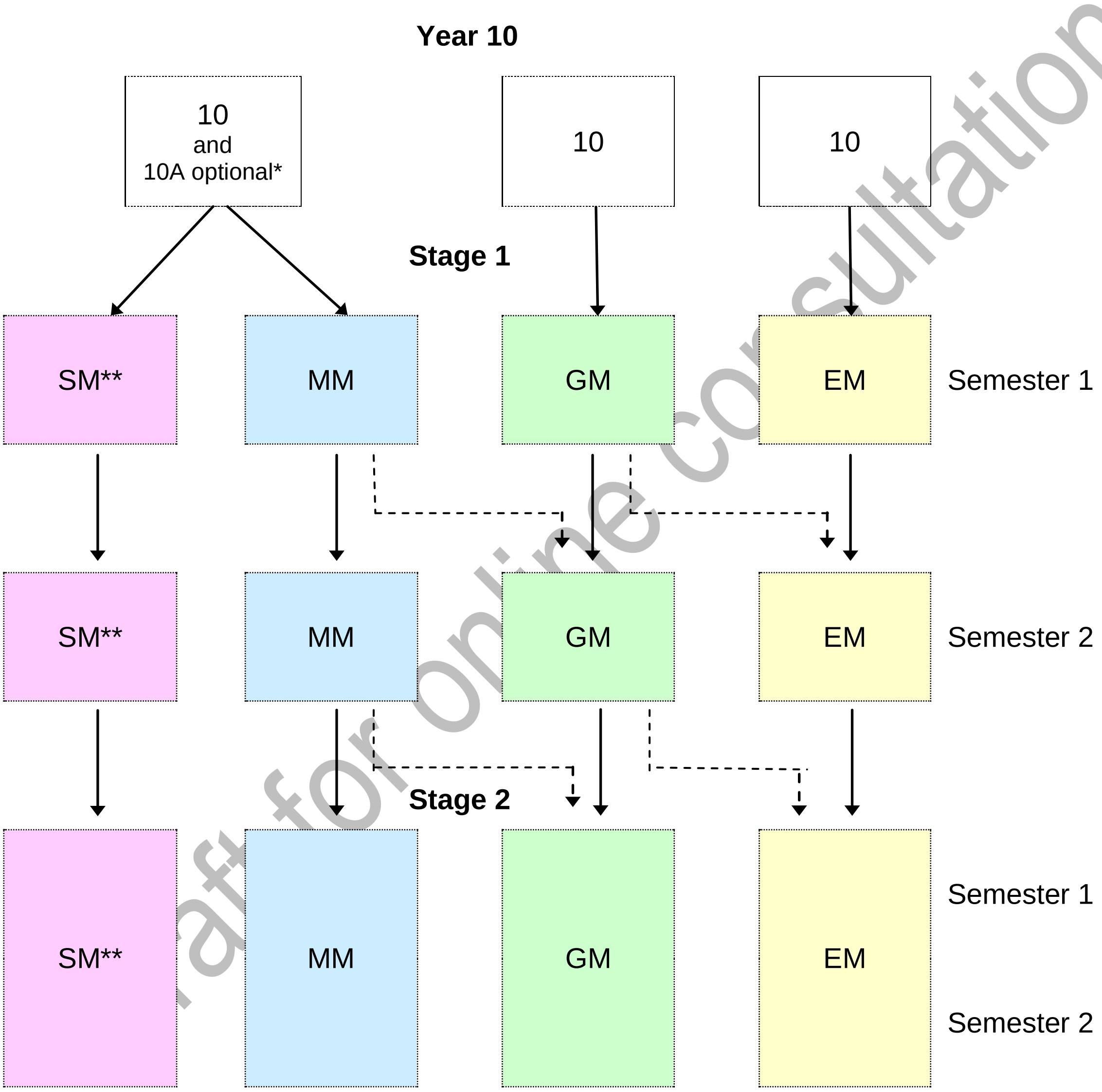
Successful completion of this subject at Stage 2 prepares students for entry to tertiary courses requiring a non-specialised background in mathematics.

## MATHEMATICAL OPTIONS

The diagram below represents the possible mathematical options that students might select as Stage 1 and Stage 2 subjects.

Solid arrows indicate the mathematical options that lead to completion of each subject at Stage 2. Dotted arrows indicate a pathway that may provide sufficient preparation for an alternative Stage 2 mathematics subject.

| | |
|---|---|
| SM | Specialist Mathematics |
| MM | Mathematical Methods |
| GM | General Mathematics |
| EM | Essential Mathematics |






Notes: * Although it is advantageous for students to study Australian Curriculum 10 and 10A in Year 10, the 10A curriculum *per se* is not a prerequisite for the study of Specialist Mathematics and Mathematical Methods. The essential aspects of 10A are included into the curriculum for Specialist Mathematics and Mathematical Methods.

** Mathematical Methods can be studied as a single subject; however, Specialist Mathematics is designed to be studied together with Mathematical Methods.

## CAPABILITIES

The capabilities connect student learning within and across subjects in a range of contexts. They include essential knowledge and skills that enable people to act in effective and successful ways.

The SACE identifies seven capabilities. They are:

- literacy
- numeracy
- information and communication technology capability
- critical and creative thinking
- personal and social capability
- ethical understanding
- intercultural understanding.

### Literacy

In this subject students develop their literacy capability by, for example:

- communicating mathematical reasoning and ideas for different purposes, using appropriate language and representations, such as symbols, equations, tables, and graphs
- interpreting and responding to appropriate mathematical language and representations
- analysing information and explaining mathematical results.

Mathematics provides a specialised language to describe and analyse phenomena. It provides a rich context for students to extend their ability to read, write, visualise, and talk about situations that involve investigating and solving problems.

Students apply and extend their literacy skills and strategies by using verbal, graphic, numerical, and symbolic forms of representing problems and displaying statistical information. Students learn to communicate their findings in different ways, using different systems of representation.

### Numeracy

Being numerate is essential for participating in contemporary society. Students need to reason, calculate, and communicate to solve problems. Through the study of mathematics, they understand and use skills, concepts, and technologies in a range of contexts that can be applied to:

- using measurement in the physical world
- gathering, representing, interpreting, and analysing data
- using spatial sense and geometric reasoning
- investigating chance processes
- using number, number patterns, and relationships between numbers
- working with graphical, statistical and algebraic representations, and other mathematical models.

### Information and communication technology capability

In this subject students develop their information and communication technology (ICT) capability by, for example:

- understanding the role of electronic technology in the study of mathematics
- making informed decisions about the use of electronic technology
- understanding the mathematics involved in computations carried out using technologies, so that reasonable interpretations can be made of the results.

Students extend their skills in using technology effectively and processing large amounts of quantitative information.

Students use ICT to extend their theoretical mathematical understanding and apply mathematical knowledge to a range of problems. They use software relevant for study and/or workplace contexts. This may include tools for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, address problems, and operate systems in particular situations.

### Critical and creative thinking

In this subject students develop critical and creative thinking by, for example:

- building confidence in applying knowledge and problem-solving skills in a range of mathematical contexts
- developing mathematical reasoning skills to think logically and make sense of the world
- understanding how to make and test projections from mathematical models
- interpreting results and drawing appropriate conclusions
- reflecting on the effectiveness of mathematical models, including the recognition of assumptions, strengths, and limitations
- using mathematics to solve practical problems and as a tool for learning
- making connections between concrete, pictorial, symbolic, verbal, written, and mental representations of mathematical ideas
- thinking abstractly, making and testing conjectures, and explaining processes.

Problem-solving in mathematics builds students' depth of conceptual understanding and supports development of critical and creative thinking. Learning through problem-solving helps students when they encounter new situations. They develop their creative and critical thinking capability by listening, discussing, conjecturing, and testing different strategies. They learn the importance of self-correction in building their conceptual understanding and mathematical skills.

### Personal and social capability

In this subject students develop their personal and social capability by, for example:

- arriving at a sense of self as a capable and confident user of mathematics through expressing and presenting ideas in a variety of ways
- appreciating the usefulness of mathematical skills for life and career opportunities and achievements
- understanding the contribution of mathematics and mathematicians to society.

The elements of personal and social competence relevant to mathematics include the application of mathematical skills for informed decision-making, active citizenship, and effective self-management. Students build their personal and social competence in mathematics through setting and monitoring personal and academic goals, taking initiative, and building adaptability, communication, and teamwork.

Students use mathematics as a tool to solve problems they encounter in their personal and working lives. They acquire a repertoire of strategies and build the confidence needed to:

- meet the challenges and innovations of a rapidly changing world
- be the designers and innovators of the future, and leaders in their fields.

### Ethical understanding

In this subject students develop their ethical understanding by, for example:

- gaining knowledge and understanding of ways in which mathematics can be used to support an argument or point of view
- examining critically ways in which the media present particular perspectives
- sharing their learning and valuing the skills of others
- considering the social consequences of making decisions based on mathematical results
- acknowledging and learning from errors rather than denying findings and/or evidence.

Areas of ethical understanding relevant to mathematics include issues associated with ethical decision-making and working collaboratively as part of students' mathematically related explorations. They develop ethical understanding in mathematics through considering social responsibility in ethical dilemmas that may arise when solving problems in personal, social, community, and/or workplace contexts.

### Intercultural understanding

In this subject students develop their intercultural understanding by, for example:

- understanding mathematics as a body of knowledge that uses universal symbols which have their origins in many cultures
- understanding how mathematics assists individuals, groups, and societies to operate successfully across cultures in the global, knowledge-based economy.

Mathematics is a shared language that crosses borders and cultures, and is understood and used globally.

Students read about, represent, view, listen to, and discuss mathematical ideas. They become aware of the historical threads from different cultures that have led to the current bodies of mathematical knowledge. These opportunities allow students to create links between their own language and ideas and the formal language and symbols of mathematics.

## ABORIGINAL AND TORRES STRAIT ISLANDER KNOWLEDGE, CULTURES, AND PERSPECTIVES

In partnership with Aboriginal and Torres Strait Islander communities, and schools and school sectors, the SACE Board of South Australia supports the development of high-quality learning and assessment design that respects the diverse knowledge, cultures, and perspectives of Indigenous Australians.

The SACE Board encourages teachers to include Aboriginal and Torres Strait Islander knowledge and perspectives in the design, delivery, and assessment of teaching and learning programs by:

- providing opportunities in SACE subjects for students to learn about Aboriginal and Torres Strait Islander histories, cultures, and contemporary experiences
- recognising and respecting the significant contribution of Aboriginal and Torres Strait Islander peoples to Australian society
- drawing students' attention to the value of Aboriginal and Torres Strait Islander knowledge and perspectives from the past and the present
- promoting the use of culturally appropriate protocols when engaging with and learning from Aboriginal and Torres Strait Islander peoples and communities.

## SACE NUMERACY REQUIREMENT

Completion of 10 or 20 credits of Stage 1 General Mathematics with a C grade or better, or 20 credits of Stage 2 General Mathematics with a C- grade or better, will meet the numeracy requirement of the SACE.

# Stage 1
# General Mathematics

# LEARNING SCOPE AND REQUIREMENTS

## LEARNING REQUIREMENTS

The learning requirements describe the essential elements of Stage 1 General Mathematics. They summarise the knowledge, skills, and understandings that students are expected to develop and demonstrate through learning in the subject.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. recognise and apply the mathematical techniques needed when analysing and finding a solution to a problem, including the forming and testing of conjectures
4. interpret results, draw conclusions, and reflect on the reasonableness of solutions in context
5. make discerning use of electronic technology
6. communicate mathematically and present mathematical information in a variety of ways.

## CONTENT

Stage 1 General Mathematics may be studied as a 10-credit or a 20-credit subject.

Students extend their mathematical skills in ways that apply to practical problem solving and mathematical modelling in everyday contexts. A problems-based approach is integral to the development of mathematical skills and the associated key ideas in this subject.

Areas studied cover a range of applications of mathematics, including: personal financial management, measurement and trigonometry, the statistical investigation process, modelling using linear functions, and discrete modelling using networks and matrices. In this subject there is an emphasis on consolidating students' computational and algebraic skills and expanding their ability to reason and analyse mathematically.

Stage 1 General Mathematics consists of the following list of six topics:

Topic 1: Investing and borrowing
Topic 2: Measurement
Topic 3: Statistical Investigation
Topic 4: Applications of Trigonometry
Topic 5: Linear Functions and their Graphs
Topic 6: Matrices and Networks.

### Programming

For a 10-credit subject students study three topics chosen from the list.

For a 20-credit subject students study six of the topics in the list.

The topics selected can be sequenced and structured to suit students' needs. The suggested order of the topics in the list is a guide only.

Each topic consists of a number of subtopics. These are presented in the subject outline in two columns as a series of 'key questions and key concepts' side by side with 'considerations for developing teaching and learning strategies'.

The 'key questions and key concepts' cover the prescribed content for teaching, learning, and assessment in this subject. The 'considerations for developing teaching and learning strategies' are provided as a guide only.

A problems-based approach is integral to the development of the computational models and associated key concepts in each topic. Through key questions students deepen their understanding of concepts and processes that relate to the mathematical models required to address the problems posed.

The 'considerations for developing teaching and learning strategies' present problems and guidelines for sequencing the development of ideas. They also give an indication of the depth of treatment and emphases required.

It is recommended that students carry out calculations by hand to develop an understanding of the processes involved, before use of electronic technology to enable more complex problems to be solved when it is more efficient to do so.

## Topic 1: Investing and Borrowing

Students discuss reasons for investing money and investigate using financial institutions and the share market as vehicles for investment of a sum of money. They calculate their expected returns from simple and compound interest investments using electronic technology (such as spreadsheets and financial packages in graphic calculators) and examine the effects of changing interest rates, terms, and investment balances. Students make comparisons between various scenarios and considerations of the limitations on the reliability of predictions made using simple and compound interest models.

Share market calculations include the costs of buying and returns from selling shares, break even prices, and returns from dividends. Students make comparisons between the returns possible from share investments and those made in financial institutions. The effects of taxation and inflation on the return from a lump sum investment are investigated to determine whether real growth has occurred. Students consider the costs of borrowing money using credit or a personal loan, by accessing calculation tools on the Internet.

## Topic 1: Investing and Borrowing

| Key Questions and Key Concepts | Considerations for Developing Teaching and Learning Strategies |
|---|---|

### Sub-topic 1.1: Investing for Interest

Why invest money in financial institutions?
Where can money be invested?

- Discussion of financial institutions
- Fees and charges
- Types of investment

Students learn about the different types of financial institutions that can be used for investment, (e.g. banks, credit unions, investment companies) the methods of investment they offer (e.g. term deposits, savings accounts) and the associated costs of investing in this way (i.e. fees and charges).

Calculations involving rates and percentages are discussed as necessary.

How is simple interest calculated, and in which situations is it used?

- Using the simple interest formula to find the
  - simple interest
  - principal
  - interest rate
  - time invested in years
  - total return

Spreadsheets are built to carry out simple interest calculations that could then be graphed leading to a discussion of the linear relationship between time and amount of interest earned. 'What if …' questions are posed to investigate the effects of changing the principal, interest rate, and time.
Students explore the formula for simple interest using examples of simple interest calculations such as for term deposits.

Students recognise that simple interest is a percentage calculation multiplied by the number of years.

How does compound interest work?

Using examples collected from financial institutions, students use a spreadsheet to examine the effect of compounding growth on an investment. The difference in the nature of this growth compared with simple interest is discussed.

How is compound interest calculated?

- Derivation of the compound interest formula
- Using the formula to find future value, interest earned, and present value

Using technology, students examine the behaviour of the graph of time versus amount for compounding interest and explore how changes to interest rate and compounding periods affect the graph.
Students are guided through the steps of the derivation of the formula for compound interest and then use it to further investigate the solutions to problems that involve finding future or present value or total interest earned on an investment.

- Effects of changing the compounding period
- Annualised rates for comparison of investments

When interest is compounded more often than annually the compound interest formula can be adapted to take this into account. Students use this formula to explore the effects of changing the compounding period on the amount of interest earned. Another way to quantify the effect of more frequent compounding is to annualise the rate, making comparisons possible.

- Using electronic technology to find
  - future value
  - present value
  - interest rate
  - time
  - comparison rates on savings

Technology provides an alternative way of solving problems. It can be used to find out how long it takes to save a certain amount, or the interest rate required. Students are given a variety of problems to solve in practical contexts. They discuss the reasonableness of relying on such calculations because of the limitations of the model used (such as the assumption of a non-changing interest rate).

Which is the better option: simple interest or compound interest?

Situations are presented that allow students to compare the graphs of simple and compound interest growth for the same principal over time. Given a larger simple interest rate and a smaller compound interest rate students use their graphs to determine when compound interest becomes the better option.

| Key Questions and Key Concepts | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| | They also calculate the simple interest rate that, over a specified number of years, will earn the same amount as a given compound interest rate. |
| **Sub-topic 1.2: Investing in Shares** | |
| How can the share market be used to make money from the money someone already has?<br>• Share market information<br>• Costs and risks<br>• Buying and selling shares | Students discuss the share market as an alternative to investing in a financial institution This discussion includes where information on share investments can be found and the associated costs and risks involved. |
| • Breakeven price<br>• Using a brokerage rate<br>$BE = \frac{b(1+1.1r)}{1-1.1r}$<br>• Using a flat fee for brokerage<br>$BE = \left(\frac{2.2f}{n}\right) + b$ | Students calculate the cost of buying shares (including brokerage and GST) and the return from selling them at a later date. They also find the break-even price. |
| • Calculation of the dividend return from shares given the percentage dividend or the dividend per share. | Dividends also provide income from share investments.<br>Using current information, students calculate the dividend income generated by a share portfolio. |
| **Sub-topic 1.3: Return on Investment** | |
| • Expressing the return on an investment as a percentage of the original investment | To be able to compare one investment with another, students express the return as percentage growth. |
| • The effect of tax and inflation on real growth of an investment | To determine whether an investment has made real growth considerations such as tax and inflation are taken into account. Inflation is introduced as the equivalent of an annual compounding model and calculated using either the formula or electronic technology. |
| **Sub-topic 1.4: Costs of Borrowing** | |
| Why do many people use credit to buy items rather than saving for them?<br>What types of credit are available?<br>What is the total cost of using credit? | Students investigate forms of credit available, such as credit cards, store cards, line of credit, and discuss their advantages and disadvantages.<br>They discuss the extra costs to the purchaser who uses credit and calculate the total cost of using a credit card or a line of credit and compare with paying cash. Students investigate the cost of, for example, buying a TV or a computer on consumer credit. |
| How much does a personal loan cost?<br>• Extra fees and charges<br>• Administration fees<br>• Interest | Using internet-based bank loan calculators, students investigate the costs and time involved in repaying personal loans and research fees, charges, and hidden costs associated with loans. |
| • When is it better to borrow than save? | The effect of the inflation rate on the price of an item is investigated to find out how long it would take for the savings to equal the cost of the item. This is expanded into a discussion on borrowing versus saving. |

## Topic 2: Measurement

Students apply measurement techniques such as estimation, units of measurement, scientific notation, and measuring devices and their accuracy. They extend their understanding of Pythagoras' theorem and use formulae to calculate the perimeter, area, and volume of standard plane and solid shapes, including triangles, quadrilaterals, circles, ellipses, prisms, pyramids, cylinders, cones, and spheres. This study is extended to compounds of these shapes. The estimation of irregular areas and volumes is considered by approximation using simple regular shapes or by applying Simpson's rule.

Students examine scales as they apply in practical contexts such as reading and making maps, plans, or models. Problems set in familiar contexts are used to develop students' understanding of the concept of rates as changes in related measurements, for instance flow rates, density, or unit pricing.

## Topic 2: Measurement

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 2.1: Application of measuring devices and units of measurement** | |
| Application of common measuring devices, the metric system, its units and conversion between them | Students apply the most appropriate device and the associated metric units to measuring in a given situation. They consider the calculation of conversions between commonly used units. |
| How should accuracy be considered in measurement?<br>• Estimation and approximation<br>• Rounding off to a given number of significant figures<br>• Calculation of absolute and percentage errors using error tolerances. | Students estimate and measure quantities associated with familiar objects. The results are rounded to a sensible accuracy.<br><br>Students are aware of the accuracy or tolerances of measurements taken using various devices and use these to calculate either absolute or percentage (relative) errors of measurement. They discuss the implications of such errors in context. |
| How are very large and very small values in measurement expressed?<br>• Scientific notation | Students correctly interpret both large and small measurements given in scientific notation on a calculator or spreadsheet. |

## Key Questions and Key Ideas

## Considerations for Developing Teaching and Learning Strategies

### Sub-topic 2.2: Perimeter and area of plane shapes

Pythagoras' theorem

Students will be familiar with Pythagoras' Theorem from previous years. They extend their understanding of its application through solving of problems set in practical contexts involving both two and three dimensional shapes.

How can knowing the perimeter and area of a two-dimensional shape help with solving a problem?

- Calculating circumferences and perimeters of standard and composite shapes (including circles, sectors, quadrilaterals and triangles)
- Calculating areas of standard and composite shapes (including circles, sectors, quadrilaterals, ovals, trapeziums and triangles)
- Converting between units of measurement for area

Students calculate perimeters and area of plane shapes. The focus is on solving practical problems set in familiar contexts, with increasing complexity of the shapes involved.

Students use appropriate units for area and are able to convert between square centimetres, square metres, square kilometres, and hectares.

How can the area of an irregular plane shape be estimated?

- Approximation using a simple mathematical shape (circle, oval, rectangle, triangle, etc)
- Simpson's rule

$$A = \frac{1}{3}w(d_1 + 4d_2 + d_3)$$

Students practice approximating irregular areas by superimposing a simple mathematical shape, such as a circle, oval, rectangle, or triangle and balancing the 'overlaps' (area included and excluded) to enable them to calculate a reasonable estimate.

The efficiency of this method is compared with another, such as counting squares on an overlaid grid.
Simpson's rule provides a useful formula for estimating the area of a shape with an irregular curved boundary.
If this rule is applied successively to an area divided into any even number of regions the formula becomes:

Simpson's rule:

$A = w/3(L_0 + 4L_1 + 2L_2 + \ldots + 4L_{n-1} + L_n)$

where:

$w$ = distance between offsets

$L_k$ = is the length of the k$^{th}$ offset

To use this method accurately $L_0$ and $L_n$ must be the measurements of the offsets at the ends of the baseline, even if these lengths are of size zero. The offsets must divide the shape into an *even* number of regions of equal width and cases where an end offset distance is zero dealt with correctly.

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 2.3: Volume and surface area of solids** | |
| How is the amount of space an object occupies or the amount of liquid a container will hold determined?<br>• Calculating volume or capacity for standard and composite solids including prisms, cylinders, cones, pyramids, and spheres. | Students take measurements of prisms (e.g. a locker, dirt in a flower bed, the interior of a shed) or cylinders (e.g. a rainwater tank, a fire extinguisher) and calculate their volumes, using formulae and correct units of measurement. Students' understanding is extended to tapered shapes such as cones and pyramids and the sphere dealt with as a special case.<br><br>Skills and techniques are developed by solving practical problems in context. Students investigate the formulae for less common solids such as the frustum of a cone or pyramid, or the cap of a sphere.<br><br>Students use appropriate units for volume and capacity and convert between cubic centimetres, cubic metres, millilitres, litres, and kilolitres. |
| • Converting between units for volume and capacity | Irregular volumes are calculated using either Simpson's rule or approximation to a regular shape to find an irregular base area that can be multiplied by an average height/depth (prismatic model) or 1/3 maximum height/depth (conical model). |
| • Estimating the volume of an irregular solid using an appropriate mathematical model.<br><br>How is the area of the outside surface of a solid shape determined?<br><br>• Calculating surface area for standard and composite solids including prisms, cylinders, cones, pyramids, and spheres. | For most of the standard solids the surface area is the sum of the areas of the shapes that comprise its 'net'. This approach is used to arrive at surface area formulae for such solids. Students use such formulae when distinguishing between 'open' and 'closed' shapes (for instances an open pipe vs a solid cylinder). The sphere is treated as a special case. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 2.4: Scale and rates** | |
| How does a scale factor work?<br>• Using a scale factor to calculate actual and scaled measurements<br>• Drawing scaled diagrams<br>• Determining the scale factor needed or used | Students work with maps and plans of different scale to calculate actual lengths or distances. They apply ratios and appropriate rounding off.<br><br>Students construct maps or plans, including those where a scale is chosen by the student (e.g. a scale plan of the school for new students). A drawing software package could be used. Ground measurements could be used to determine the scale of a printout of a Google map of a local area. |
| • Scaling areas and volumes | Students obtain a scale factor and use it to solve scaling problems involving the calculation of areas of similar figures or surface areas and volumes of similar solids. An understanding of how scaling works when extended from simple linear measurement to two and three dimensions could be provided by considering the problems that can arise from testing scaled models in engineering or investigating the change in the cross sectional area of the leg bones of animals as their weight increases. |
| What is a rate? What does it measure?<br>• Rates of change with time, particularly speed and flow rates.<br>• Other rates, particularly density.<br>• Converting between units for a rate. | Students investigate the idea of a rate as the change of one measurement with respect to another in practical situations. Speed and density are the most obvious of these; however, items/factors such as the 'gsm' rate for the thickness of paper or card, or flow rate from a tap provide useful contexts for investigation.<br><br>Conversion between units for rates (e.g. m/sec and kph) is an important aspect of this subtopic. |

## Topic 3: Statistical Investigation

This topic begins with the consideration of the structure of the process of statistical investigation from the collection of data using various methods of sampling, through its analysis using measures of central location and spread, to the formation of conjectures and the drawing of conclusions based on that analysis.

In sampling there is emphasis on the importance of eliminating bias as well as ensuring the validity and reliability of results. Analysis of data incorporates its representation in tabular and graphical form (stem-and-leaf plots, box-and-whisker diagrams, and histograms) and the calculation of summary statistics from the sample.

Students learn to form conjectures that are supported or refuted by a logical argument, using justification from the results of their analysis. The suitability of the statistical tools and measures used in the solution of the problems is emphasised throughout this topic. Electronic technology is used to aid in the statistical investigation process.

## Topic 3: Statistical Investigation

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
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| **Sub-topic 3.1: The statistical investigation process** | |
| What are some examples of situations in which statistics are used to analyse and investigate problems?<br><br>The 'statistical process'<br>• Identifying the problem<br>• Formulating the method of investigation<br>• Collecting data<br>• Analysing the data<br>• Interpreting the results and forming a conjecture<br>• Considering the underlying assumptions | Students collect examples of data and statistics reported in the media.<br><br>Students consider the statistical process that underlies the production of the examples they have collected. Individual items are analysed with a view to identifying the context, the problem being solved or investigated, the statistics used, and the data collected.<br><br>Students recognise the conjecture they are being asked to accept, and question the underlying assumptions that might have been made in the analysis. Useful contexts are provided by advertisements that use statistics.<br><br>Students discuss, whether or not they are convinced with the information and arguments that are presented to them.<br><br>In the process of analysing items, students read and interpret data presented in a variety of ways. |
| **Sub-topic 3.2: Sampling and collecting data** | |
| What is a sample and what is the purpose of sampling? | Students handle raw data with a clear purpose. This may be in the form of verifying a claim (e.g. 'on average the eggs we buy from the supermarket weigh 55 grams') or supporting a conjecture about the outcome of an experiment. The focus is on strengthening the statistical arguments used to support conjectures.<br><br>Students develop the understanding that sampling is undertaken to reduce the expense in cost and/or time of assessing an entire population. |
| What is bias and how can it occur in sampling? | They discuss the ways in which bias could affect the reliability of a sample yielding statistics that reflect those of the population. |
| What methods of sampling are there?<br>• Simple random, stratified and systematic sampling methods | Using appropriate sampling methods students select samples of different sizes from a single large population for which the summary statistics are known. As the statistical investigation process is developed through the next three subtopics the effects of sample size are discussed. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 3.3: Classifying and organising data** | |
| Categorical data<br>• Ordinal<br>• Nominal<br><br>Numerical data<br>• Discrete<br>• Continuous | Students become aware of the different measurement levels associated with categorical and numerical data and that there are appropriate ways to analyse and present these different levels. Specifically:<br>• nominal or ordinal data can be summarised in a table of counts or proportions from which a bar chart or pie chart can be drawn<br>• numerical data can be summarised in a frequency distribution table from which a dot plot, stem plot, or histogram can be drawn. |
| How is it appropriate to organise and display data of the different types?<br>• Categorical data – tables and bar or pie charts<br>• Numerical data – dot plot, stem plot, histogram | Students examine a range of types of data set, and use technology to present summaries of data whenever possible. They discuss the advantages and disadvantages of the various choices, and select the most appropriate form of presentation for a particular set of data. Misleading representations are discussed. |
| What is an outlier? How should outliers be dealt with? | Outliers would be identified visually in numerical data at this point (with a more formal definition to be given later, once measures of location and spread have been introduced). Decisions are made about how best to deal with them, remembering that only values that can be justified as invalid should be removed from the data. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 3.4: The shape, location and spread of distributions of numerical data** | |
| What does the distribution of data within a data set look like? | Students identify the shapes of distributions of data, using stem plots, dot plots, or histograms. For single sets of data (e.g. the 'average egg' claim), students should look for the placement of the peak in the graph. For related sets of data (e.g. eggs from black v. white hens), they also look for differences in the characteristics of the shapes of the distributions — symmetry, skewedness, bimodality, etc. |
| What is meant by 'average'?<br>• Measures of central location (median and mean) | Students examine sets of data considering the appropriateness of using median or mean (or possibly modal class) as a measure of 'average'. They are reminded that measures of central location are valid for use only with data measured on an interval scale. |
| How do you decide on the most appropriate measure of 'average'?<br><br>When can these measures become unreliable or misleading? | Discussion is supported by carefully chosen examples of what can distort the different measures of the centre of a distribution. They allow students to choose the one most appropriate for a given purpose and a given set of data. The effect of outliers on measures of centre is discussed. |
| Do sets of data with the same 'average' necessarily tell the same story?<br>• Box-and-whisker plots | Students become aware that the centre, on its own, is of limited use as the descriptor of a distribution, but that it can be used to compare two sets of data or to compare a single set of data with a standard. The 'egg' examples could be used here.<br>Supported by carefully chosen examples and appropriate visual representations, students discuss the differences that can still exist when the 'average' is the same for two or more sets of data. Box plots are reviewed or introduced at this point and the boundaries for outliers calculated using LB = $Q_1$-1.5x IQR, UB = $Q_3$+1.5xIQR. |
| • Measures of spread (range, interquartile range, standard deviation)<br>• Outliers | The various measures of spread are introduced and their limitations discussed, particularly with respect to the influence of outliers. Note that, although it is useful to show how the standard deviation is calculated by hand so that students understand how it is derived, they use electronic technology to find this value in any assessment task. |
| What influence does sample size have on the reliability of findings?<br>• Sample statistics compared with population parameters | Given some sample statistics students can compare these values with known population parameters for different sized samples to gain an understanding of the possible impact of sample size on the reliability of sample statistics as predictors of population parameters. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 3.5: Forming and supporting conjectures across two or more groups** | |
| How do the statistical techniques and measures learnt help to argue whether a claim is true or false?<br>Analysis of numerical data:<br>• graphical representation<br>• dealing with outliers<br>• shape of the distribution(s)<br>• measures of centre and spread<br>• argument to support the conjecture. | This subtopic draws all the threads of the statistical process together. Students investigate questions of interest using real data.<br><br>For numerical data, graphical tools for comparison include back-to-back stem plots, box plots with a common scale, or superimposed histograms. |
| Analysis of categorical data:<br>• table of counts<br>• graphical representation<br>• identification of the mode<br>• calculation of proportions<br>• argument to support the conjecture. | For categorical data, the process is arguably simpler as the ‘claim’ will be proportional (e.g. ‘Most students use Brand X toothpaste’ or ‘Red cars are more popular than blue ones’).<br><br>In arguing the truth of a conjecture students consider the origin of the data and the sampling methods used (if they are known). Students understand that if the conjecture is supported by the sample this does not automatically make it true for the population from which the sample was taken. |

## Topic 4: Applications of Trigonometry

This topic focuses on the calculations involved in triangle geometry and their many applications in practical contexts such as construction, surveying, design, and navigation. An understanding of similarity and right triangle geometry leads students to the development of formulae for the calculation of the area of a triangle.

Non-right triangle trigonometry is introduced through the derivation of the cosine rule from Pythagoras' theorem, and the sine rule from the triangle area formula, $A = \frac{1}{2} ab\text{Sin}\theta$. Students investigate problems that involve solving for unknown sides and angles in triangles found in both two and three dimensional situations. They consider cases where the data presented in sine rule problems may be ambiguous.

## Topic 4: Applications of Trigonometry

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 4.1: Similarity** | |
| In what kinds of problems are triangles important? | Plane figures form the basis of two-dimensional Euclidean geometry and triangles are the simplest of these. The importance of understanding the mathematics of triangles can be demonstrated by considering a variety of problems in contexts such as construction, design, surveying, and navigation. |
| How many measurements are required to determine a triangle uniquely? | Students are already familiar with the names and properties of different types of triangles. Through practical investigation they discover that, in most instances, three measurements involving side lengths and/or angles will determine a unique triangle. The exceptions to this (i.e. AAA and SSA) are discussed using specific examples. |
| Under what conditions can two triangles be proved to be similar? | Students understand that similar figures are in proportion to one another – ie one is an enlargement of the other with a scale factor relating corresponding measurements.<br>The conditions for similarity of triangles are discovered through modification of the rules found above.<br><br>Constructions are done by hand and/or using an interactive geometry software package to demonstrate to students the (in)validity of establishing similarity using different sets of conditions. |
| How can similarity be used to solve problems? | Students solve problems set in practical contexts by establishing similarity, setting up a proportion, and solving for the unknown side. |
| **Sub-topic 4.2: Right triangle geometry** | |
| What mathematical tools are there for solving problems involving right-angled triangles?<br>• Pythagoras' theorem<br>• Trigonometric ratios | Pythagoras' theorem is also covered in Topic 2.<br>Students' understanding of the trigonometric ratios for right triangles is consolidated. The idea of similarity is critical to understanding why each angle has its own unique values of sine, cosine, and tangent.<br>Problems are presented in 2D and 3D contexts and with practical activities where appropriate, for example:<br>• finding the height of an object, using an inclinometer<br>• finding the angle of inclination of the Sun<br>• determining whether or not a volleyball court is truly rectangular<br>• calculating the length of ladder needed to safely reach an otherwise inaccessible spot<br>• calculating the vertical angle of a cone given its diameter and height |

## Key Questions and Key Ideas

## Considerations for Developing Teaching and Learning Strategies

### Sub-topic 4.3: Area of triangles

How is the area of a non-right triangle found if the perpendicular to a side cannot be measured easily or accurately?

$Area = \frac{1}{2}ab \sin C$

Using right angled triangles and trigonometry students are led through the derivation of the formula $A = \frac{1}{2}ab.\sin C$.

A practical comparison measuring and finding the area of one triangle using each of the three angles is a useful exercise to demonstrate the veracity of the formula. It forms the basis of a discussion of the significance of errors of measurement.

How can the area of a triangle be determined from its three sides?

- Heron's rule

$Area = \sqrt{s(s-a)(s-b)(s-c)}$

$$\text{where } s = \frac{a+b+c}{2}$$

Heron's rule is applied to the side measurements of the triangle above to show it as an alternative way of calculating area. (The derivation of Heron's rule is not required in this course but could be shown to students where appropriate.)

A variety of practical and contextual problems are posed requiring students to decide which rule to use and/or which measurements to take to find a specified area.

## Sub-topic 4.4: Solving problems with non-right triangles

| | |
|---|---|
| How are problems in which the triangles involved are not right-angled solved? | The need for tools to deal with non-right triangles are emphasised by posing problems in contexts such as surveying, building, navigation, and design. Students are asked how they would find the answers to these problems, using the skills they have learnt. The validity and/or shortcomings of methods such as scale drawing and trial and error is discussed. |
| The cosine rule<br>• Solving for the third side when two sides and the included angle are known<br>• $a^2 = b^2 + c^2 - 2bc \cos A$ | Derivation of the cosine rule, using Pythagoras' theorem, is shown. Students recognise that the cosine rule is a 'generalised' version of Pythagoras' theorem with a 'correction factor' for angles that are larger or smaller than 90°. Students to understand why the cosine of obtuse angles is negative (alternatively, calculation for a few such angles would demonstrate this fact). |
| • Solving for angles when the three sides are known | Students rearrange the formula into a form that can be used to find unknown angles.<br><br>Problems requiring the finding of an unknown side or angle using the cosine rule are drawn from real situations and posed in context. |
| The sine rule<br>• Solving triangles where two sides and the non-included angle are known<br>• $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$<br>• Solving triangles where two angles and one side are known | The derivation of the sine rule from the area formula (or otherwise) is shown.<br>Confirmation of the sine rule by direct measurement is useful.<br><br>Students find the solution of contextual problems drawn from real situations for an unknown side or angle, using the sine rule.<br><br>The problems posed for finding an unknown side or angle using the sine rule are drawn from real situations and the answers interpreted in context.<br><br>Ambiguous cases where there are two possible solutions from the given information are presented for discussion.<br>Situations are discussed where, due to rounding or because contradictory data has been given in the question, the sine or cosine value calculated is greater than one and hence no angle can be found. |
| • Solving problems involving direction and bearings. | Students understand bearings and how to interpret them so that practical problems involving navigation and angles of elevation and depression can be solved by applying trigonometry. |

## Topic 5: Linear Functions and their Graphs

This topic focuses on developing the process of mathematical modelling. It examines linear functions through a study of the various forms in which such relationships can be represented –contextual, numerical, graphical and, in particular, algebraic. Students identify the links that allow them to move between these representations to analyse and solve problems, and make predictions. To deepen their understanding and improve their facility with these concepts students experience applications of linear functions in a wide variety of contexts. The investigation of piece-wise linear and step functions extends this study to some familiar applications, such as the calculation of income tax.

# Topic 5: Linear Functions and their Graphs

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 5.1: Linear equations** | |
| How can problems that involve linear functions be represented? | Mathematics is used to create models of situations we want to study. These models are useful for studying relationships, observing patterns, and making predictions. Linear functions are the simplest of these models. The functions used in this subtopic are drawn from a range of practical contexts (e.g. taxi fares, simple interest, water rates, timed telephone call charges). |
| • Contextual description | Students begin this study by considering examples such as the way a plumber, who has a callout fee of $40 and a rate of $25 per hour, calculates the charge for a job. How can we use this information to solve problems such as 'How much should be charged for a job taking 4 ¼ hours?' or 'If the charge is $520 how long did the job take?' Students provide estimated answers without requiring a formal approach. The posing of similar problems in more complex contexts and with less simple parameters lead them to ask 'Is this the most efficient way of solving these problems?' |
| In what other ways can such a problem be represented?<br>• Numerical table of values | Students create a systematic table of values and identify the growth pattern in the plumber's charge values. The labels for the two columns in the table are used to introduce the idea of independent and dependent variables. By organising the data in this way students answer the questions posed above more confidently. The usefulness and limitations of using the table for finding interpolated and extrapolated values is discussed. |
| • Graphical representation<br>- Slope and intercepts in context<br>- Determining *x* or *y* value from a linear graph given the other corresponding value. | The pairs of values in the table are plotted onto a graph. The linearity of the graph is discussed and terms such as slope and axis intercepts are interpreted in context. Students use the graph to determine answers to the questions posed above and compare them with the other answers found. Limitations to the accuracy and efficiency of finding solutions when using a graph to find interpolated and extrapolated values are discussed. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| • Algebraic formula<br>- Developing a linear formula from a word description<br>- Substitution and evaluation<br>- Rearrangement of linear equations<br>- Solving linear equations | Students express the relationship between hours worked (*x*) and amount charged (*y*) as a verbal rule and are able to appreciate its translation into an algebraic equation – namely $y = 25x + 40$. By substituting for either variable and rearranging (if necessary) the problems posed at the beginning of the topic, and any others, can be solved efficiently. |
| What are the links between the four ways of representing a linear relationship? | Students understand the links between the four representations of the problem being studied, namely:<br>• The two parameters ($40 callout fee, $25 per hour rate) in the contextual description<br>• The initial value (for $x=0$) and the constant increment (+$25) in the *y*-values in the table<br>• The *y*-intercept and slope of the graph<br>• The constant term and coefficient of *x* in the formula,<br>This allows translation between any two of these forms when solving problems.<br><br>Students are exposed to a wide variety of problems posed in different forms and contexts to strengthen their understanding of the representations of the linear model and the links between them. Once students have gained an understanding of the concepts, electronic technology is introduced to aid the construction of tables and graphs. |
| **Sub-topic 5.2: Simultaneous linear equations** | |
| When a problem has two independent variables how much information is required to determine a unique solution? | Students realise that information given in a problem may not always produce a unique answer. For example, if told a cycle shop display containing bikes and trikes has 9 cycles, a student cannot determine how many of them are bicycles and how many are tricycles. They can, however, narrow it down to a limited set of possibilities.<br>It is only with a further piece of information (for instance that the display also has 23 wheels) that students can determine exactly how many bicycles and tricycles are in the shop display. |
| How can contextual problems involving simultaneous linear equations be solved efficiently?<br>• Where two lines meet on a graph (without electronic technology) | Students begin by trying to solve such a problem by trial and error, listing numerical possibilities for each constraint and finding a pair that match. However, by considering the graphical and algebraic representations of the linear relationships involved more efficient methods of solution are demonstrated. |
| • Algebraic manipulation | Students solve pairs of simultaneous linear equations using algebraic methods (elimination and/or substitution). |
| • Using electronic technology:<br>- Graphically<br>- Using the equation solver functionality | Electronic technology (either by graphing or using the equation solver functionality) is used once students understand the methods involved. Students rearrange linear relationships into appropriate forms for the entry of data into the technology they are using. |
| • Non-unique solutions | Situations in which a non-unique solution arises, due to insufficient information (two equivalent equations), or inconsistent information (parallel lines), given in the original problem, are examined both graphically and algebraically. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 5.3: Piece-wise linear and step functions** | |
| What are piecewise linear relationships?<br>• Piecewise linear functions<br>• Step Functions | A piecewise function is one composed of several straight-line sections. The different types of piecewise functions are considered in context. Examples include:<br>- Calculating income tax with changing tax brackets<br>- Calculating parking fees according to a scale based on a charge for each number of hours (or part thereof)<br>- Calculating the total cost of purchasing goods where the cost per item changes over intervals (eg Bulk discounts when purchasing electronic components by mail order). |
| In what contexts do they occur?<br>• Construction of the algebraic formulae for piecewise linear and step functions<br>• Graphing these relationships using electronic technology<br>• Solving problems using both algebraic and graphical representations | The variables in these scenarios can be either continuous or discrete. The continuity of the functions and ways of dealing with the issues that arise in the case of discontinuous (step) relationships is discussed.<br><br>For example:<br>- How is the graph drawn so that the same parking time does not attract two different fees?<br>- Is it possible that buying six electronic parts could be cheaper than buying five? |

## Topic 6: Matrices and Networks

This topic introduces students to discrete mathematics through the application of matrices and graph theory to solving problems in familiar contexts. Three different applications of matrices are studied: costing and stock management, connectivity of networks, and transition problems.

The first application, costing and stock management, allows matrix arithmetic to be learnt in a context where it can be logically developed. The second application introduces the concepts and terminology of networks and shows how matrices are used to examine the efficiency or reliability of a network system by considering the number of subsidiary paths it contains. In the third application, transition matrices, future trends are predicted in a situation where things change state over time with known probabilities.

In each case students examine problems set in a variety of contexts, and discuss the appropriateness of the models and the usefulness of the solutions found. Electronic technology is used extensively for calculations involving matrix multiplication and for investigating the effects on the outcomes of altering initial parameters.

## Topic 6: Matrices and Networks

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 6.1: Matrix arithmetic and costing applications** | |
| What is a matrix?<br><br>How is information organised in a matrix?<br>• Columns and rows in a matrix<br>• Order (or dimensions) of a matrix | The introduction of matrices and their arithmetic is carried out using concrete examples. For instance, suppose that a business has three different outlets from which its stock is sold and that this stock comprises four different components. The amount of stock that has been delivered to each outlet in a particular month, for instance, can be efficiently stored, displayed, changed, or retrieved if it is kept in the form of a labelled rectangular array – i.e. a matrix.<br><br>The definition of the order of a matrix as the number of rows by the number of columns is explained. |
| In what ways can costing and stock information in matrix form be manipulated?<br>• Adding and subtracting matrices<br>• Multiplication by a scalar<br>• Matrix multiplication<br>- by a row or column matrix<br>- using matrices of higher order<br>- multiplying by a row or column matrix of 1's. | Initially students carry out all matrix arithmetic by hand to develop a clear understanding of the processes.<br><br>The need for two matrices to be of the same order for them to be added or subtracted is quite common if handled in a context such as stock control. Each corresponding position in the matrices represents something unique and the way to combine the numbers when adding the matrices is obvious. Similarly, multiplying any matrix by a number (scalar) is clear in this context.<br><br>The concept of matrix multiplication is complex. By using an example such as 'Column matrix A represents the number of two different items (say chops and sausages ordered by Mario for a barbeque) and matrix B is a row matrix containing the cost of each item', the idea of multiplying pairs of numbers and summing the result to find the total cost is sensible.<br><br>By looking at the structure of the calculations with respect to the labels on rows and columns of the two matrices it can be seen how the chop and sausage labels are matched up and then disappear in the final answer, eg Cost of a chop x no. of chops for Mario + cost of a sausage x no. of sausages for Mario = Total cost for Mario.<br><br>Judicious use of technology is used for matrix calculations once students are familiar with how they are carried out.<br><br>Emphasis is placed on the acceptance of the convention of why the costs are given in a row matrix and not a column matrix in the case of the last example. Examples are also used where A is transposed and the 'cost' matrix is a column matrix.<br><br>Multiplying a rectangular matrix A on the left by a row matrix of 1's (of appropriate size) has the useful property of summing the elements in each column of A. Similarly multiplying on the right by a column matrix of 1's sums the rows. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| • Using electronic technology to do matrix arithmetic | Having understood the processes of matrix arithmetic it is appropriate for students to use electronic technology to carry out most calculations, however they should be judicious in its use and perform calculations by hand this would be more efficient. |
| How can matrices be used to solve problems in costing and inventory control? | Students investigate costing and inventory problems that are set in a variety of practical contexts that require the use of a range of matrix arithmetic operations. |
| **Sub-topic 6.2: Application of matrices to network problems** | |
| What information is given in a network diagram? | Students examine and work with network diagrams drawn from a variety of contexts. These could include, for example, flow charts, precedence diagrams, maps, family trees, results of a sporting competition, and social relationships. |
| • Reading information from a network diagram | By identifying the context students interpret the information being presented in a weighted and directed network diagram (e.g. distance between nodes, time of travel, direction of travel, capacity of an arc, winning player, social influence) and answer specific questions about a situation. |
| • Deducing relationships | Students realise that a network shows relationships and interconnections that are not always spatial. A study of a precedence network of jobs that make up a complex task is used as a case in point. |
| • Using appropriate terminology | The correct terminology is taught where it is relevant to the problems studied (e.g. arcs, nodes, directed and undirected networks, trees, circuits, and Euler paths) to enable consistent and concise communication in the discussion of networks and network problems. |
| How can a matrix be used to show the connections in a network?<br>• Connectivity matrices | Students convert from a directed network diagram to its representation as a connectivity matrix of zeroes and ones (usually), and vice versa. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| How do matrix operations help to find the number of indirect connections in a network?<br>• Powers of matrices and multi-stage connections | By looking at the structure of matrix multiplication it can be seen that squaring the connectivity matrix counts the number of directed two-step connections between any pair of vertices in the network.<br>It can thus be induced that higher powers of the connectivity matrix similarly count the number of higher order multi-step paths. |
| • Limitations of using higher powers | Although knowing how many n-step paths there are between a pair of vertices (A and B, say) is of some use, the information is limited because it is not known, without reference to the network itself, what these paths are, nor whether they are sensible (for example a 3-step path ABAB is not efficient when the 1-step path AB would suffice). |
| Of what use are weighted sums of the powers of connectivity matrices?<br>• Measures of efficiency or redundancy<br>• Prediction in dominance relationships | In a communications network, sums of the powers of the connectivity matrix can be used to examine how well the network is connected or to find the minimum number of steps required for there to be a path between every pair of nodes. They are used to examine the effects of adding or removing one or more arcs in the network.<br>In a dominance situation (such as a round robin sporting tournament) weighted sums are used to rank the power of the players and predict the likely outcome of unplayed matches. |
| • Reasonableness of weightings and limitations of the model | Due to underlying assumptions there are limitations to using matrix models in situations as those described above. The elements of a connectivity matrix show whether connections exist but do not take account of any other qualitative or quantitative properties of those connections.<br>Different weightings in a weighted sum could give different results, so their significance needs to be considered in the context of the problem.<br><br>In a dominance situation, powers higher than 2 allow for cycles (non-zero elements on the main diagonal) which may not be appropriate and therefore need to be dealt with carefully. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 6.3: Application of matrices to transition problems** | |
| What is a transition matrix and what are its properties?<br>• 2 x 2 systems | When consumers buy different brands of a certain product, how can a company determine what its market share is likely to be in the long term? Although there is an element of unpredictability about what an individual might do from one purchase to the next, matrix methods can be used to predict overall trends if something is known about the buying patterns of the whole population.<br><br>Beginning with a simple 2 x 2 situation students construct the matrix (T) of transition percentages (as decimals) and discuss the reason for the necessity that the rows add to one. |
| How can future trends be predicted? | It can be helpful to demonstrate with physical objects (such as counters) how the consumers move from one brand to another over time given the transition proportions in T.<br><br>The number of consumers who are buying each brand at a particular time can be represented as a row matrix (R). The matrix product RT will then give the number of consumers expected to buy each brand next time they buy the product. By continuing to multiply by T trends can be predicted further into the future. |
| What happens in the long run in a transition model?<br>• The steady state | Multiplication of R by successive powers of T will show a convergence to a 'steady state', where the market shares no longer change. It should be noted that although the proportions of the population buying each brand are no longer changing, there are still consumers switching between brands. Through investigation students establish that the distribution of customers in R has no effect on the final market shares in the steady state - it is the proportions in T alone that determine the steady state behaviour.<br><br>Calculating high powers of the matrix T where the rows become identical will indicate the proportions of the population who are buying each brand in the steady-state market. |
| What effect do changes to the initial conditions have on the steady state? | Students investigate what happens to the steady state if the probabilities in the initial transition matrix are changed. The motivation for this change is kept in context (e.g. one company mounts a strong advertising campaign, another receives adverse publicity, or the price of one brand changes significantly). |
| • 3 × 3 or higher systems | Once students have established an understanding of the 2x2 transition matrix model in the market shares context, the problems are expanded to matrices of order 3 and higher and to a variety of other contexts. |
| What are the limitations of the transition matrix model? | In the matrix model it is necessary to assume that the transition probabilities are fixed over a significant period of time to reach the steady state. It is also assumed that the same customers are involved every time (i.e. it is a closed system) and that they all make a purchase in every time period. Quite often at least one of these assumptions is unrealistic. It is important for students to realise that although the model is mathematical it is not necessarily accurate. |

# ASSESSMENT SCOPE AND REQUIREMENTS

Assessment at Stage 1 is school based.

## EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 1 General Mathematics.

Assessment Type 1: Skills and Applications Tasks
Assessment Type 2: Mathematical Investigation

For a 10-credit subject, students should provide evidence of their learning through four assessments. Each assessment type should have a weighting of at least 20%. Students undertake:

- at least two skills and applications tasks
- at least one mathematical investigation.

For a 20-credit subject, students should provide evidence of their learning through eight assessments. Each assessment type should have a weighting of at least 20%. Students undertake:

- at least four skills and applications tasks
- at least two mathematical investigations.

**It is anticipated that from 2018 all assessments will be submitted electronically.**

## ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by teachers to:

- clarify for the student what he or she needs to learn
- design opportunities for the student to provide evidence of his or her learning at the highest level of achievement.

The assessment design criteria consist of specific features that:

- students need to demonstrate in their evidence of learning
- teachers look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

### Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships
CT2 Selection and application of techniques and algorithms to solve problems in a variety of contexts
CT3 Application of mathematical models
CT4 Use of electronic technology to find solutions to mathematical problems.

### Reasoning and Communication

The specific features are as follows:

RC1 Interpretation and evaluation of mathematical results, with an understanding of their reasonableness and limitations
RC2 Knowledge and use of appropriate mathematical notation, representations, and terminology
RC3 Communication of mathematical ideas, and reasoning, to develop logical arguments
RC4 Forming and testing of conjectures *

* In this subject the development and testing of conjectures (RC4) is not intended to include formal mathematical proof.

## SCHOOL ASSESSMENT

### Assessment Type 1: Skills and Applications Tasks

For a 10-credit subject, students complete at least two skills and applications tasks. For a 20-credit subject, students complete at least four skills and applications tasks.

Skills and applications tasks are completed under the direct supervision of the teacher.

Students find solutions to mathematical problems that may:
- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require students to demonstrate an understanding of relevant mathematical concepts and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information to find solutions to routine and some analytical and/or interpretative problems.

Students provide explanations and use correct mathematical notation, terminology, and representation throughout the task. They may be required to use electronic technology appropriately to aid and enhance the solution of some problems.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:
- concepts and techniques
- reasoning and communication.

### Assessment Type 2: Mathematical Investigation

For a 10-credit subject, students complete at least one mathematical investigation. For a 20-credit subject, students complete at least two mathematical investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers may give students a clear, detailed, and sequential set of instructions for part of the investigation or to initiate the investigation, or may provide guidelines for students to develop contexts, themes, or aspects of their own choice. Teachers should give some direction about the appropriateness of each student’s choice and guide and support students’ progress in a mathematical investigation.

If an investigation is undertaken by a group, students explore the problem and gather data together to develop a model or solution individually. Each student must submit an individual report.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use a variety of mathematical and other software (e.g. statistical packages, spreadsheets, CAD, accounting packages) to enhance their investigation.

In a report, students form and test conjectures, interpret and justify results, summarise, and draw conclusions. Students are required to give appropriate explanations and arguments.

A report on the mathematical investigation may take a variety of forms, but would usually include the following:

- an outline of the problem to be explored
- the method used to find a solution
- the application of the mathematics, including
    - generation or collection of relevant data and/or information, with a summary of the process of collection
    - mathematical calculations and results, using appropriate representations
    - discussion and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem
- a bibliography and appendices, as appropriate.

The format of an investigation report may be written or multimodal.

Each investigation report should be a maximum of 8 pages if written, or the equivalent in multimodal form.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

## PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills and understanding that teachers refer to in deciding, on the basis of the evidence provided, how well a student has demonstrated his or her learning.

During the teaching and learning program the teacher gives students feedback on, and makes decisions about, the quality of their learning, with reference to the performance standards.

Students can also refer to the performance standards to identify the knowledge, skills, and understanding that they have demonstrated and those specific features that they still need to demonstrate to reach their highest possible level of achievement.

At the student's completion of study of a subject, the teacher makes a decision about the quality of the student's learning by:

- referring to the performance standards
- taking into account the weighting given to each assessment type
- assigning a subject grade between A and E.

Teachers can use a SACE Board school assessment grade calculator to help them to assign the subject grade. The calculator is available on the SACE website ([www.sace.sa.edu.au](http://www.sace.sa.edu.au))

## Performance Standards for Stage 1 General Mathematics

| | Concepts and Techniques | Reasoning and Communication |
|---|---|---|
| **A** | Comprehensive knowledge and understanding of concepts and relationships.<br>Highly effective selection and application of techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.<br>Successful development and application of mathematical models to find concise and accurate solutions.<br>Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | Comprehensive interpretation and critical evaluation of mathematical results, with an in-depth understanding of their reasonableness and possible limitations.<br>Proficient and accurate use of appropriate mathematical notation, representations, and terminology.<br>Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments. |
| **B** | Some depth of knowledge and understanding of concepts and relationships.<br>Mostly effective selection and use of mathematical techniques and algorithms to find accurate solutions to routine and some complex problems in a variety of contexts.<br>Attempted development and successful application of mathematical models to find accurate solutions.<br>Mostly appropriate and effective use of electronic technology to find accurate solutions to routine and some complex problems. | Mostly appropriate interpretation and some critical evaluation of mathematical results, with some depth of understanding of their reasonableness and possible limitations.<br>Mostly accurate use of appropriate mathematical notation, representations, and terminology.<br>Mostly effective communication of mathematical ideas and reasoning to develop logical arguments. |
| **C** | Generally competent knowledge and understanding of concepts and relationships.<br>Generally effective selection and use of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts.<br>Application of mathematical models to find generally accurate solutions.<br>Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | Generally appropriate interpretation and evaluation of mathematical results, with some understanding of their reasonableness and possible limitations.<br>Generally appropriate use of mathematical notation, representations, and terminology with some inaccuracies.<br>Generally effective communication of mathematical ideas and reasoning to develop some logical arguments. |
| **D** | Basic knowledge and some understanding of concepts and relationships.<br>Some effective selection and use of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts.<br>Some application of mathematical models to find some accurate or partially accurate solutions.<br>Some appropriate use of electronic technology to find some accurate solutions to routine problems. | Some interpretation and attempted evaluation of mathematical results, with some awareness of their reasonableness and possible limitations.<br>Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.<br>Some communication of mathematical ideas with attempted reasoning and/or arguments. |
| **E** | Limited knowledge or understanding of concepts and relationships.<br>Attempted selection and limited use of mathematical techniques or algorithms, with limited accuracy in solving routine problems.<br>Attempted application of mathematical models, with limited accuracy.<br>Attempted use of electronic technology, with limited accuracy in solving routine problems. | Limited interpretation or evaluation of mathematical results, with limited awareness of their reasonableness and possible limitations.<br>Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.<br>Attempted communication of mathematical ideas, with limited reasoning. |

## ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments. This policy is available on the SACE website (wwww.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement in the school assessment are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 1 are available on the SACE website (www.sace.sa.edu.au).

# SUPPORT MATERIALS

## SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

## ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).

# Stage 2
# General Mathematics

## LEARNING SCOPE AND REQUIREMENTS

### LEARNING REQUIREMENTS

The learning requirements summarise the knowledge, skills, and understanding that students are expected to develop and demonstrate through learning in Stage 2 General Mathematics.

In this subject, students are expected to:

1. understand mathematical concepts, demonstrate mathematical skills, and apply mathematical techniques
2. investigate and analyse mathematical information in a variety of contexts
3. recognise and apply the mathematical techniques needed when analysing and finding a solution to a problem, including the forming and testing of conjectures
4. interpret results, draw conclusions, and reflect on the reasonableness of solutions in context
5. make discerning use of electronic technology to solve problems
6. communicate mathematically and present mathematical information in a variety of ways.

### CONTENT

Stage 2 General Mathematics is a 20-credit subject.

Stage 2 General Mathematics offers students the opportunity to develop a strong understanding of the process of mathematical modelling and its application to problem solving in everyday workplace contexts.

A problems-based approach is integral to the development of both the models and the associated key ideas in the topics. These topics cover a range of mathematical applications including: linear and non-linear functions, statistics, finance and optimisation.

Stage 2 General Mathematics consists of the following list of five topics:

1. Modelling with Linear Relationships
2. Modelling with Non-linear Relationships
3. Statistical Models
4. Financial Models
5. Discrete Models

Topics 1 to 4 are presented in this order to allow for the sequential development of some models; however there is flexibility in the sequence of the teaching of subtopic of Topic 5.

Each topic consists of a number of subtopics. These are presented in the subject outline in two columns as a series of ‘key questions and key ideas’ side-by-side with ‘considerations for developing teaching and learning strategies’.

A problems-based approach is integral to the development of the computational models and associated key ideas in each topic. Through key questions teachers develop the key concepts and processes that relate to the mathematical models required to address the problems posed.

The ‘considerations for developing teaching and learning strategies’ present suitable problems and guidelines for sequencing the development of ideas. They also give an indication of the depth of treatment and emphases required.

The ‘key questions and key ideas’ cover the prescribed content for teaching, learning, and assessment in this subject. It is important to note that the considerations for developing teaching and learning strategies are provided as a guide only. Although the material for the external examination  will be based on the ‘key questions and key ideas’ outlined in the five topics, the applications described in the ‘considerations for developing teaching and learning strategies’ may provide contexts for examination questions.

General Mathematics prepares students for entry to tertiary courses requiring a non-specialised background in mathematics.

## Topic 1: Modelling with Linear Relationships

Students consider arithmetic sequences as discrete models of linear growth. They draw parallels with, and review the concepts of, continuous linear functions studied in Stage 1, Topic 5. Linear programming is introduced as a major application of linear functions. The solution of problems involving the interaction of two variables is investigated in depth. At first, students solve problems set in realistic contexts with which they are familiar. They are encouraged to solve these problems by trial and error before being presented with the linear programming algorithm, so that they can appreciate the dynamic nature of the problems.

Problems are posed in everyday contexts. Examples of situations where these techniques are used are discussed with, or researched by, students.

Students investigate the effects on the optimal solution of changing the initial parameters in some problems. For instance, what effect will changing the objective function or the constraints have on cost, profit, or wastage? In situations where there are multiple optimal solutions students discuss the merits of the different choices beyond the value of the objective function. Students explore to situations involving both discrete and continuous variables and understand how to deal appropriately with an optimal solution that is not achievable in a context where only discrete values of the variables are acceptable.

Students use electronic technology to support the efficient investigation of linear programming scenarios.

## Topic 1: Modelling with Linear Relationships

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 1.1: Linear models** | |
| What are arithmetic sequences and where do they occur?<br>• Recursion formulae | Students examine examples of sequences, some drawn from practical contexts, which display growth by the addition of a constant (positive and negative). Given a starting point they generate recursive formulae to predict the next term in a sequence from the one that precedes it. |
| What are the different ways an arithmetic sequence can be represented and what are the links between them?<br>• Contextual description<br>• Numerical sequence<br>• Graph<br>• Algebraic formula<br>- Finding the value of a term<br>- Finding the position of a term in the sequence | From studying linear functions in Stage 1 General Mathematics students will already be familiar with the four ways of representing relationships and the links between them. This key question is used to review these concepts and the links between them for arithmetic sequences.<br><br>The direct formula for calculating terms in the sequence are deduced and are seen to be useful for finding specified terms or determining whether a particular value is a member of the sequence.<br><br>Students use the tools above to solve problems set in practical contexts.<br><br>Simple interest grows as an arithmetic sequence when a bank pays interest only on money held in an account for the full period of time. |
| How can arithmetic sequences be used to model and analyse practical situations involving linear growth and decay?<br>• Simple interest<br>• Other models | Other contexts where discrete values might occur include taxi fares and parking fees that often charge in discrete steps, hence generating arithmetic sequences of values. |
| What changes if the independent variable is continuous?<br>• Linear functions<br>• Simple interest and other models | Students make the links between the discrete (arithmetic sequences) and continuous (linear functions) models of situations with a constant growth rate. They understand the differences between them in the graphical and algebraic representations.<br>Simple interest is revisited as a continuous model. Students understand the change in conditions allowed by the bank for this to be appropriate.<br>Other models could include, for example, charges for work by a tradesman, or cooking times for weights of a roast. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 1.2: Linear programming** | |
| How can linear functions be used to optimise a situation where we have control of two variables? | Students consider what might be optimal (e.g. maximum profit, minimum cost, efficient use of materials) in a specific situation where two variables are involved. For example:<br>• The Year 12 students have some materials left over from the making of their school jumpers. They decide to use these materials to make two types of stuffed toy, which they will sell to raise money for their formal. What is the best way to use the materials?<br>This problem requires a combination of two things as its solution. By suggesting possible combinations and testing for their feasibility, students gain an appreciation of the dynamic nature of the problem. The number made of one type of toy will affect the materials available to make the other.<br>At this point a better way of organising the information is introduced. If different combinations of the two toy designs that will completely use up one of the materials are plotted on a grid, the straight-line relationship becomes apparent and its linear function is deduced. By testing points from either side of the line in the equation the idea of graphing inequalities is developed and the feasible region is constructed.<br>When the possible solutions have been found, students use trial and error to seek the best solution for several different objective functions. They deduce for themselves that these solutions always occur on the boundary of the region. To find the corner points of the feasible region the solution of simultaneous equations are used. |
| • Setting up constraints and objective function<br>• Graphing the feasible region<br>• Finding the optimal solution<br>• Considering wastage | The solution process can be formalised as a set of steps which are carried out by hand as well as with the aid of electronic technology:<br>• Formulate and graph the constraints with appropriate labels<br>• Formulate the objective function<br>• Identify the feasible region and calculate its vertices<br>• Evaluate the objective function at each vertex<br>• Compare the values to find the optimal solution.<br>Students identify which of the originally available resources are used up in the optimal solution and calculate the 'wastage' or what is left of the rest. |
| How do we deal with an optimal solution that is not achievable because only discrete values are allowed? | In some contexts the variables can only take certain discrete values (e.g. whole numbers of garments or half tablets). The optimal solution given by the algorithm may need to be adjusted to be achievable. In such a case students test the discrete points that surround the optimal point to find the best solution that is possible, being careful to stay within the feasible region. |
| What happens to the optimal solution if the original parameters change? | Students make changes to the objective function and/or constraint parameters to investigate the effects on the optimal solution and wastage. |

## Topic 2: Modelling with Non-linear Relationships

Students investigate the growth of geometric sequences and compare it with the linear growth model of arithmetic sequences studied in the previous topic. They examine the geometric growth model in its four representations – contextual, numerical, graphical and algebraic, and extend this to the continuous exponential function model.

Applications of exponential functions set in a variety of social, scientific, and practical contexts, such as population growth and radioactive decay, are studied with a view to solving problems and making predictions.

The use of logarithms is not required in this subject. All problems involving the solution for values of a power are carried out using electronic technology, either graphically or using the equation solver facility. Students examine the exponential nature of the growth of compound interest and inflation encountered in Stage 1, Topic 1. Solving problems in these contexts without the use of the financial package in the graphic calculator reinforces the underlying exponential model.

The development of the geometric series model provides an introduction to the annuity model that will be developed further in Topic 4: Financial Models.

## Topic 2: Modelling with Non-linear Relationships

**Key Questions and Key Ideas** | **Considerations for Developing Teaching and Learning Strategies**

### Sub-topic 2.1: Geometric Sequences

What are geometric sequences?

Students examine examples of sequences, some drawn from practical contexts, which display growth by the multiplication of a constant (both greater and less than one). They generate recursive formulae to predict the next term in a sequence from the one that precedes it.

How does their growth differ from arithmetic sequences?

From the numerical values in the sequences students observe and discuss the difference in the nature of the growth of a geometric sequence as opposed to an arithmetic sequence.

What are the different representations like for a geometric sequence and how do we move between them?

- Features of the graph
- Derivation of the direct formula for the terms in a geometric sequence

Using contextual examples students revisit the four ways of representing a mathematical model:

- Contextual
- Numerical
- Graphical
- Algebraic

They note the specific features associated with the geometric model, namely the y-intercept, asymptotic behaviour and shape of the graph, and the form of the algebraic formula

$$T_n = a_1 . r^{n-1} = \left(\frac{a_1}{r}\right) . r^n ,$$

as well as the links between them.

## Key Questions and Key Ideas

## Considerations for Developing Teaching and Learning Strategies

### Sub-topic 2.2: Exponential Functions

What is the continuous equivalent of a geometric sequence?

- Exponential functions

In most contexts involving geometric growth the variables are more appropriately considered as continuous and the transition from the geometric sequence to an exponential function of the form $y = a \, . \, b^x$ is easily made.

Where can exponential growth or decay be found?
How can the model be used to solve problems?

- Compound interest

There are many contexts in which exponential growth (or decay) can be found. One context immediately useful is compound interest, and the work done in this subtopic will provide links with the financial models studied in Topic 4 – Financial Models.

Students derive the formula for compound interest and investigate how changes to the initial conditions, such as the compounding period, affect the growth and the future value.

Problems requiring the finding of present and future values, time, and interest rate are solved with electronic technology either using a graph or an equation solver (the use of the TVM (financial) package is addressed in Topic 4). Algebraic techniques that use logarithms and $n^{th}$ roots are not required.

- Other growth contexts (Population growth, Inflation, etc)
- Decay contexts (Radioactive decay Depreciation, cooling etc)

Other problems involving both growth and decay are posed in a variety of contexts. These problems begin with a situation in which the algebraic model is either given or can be deduced. Once the model has been determined it is used to find values for one variable given the value of the other using either a graph or an equation solver where necessary. The algebraic solution of exponential equations using logarithms is not required.

- Finding percentage growth (or decay)

Students express the parameters of an exponential model as a percentage gain or loss per time period, which can be interpreted in the context of the problem.

Geometric series

$$F_v = P_{mt}\left(\frac{r^n - 1}{r - 1}\right)$$

The compound interest model is extended to the idea of a regular savings annuity by considering the idea of a geometric series. The use of spreadsheets shows students how an annuity works.

Students see the derivation of the formula and use it to solve simple future value problems by direct substitution. They become aware that for anything more complex (such as finding *n* or *r*) using the TVM (financial) package in electronic technology is a more efficient tool that will be explored in Topic 4 - Financial Models.

## Topic 3: Statistical Models

The linear and exponential growth behaviour studied in the previous topics are observed in bivariate data. By using statistical tools such as scatterplots and regression to analyse this data, students develop algebraic models used for predictive purposes.

Trends in times series data, such as the monthly occupancy rate for a hotel over a period of years, are analysed and modelled using the tools of regression. Data smoothing and seasonal adjustment are applied, where appropriate, to refine the models and make more reliable forecasts of future figures.

The normal distribution is an important mathematical model for making predictions in many social, industrial, and scientific contexts. Students investigate the characteristics and nature of the normal distribution, both through data simulation and graphical representation of the normal probability density function. They use this model to solve problems and make predictions in a range of contexts where data is expected to be approximately normally distributed.

Students use electronic technology to support both calculations and presentation of their work throughout this topic.

## Topic 3: Statistical Models

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 3.1: Bivariate Statistics** | |
| How can bivariate data be modelled when the relationship appears linear but is not perfect?<br>• The statistical investigation process<br>• Explanatory & response variables<br>• Scatter plots<br>• Association | Students examine example sets of paired data and discuss the steps involved in the process of statistical investigation.<br><br>For these data sets students identify the explanatory and response variables from the context (or recognise when two variables may be co-dependent - for instance, marks on Maths and English tests). They construct an appropriately scaled scatterplot and use it to describe the direction (positive/negative), form (linear, non-linear) and strength (strong/moderate/weak) of any association observed between the variables. |
| • Correlation coefficients | Students use electronic technology to calculate the values of $\boldsymbol{r}$ and $\boldsymbol{r^2}$ and use them to assess the strength of a linear association in terms of the explanatory variable. As a guide $r^2 \geq 0.7$ is sufficiently large to proceed with using a least squares regression line for prediction. |
| • The effects of outliers | Outliers are identified visually from the scatterplot. Removing them from the data can strengthen the correlation and improve the way the line of best fit predicts; however there needs to be careful consideration of whether it is appropriate to do this. Removal of outlying data should only be made with reasonable justification. |
| • Causality | Students discuss whether a strong correlation is enough to imply that there is a causal link between the variables. The other possible explanations are coincidence or that both variables are changing in response to a third. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
| --- | --- |
| • Linear regression<br>- $y = ax + b$<br>- interpretation of the values of ‘a’ and ‘b’ | If the correlation is strong enough and the trend is linear it is appropriate to find, using electronic technology, the equation of the least squares line of best fit (linear regression). Students interpret the values of the parameters ‘a’ and ‘b’ in the context of the problem. |
| • Residual plots | A residuals plot is used to decide if a straight line is the best model for the data. A pattern observed in the residuals plot indicates that the trend in the data is, in fact, curved and a nonlinear model might fit the data better. Large residual values also indicate that the linear model may not be appropriate. |
| • Exponential regression<br>- $y = a.b^x$<br>- interpretation of the values of ‘a’ and ‘b’ | If the scatterplot and/or the residuals plot indicate that a linear model may not be appropriate students can use electronic technology to test whether an exponential model works for the data.<br><br>When fitting an exponential model to data students follow the same protocols as they use with fitting a linear model and interpret the parameters of the equation in the context of the problem.<br><br>Students are made aware that electronic technology applies a transformation to the data using *x* versus log *y* so that a linear regression can be used to find the exponential regression equation and calculate a correlation coefficient; however it is not required that they learn to use logarithms or carry out this transformation on the data themselves.<br><br>Note: Electronic technology will usually give the option to express the exponential equation in either $y = a \, . \, b^x$ or $y = a \, . \, e^{kx}$ form. It is not required that students use the second form. For some students it may be considered as a useful extension to discuss Euler’s constant (and show that it can arise from considering ‘continuous compounding’) and perform a calculation to show that $b = e^k$ so the models are equivalent. |
| • Interpolation and extrapolation, reliability and interpretation of predicted results | Once the model for the relationship between the variables has been determined, values of the explanatory variable are used to predict values for the response variable (and vice versa), either between the known data limits (interpolation) or outside the known data limits (extrapolation). The reliability of such predictions is discussed with reference to the correlation statistic and the residual analysis and, in the case of extrapolation, to the validity of assuming that the conjectured trend will continue beyond the data limits. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 3.2: Time Series Analysis**<br>What is a time series?<br>• Trend<br>• Irregular fluctuations (noise)<br>• Seasonal variation | A time series is a collection of observations of data measured at regular time intervals. Such a series can be decomposed into three components: the trend (long term direction), the seasonal movements (systematic, cyclic or calendar related) and the irregular fluctuations (unsystematic, short term, random).<br><br>Examples of seasonal variation include the sharp escalation in retail sales that occurs around the Christmas period, or an increase in water consumption in summer due to warmer weather.<br><br>Students examine sets of real time series data from a variety of contexts with a view to identifying these components. Possible contexts include share prices, indices such as the CPI or AOI, population statistics, occupancy rates in a hotel, and mean monthly temperature. |
| What can we do to analyse and make forecasts from time series data which exhibits 'noise' or irregular variation but seems to have an overall linear or exponential trend?<br><br>• Smoothing irregular variations by using moving averages | Data that has significant irregular variation can make it difficult to identify the true overall trend. To see the trend more clearly a moving average can be calculated that has the effect of smoothing out the irregular variations. The trend line found for the 'smoothed' data would be expected to have a stronger correlation than the original raw data.<br><br>If the data contains a cyclic element the number of data points taken for the moving average should match the cycle. An example of this would be CPI figures measured quarterly over several years. There are four values in the cycle of a year and so a centred four-point moving average would be used to smooth the data.<br>Note: moving average calculations are carried out using electronic technology such as a spreadsheet or purpose designed software rather than by hand. |
| • Identifying seasonal variation | Seasonality in a time series can be identified by regularly spaced peaks and troughs that have a consistent direction and approximately the same magnitude, relative to the trend.<br><br>Seasonal adjustment is the process of estimating and then removing from a time series influences that are systematic and calendar related. |
| • Using a seasonal index to adjust for periodic fluctuations<br><br>• Forecasting | To make seasonal adjustments using an additive model the steps below are followed:<br>1. The moving average value is subtracted from the raw data value for each of the time periods within the moving average (this finds the individual seasonal errors).<br>2. These errors are then averaged for each 'season' (each quarter in the CPI example above) to give a seasonal index number.<br>3. A trend line equation is found for the moving averages.<br>4. A forecast for a particular time period can then be made by finding the predicted value from the equation (using the appropriate 'time number') and adding the relevant seasonal index to give a seasonally adjusted prediction.<br><br>Students implement the statistical investigation process to answer questions that involve the analysis of time series data. Data is chosen carefully as too high a 'noise to signal' ratio can render the analysis invalid. |

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 3.3: The normal distribution** | |
| What is the normal distribution and how does it arise?<br>• Parameters $\mu$ (mean) and $\sigma$ (standard deviation)<br>• Bell shape and symmetry about the mean<br>• Discussion of the Normal probability density function | Students experience a range of data sets that illustrate both normal and non-normal characteristics leading to a discussion of the characteristic shape of the bell curve and its symmetry about the mean.<br>By graphing the mathematical formula of the normal probability density function $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)}$ for various values of $\mu$ and $\sigma$ students can see the effects of these parameters on the spread and location of the graph. Students appreciate that this function uses an exponential variable but behaves in a very different way to $y = a.b^x$ studied in the last topic. |
| Why do so many observed sets of data appear normally distributed?<br>• Quantities that arise as the sum of a large number of independent random variables can be modelled as normal distributions | To investigate one explanation for why and where normal distributions occur, students experience the building of a spreadsheet that generates a large amount of data from the sum of a finite number of random numbers. This is set in the context of simulating the production of something like brick pavers where several factors in the production process introduce random variations into the final quantity being measured (e.g. length or weight). By building the factors in one at a time students observe the emergence of a bell-shaped curve in the distribution of the measurements. |
| Why are normal distributions important?<br>• The variation in many quantities occurs in an approximately normal manner<br>• Normal distributions may be used to make predictions and answer questions that relate to such quantities. | Other contexts where the same underlying process might be expected to occur are discussed, supported by data samples. Some possibilities are: volume of soft drink in a can, height of humans of the same age and gender, weight of jelly beans, and lifetime of batteries. |
| What are the characteristics of the normal distribution and how can they be used for prediction?<br>• 68% : 95% : 99.7% rule<br>• Calculation of area under the curve, looking at the position of one, two, and three standard deviations from the mean | A refinement of the spreadsheet mentioned above allows students to see the features of normal distributions unfold.<br>Students understand the '68:95:99.7%' rule and use it to make predictions, given the mean and standard deviation, of approximate proportions or probabilities in context. These calculations are done both with and without the use of electronic technology. |
| • Calculation of non-standard proportions | Using electronic technology students find other proportions or probabilities that relate to non-integral multiples of the standard deviation from the mean. |
| • Calculation of values on the distribution given the area under the curve | Students reverse the process to find values on the distribution that bound a given area under the curve ('inverse normal' problems). |

## Topic 4: Financial Models

In this topic the focus is on the annuity model (initially developed in Topic 2) and its applications to investing and borrowing money. The broad areas of consideration are:

- saving money for a future need by making regular deposits
- repayment of a reducing balance loan
- receiving an income stream from a lump sum investment.

Students investigate the different types of saving plans such as superannuation and long-term deposits. They consider the effects of bank and government charges, taxation, and inflation on savings plans. Students consider the costs of borrowing money taking into account a range of variables (for example: repayments, interest rates, term of the loan, compounding interval). They discuss mortgages, personal loans, pension annuities, and interest-only loans with sinking funds.

Students use the annuity model to investigate strategies for minimising interest paid on a loan or maximising the interest earned on an investment. The nature of the calculations involved requires use of electronic technology (via the graphic calculator financial package or spreadsheets) to aid efficient investigation.

## Topic 4: Financial Models

### Key Questions and Key Ideas

### Considerations for Developing Teaching and Learning Strategies

### Sub-topic 4: Models for saving

How is the compound interest model used to plan for the future?

- Finding FV, PV, n & I

Students use electronic technology to solve problems involving compound interest.

How can the compound interest model be improved to make it more realistic and flexible?
What mathematics is used in calculating future-value annuities?

- Future value
- The regular deposit
- The number of periods
- The interest rate
- The value of the accumulating savings after a given period
- Total interest earned

The concept of annuities, introduced in Geometric Series in Topic 2, is extended using electronic technology to solve more complex problems involving regular savings annuities.

Students investigate ‘What if …’ questions with varying future values, payments, rates, and times. They discuss the limitations to the reasonableness of their results given the underlying assumptions made in the model.

Contexts include superannuation and long term deposits.

What factors should be considered when selecting an investment?

- Interest as part of taxable income, including calculations
- The effects of inflation, including calculations
- Institution and government charges

The return from any investment is subject to many factors that can combine to erode the overall return. Students become aware of the impact of these factors when managing investments.

Students consider the effect of inflation on long-term investments.

How can different investment structures be compared?

- Comparison of two or more investments involving nominal and/or flat interest by conversion to an equivalent annualised rate (effective rate).

Investment structures can differ in the type of interest paid, fee structure, or their rate of compounding the rates, and are not always immediately comparable. Effective annualised compounding rates allow for a common basis on which to make such comparisons.

Students consider the importance of effective rates in relation to advertisements for saving schemes.

How can a regular income be provided from savings?

- Annuities
- Superannuation

Students consider how, by reversing the annuity savings model, a lump sum deposit can be used to provide a regular income.

A variety of “What if .....?” scenarios are investigated and the limitations of the model discussed.

**Key Questions and Key Ideas** | **Considerations for Developing Teaching and Learning Strategies**

## Sub-topic 4.2: Models for Borrowing

If money must be borrowed, how much will it cost?

Most students, at some stage in their lives, will borrow a significant sum of money. They consider how much this could cost in the long run to determine whether it would be better to save the money and pay cash, or borrow and repay the debt.

Loans are structured in different ways, often according to their purpose. Students obtain information from lending institutions about the types of personal, business, and home loans available.

- Interest-only loans and sinking funds
- Reducing balance loans
  - Finding the repayment for a given loan
  - Calculating total interest paid
  - The size of an outstanding debt after a given time

Interest-only loans and sinking funds may be used by property investors or businesses. Students carry out the calculations associated with taking out an interest-only loan, and using a sinking fund to repay the principal. They discuss the possible advantages or disadvantages of using such a model.

Students use electronic technology to calculate various aspects of a loan. This process often highlights the real costs of a loan, particularly in its early stages.

How could the amount of interest paid on a loan be reduced?
Finding the effect of:

- increasing the frequency of payments
- increasing the value of the payments
- Reducing the term of the loan
- paying a lump sum off the principal owing
- changing interest rates
- offset accounts

Many different factors that affect the cost of a loan are under the control of the borrower. Students investigate the effects that changing one or more of these factors may have on the total interest paid.

Discussion includes the reasonableness and/or affordability of the various interest minimisation strategies considered.

Is the nominal rate of interest quoted by a bank what is really being paid on a loan?

- Discussion of loan interest rates, including variable rate, fixed rate, and others
- The interest paid
- Calculation of the comparison rates for two or more loans to determine the most appropriate option

When they compare loan options, students investigate comparison rates that take into account the fees and charges connected with the loan as well as the compounding period. Discussion focuses on the importance of factors other than the interest rate when choosing a loan.

## Topic 5: Discrete Models

This topic continues the study of discrete models begun with matrix applications in Stage 1, Topic 6. The focus is on network applications to the solution of problems involving shortest connections, maximum flow, and critical path analysis. Students encounter a range of optimisation problems that can be represented in the form of a network. To demonstrate the diversity of discrete models students investigate ‘assignment problems’ and learn the application of the Hungarian algorithm to their solution.

In all the applications, before being presented with solution algorithms, students attempt to solve problems set in familiar contexts by trial and error, so that they can appreciate the nature and complexity of the problems. Once the algorithms have been introduced, new problems are posed in broader contexts so that students understand that, although the calculations are relatively simple, the methods they are learning underpin some powerful techniques.

As part of their study students investigate the effects of changing the initial conditions or parameters of problems with a view to improving the solutions. For instance, in a critical path problem, which jobs could be shortened to improve the minimum completion time? In a problem of maximum flow, which connections could be added or upgraded to improve the flow?

The arithmetic computations required for the solution of the problems presented in this topic are conducted without electronic technology.

# Topic 5: Discrete Models

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|

## Sub-topic 5.1: Network problems

What are networks?

- Definition of a network and network terminology

Students consider the ideas and terminology used when constructing and reading information from network diagrams. Examples are drawn from practical contexts.

Students are assisted to see how the information in problems can be represented in network form.

How can networks be used to represent situations in which there is a problem to be solved?

- Connectivity networks
- Flow networks
- Precedence networks

For example:

- We have to drive between two given places. Which is the best way to go? Why?
- The local council is planning road upgrades because a lot of traffic passes through our area on its way to somewhere else. Which are the best roads to upgrade, and why?
- Some friends have decided to make pizzas to eat while they watch their favourite program on TV. When is the best time to start this task so the pizzas are ready when the TV program starts?
- Students have asked for drinking fountains to be installed in specific places at the school. What is the best way to connect them all to the rainwater supply?

How many paths are there through a directed network?

- With and without restrictions

By beginning with using trial and error to find the number of paths through a directed network students appreciate the efficiency of using the algorithm. The problems are extended to include restriction. For example, avoiding a node or an arc (e.g. because of an accident or a burst water main) or having to use a specified node or arc (e.g. because someone has to be picked up on the way).

What is the shortest path through a network?

- With and without restrictions

Students consider weighted networks where each arc incurs a 'cost'. The idea of an optimal or shortest path which uses the least 'cost' is explored and the algorithm for finding it applied. Students interpret the meaning and understand the limitations of the answers gained using this kind of simplified mathematical model.
The idea of a 'tree' being a connected network with no circuits (i.e. no redundancy) is explored.

What is the cheapest way to connect up a set of points if there is more than one option available?

- Spanning trees -'greedy' and Prim's algorithms to find the minimum spanning tree in a connectivity network

Students explore both scaled and unscaled representations of minimum spanning tree problems when seeking a solution. More than one algorithm is available and students consider which might be best in a given situation.
Extensions to these problems take practical considerations into account. For instance, what if a connection cannot be made in a straight line? What happens to the best solution if extra nodes are connected to the system later? Is the optimal solution practical if there are limitations on how far any node in the network can be from a 'source' node?

What is the maximum flow that can be achieved through a network of conduits?

- Use of an algorithm to find maximum flow

The flow considered could be freight, people, water, telephone calls, internet connections, or traffic.
The algorithm using the exhaustion of paths is easier than the Dedekind 'cuts' method for all but the simplest networks. The 'cuts' method is, however, useful when considering upgrades to a system of flow. Extensions of these problems would deal mainly with upgrading a system by either creating a new connection or improving an existing one.

| Key Questions and Key Ideas | Considerations for Developing Teaching and Learning Strategies |
|---|---|
| **Sub-topic 5.2: Critical Path analysis** | |
| If a job requires the completion of a series of tasks with set precedence, what is the minimum time in which this job can be finished?<br><br>• Precedence tables | Students become acquainted with the idea of precedence in the flow of jobs that make up a complex task. Through a practical example, they explore how to create a precedence table that indicates which other jobs must be completed before a given job can start. |
| • Drawing networks | From a precedence table, a network can be drawn to represent the task. For straightforward networks this can be done by trial and error, however students may benefit from being taught how to use a bipartite graph to work out the order in which to construct the nodes. |
| • Dummy links | It is sometimes necessary to use 'dummy' arcs in the network to show a given precedence correctly (e.g. when job E requires both A and B to be complete but job C requires only B to be complete). Students gain an understanding that two different-looking networks may be topologically identical. |
| For which of the tasks is it critical that there is no delay?<br>• Forward and backward scan<br>• Minimum completion time<br>• Critical path<br>• Earliest and latest starting times for individual tasks<br>• Slack time | Once a network representation is available for a problem, students can determine the minimum completion time and critical jobs by finding the longest path through the network. They discuss the amount of leeway available in the starting time for a given job in the network, and what happens if time for a specific job is shortened or lengthened; they look for ways of reducing the minimum completion time in the context of a specific problem. Students discuss the reasonableness of their results and any limitations to the model in the context of the problem. |

## Sub-topic 5.3: Assignment problems

| | |
|---|---|
| What are 'assignment' problems? | Assignment problems deal with allocating tasks in a way that minimises costs. For example if the times in which four swimmers each do 50 metres of each of the four different strokes are known, how should they be placed in a medley relay to minimise the total time for them to complete the race? |
| How can assignments be arranged to give the optimum result?<br>• The Hungarian algorithm<br>- Finding minimum cost | The Hungarian algorithm consists of a set of iterative steps to find the optimum solution of an assignment problem.<br>1. Reduce the array of costs in rows and then columns by subtraction of the minimum value.<br>2. Cover the zero elements with the minimum number of lines. If this minimum number is the same as the order of the array go to step four.<br>3. Let M be the minimum uncovered element. The array is further reduced by subtracting m from all uncovered elements and adding m to any element covered by two lines. Return to step 2.<br>4. There is an optimum solution using only zeros in the augmented array. Apply this pattern to the original array. |
| - Finding maximum profit | https://www.youtube.com/watch?v=dQDZNHwuuOY is a YouTube clip demonstrating the Hungarian algorithm<br><br>The algorithm can be adapted to finding the assignment that gains the maximum profit by effectively minimising profit lost. |
| - Non-square arrays | In some problems the array is rectangular rather than square, for example when there are six possible contenders for the four positions in the medley relay swim team. In such cases the array is 'squared up' by adding dummy rows or columns before applying the algorithm. |

## ASSESSMENT SCOPE AND REQUIREMENTS

All Stage 2 subjects have a school assessment component and an external assessment component.

### EVIDENCE OF LEARNING

The following assessment types enable students to demonstrate their learning in Stage 2 General Mathematics:

*School Assessment (70%)*
- Assessment Type 1: Skills Assessment Tasks (45%)
- Assessment Type 2: Mathematical Investigation (25%)

*External Assessment (30%)*
- Assessment Type 3: Examination (30%)

Students provide evidence of their learning through eight assessments, including the external assessment component. Students undertake:
- five skills and applications tasks
- two mathematical investigations
- one examination.

**It is anticipated that from 2018 all school assessments will be submitted electronically.**

## ASSESSMENT DESIGN CRITERIA

The assessment design criteria are based on the learning requirements and are used by:

- teachers to clarify for the student what he or she needs to learn
- teachers and assessors to design opportunities for the student to provide evidence of his or her learning at the highest possible level of achievement.
- The assessment design criteria consist of specific features that:
- students should demonstrate in their learning
- teachers and assessors look for as evidence that students have met the learning requirements.

For this subject, the assessment design criteria are:

- concepts and techniques
- reasoning and communication.

The specific features of these criteria are described below.

The set of assessments, as a whole give students opportunities to demonstrate each of the specific features by the completion of study of the subject.

### Concepts and Techniques

The specific features are as follows:

CT1 Knowledge and understanding of concepts and relationships

CT2 Selection and application of techniques and algorithms to solve problems in a variety of contexts

CT3 Development and application of mathematical models

CT4 Use of electronic technology to find solutions to mathematical problems

### Reasoning and Communication

The specific features are as follows:

RC1 Interpretation and evaluation of mathematical results with an understanding of their reasonableness and limitations

RC2 Knowledge and use of appropriate mathematical notation, representations, and terminology.

RC3 Communication of mathematical ideas and reasoning, to develop logical arguments

RC4 Forming and testing of valid conjectures *

* In this subject the development and testing of conjectures (RC4) is not intended to include formal mathematical proof.

## SCHOOL ASSESSMENT

### Assessment Type 1: Skills and Applications Tasks (45%)

Students complete five skills and applications tasks which taken as a whole cover content from all topics.

Skills and applications tasks are completed under the direct supervision of a teacher.

Students find solutions to mathematical questions that may:
- be routine, analytical, and/or interpretative
- be posed in a variety of familiar and new contexts
- require discerning use of electronic technology.

In setting skills and applications tasks, teachers may provide students with information in written form or in the form of numerical data, diagrams, tables, or graphs. A task should require the student to demonstrate an understanding of relevant mathematical ideas, facts, and relationships.

Students select appropriate techniques or algorithms and relevant mathematical information and apply them to find solutions to routine, analytical, and/or interpretative problems. Some of these problems should be set in contexts, for example: social, scientific, economic or historical.

Students provide explanations and arguments, and use notation, terminology, and representation correctly throughout the task. They may be required to use electronic technology appropriately to aid and enhance the solution of some problems.

For test situations students may, at the teacher's discretion, take into the room one sheet of paper (up to A4 size), with handwritten notes on one side only.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:
- concepts and techniques
- reasoning and communication

### Assessment Type 2: Mathematical Investigation (25%)

Students complete two investigations.

Students investigate mathematical relationships, concepts, or problems, which may be set in an applied context. The subject of a mathematical investigation may be derived from one or more subtopics, although it can also relate to a whole topic or across topics.

A mathematical investigation may be initiated by a student, a group of students, or the teacher. Teachers may give students a clear, detailed, and sequential set of instructions for part of the investigation or to initiate the investigation, or may provide guidelines for students to develop themes or aspects of their own choice. Teachers should give some direction about the appropriateness of each student's choice and guide and support students' progress in an investigation.

If a mathematical investigation is undertaken by a group, students explore the problem and gather data together to develop a model or solution individually. Each student must submit an individual report.

Teachers may need to provide support and clear directions for the first investigation. However, the second investigation must be less directed and set within more open-ended contexts.

Students demonstrate their problem-solving strategies as well as their knowledge, skills, and understanding in the investigation. They are encouraged to use mathematical and other software (e.g. statistical packages, spreadsheets, Computer Algebra Systems, accounting packages) to enhance their investigation. The generation of data and the exploration of patterns or the changing of parameters may provide an important focus. Notation, terminology, forms of representation of information gathered or produced, calculations, and results are important considerations.

In the report, they form conjectures, interpret and justify results, and draw conclusions. They support this process by giving appropriate explanations and arguments.

The report may take a variety of forms, but would usually include the following:

- an outline of the problem and context
- the method required to find a solution, in terms of the mathematical model or strategy used
- the application of the mathematical model or strategy, including
  - relevant data and/or information
  - mathematical calculations and results using appropriate representations
  - the analysis and interpretation of results, including consideration of the reasonableness and limitations of the results
- the results and conclusions in the context of the problem
- a bibliography and appendices, as appropriate.

The format of an investigation report may be written or multimodal.

Each investigation report should be a maximum of 12 pages if written, or the equivalent in multimodal form.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication

## EXTERNAL ASSESSMENT

### Assessment Type 3: Examination (30%)

Students undertake a 2-hour external examination in which they answer questions on all of the topics. The examination is divided into two parts:

The examination is based on the key questions and key ideas in the topics above. The considerations for developing teaching and learning strategies are provided as a guide only, although applications described under this heading may provide contexts for examination questions.

The examination consists of a range of problems, some focusing on knowledge, routine skills, and applications, and others focusing on analysis and interpretation. Students provide explanations and arguments, and use correct mathematical notation, terminology, and representation throughout the examination.

- Part 1 (40%, 50 minutes) : Calculations without electronic technology covering aspects of:
    - Topic 1: Modelling with Linear Relationships
    - Topic 3: Statistical Models
    - Topic 5: Discrete Models

- Part 2 (60%, 70 minutes): Calculation with discerning use of electronic technology covering aspects of:
    - Topic 1: Modelling with Linear Relationships
    - Topic 2: Modelling with Non-linear Relationships
    - Topic 3: Statistical Models
    - Topic 4: Financial Models

Students have 10 minutes in which to read both Part 1 and Part 2 of the examination. At the end of the reading time, students begin their answers to Part 1. For this part, students do not have access to electronic technology (graphics or scientific calculators).

At the end of the specified time for Part 1, students stop writing. Students submit Part 1 to the invigilator.

Students have access to Board-approved calculators for Part 2 of the examination. The invigilator coordinates the distribution of calculators (graphics and scientific). Once all students have received their calculators, the time allocated for Part 2 begins, and students resume writing their answers.

The SACE Board will provide a list of approved graphics calculators for use in Assessment Type 3: Examination that meet the following criteria:

- have flash memory that does not exceed 5.0 MB (this is the memory that can be used to store add-in programs and other data)
- can calculate derivative and integral values numerically
- can calculate probabilities
- can calculate with matrices
- can draw a graph of a function and calculate the coordinates of critical points using numerical methods
- solve equations using numerical methods
- do not have a CAS (Computer Algebra System)
- do not have SD card facility (or similar external memory facility).

Graphics calculators that currently meet these criteria, and are approved for 2017, are as follows:
*Casio fx-9860G AU*
*Casio fx-9860G AU Plus*
*Hewlett Packard HP 39GS*
*Sharp EL-9900*
*Texas Instruments TI-83 Plus*
*Texas Instruments TI-84 Plus*
*Texas Instruments – TI 84 Plus C –silver edition*
*Texas Instruments – TI 84 Plus CE*

Other graphic calculators will be added to the approved calculator list as they become available.

Students may bring two graphics calculators or one scientific calculator and one graphics calculator into the examination room.

There is no list of Board-approved scientific calculators. Any scientific calculator, except those with an external memory source, may be used.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- concepts and techniques
- reasoning and communication.

## PERFORMANCE STANDARDS

The performance standards describe five levels of achievement, A to E.

Each level of achievement describes the knowledge, skills, and understanding that teachers and assessors refer to in deciding how well a student has demonstrated his or her learning, on the basis of the evidence provided.

During the teaching and learning program the teacher gives students feedback on their learning, with reference to the performance standards.

At the student's completion of study of each school assessment type, the teacher makes a decision about the quality of the student's learning by:

- referring to the performance standards
- assigning a grade between A+ and E- for the assessment type.

The student's school assessment and external assessment are combined for a final result, which is reported as a grade between A+ and E-.

## Performance Standards for Stage 2 General Mathematics

| | Concepts and Techniques | Reasoning and Communication |
|---|---|---|
| **A** | CT1 Comprehensive knowledge and understanding of concepts and relationships.<br>CT2 Highly effective selection and application of techniques and algorithms to find efficient and accurate solutions to routine and complex problems in a variety of contexts.<br>CT3 Successful development and application of mathematical models to find concise and accurate solutions.<br>CT4 Appropriate and effective use of electronic technology to find accurate solutions to routine and complex problems. | RC1 Comprehensive interpretation and critical evaluation of mathematical results, with an in-depth understanding of their reasonableness and possible limitations.<br>RC2 Proficient and accurate use of appropriate mathematical notation, representations, and terminology.<br>RC3 Highly effective communication of mathematical ideas and reasoning to develop logical and concise arguments.<br>RC4 Formation and testing of appropriate conjectures, using sound mathematical evidence. |
| **B** | CT1 Some depth of knowledge and understanding of concepts and relationships.<br>CT2 Mostly effective selection and use of mathematical techniques and algorithms to find accurate solutions to routine and some complex problems in a variety of contexts.<br>CT3 Attempted development and successful application of mathematical models to find accurate solutions.<br>CT4 Mostly appropriate and effective use of electronic technology to find accurate solutions to routine and some complex problems. | RC1 Mostly appropriate interpretation and some critical evaluation of mathematical results, with some depth of understanding of their reasonableness and possible limitations.<br>RC2 Mostly accurate use of appropriate mathematical notation, representations, and terminology.<br>RC3 Mostly effective communication of mathematical ideas and reasoning to develop logical arguments.<br>RC4 Formation and testing of mostly appropriate conjectures, using some mathematical evidence. |
| **C** | CT1 Generally competent knowledge and understanding of concepts and relationships.<br>CT2 Generally effective selection and use of mathematical techniques and algorithms to find mostly accurate solutions to routine problems in a variety of contexts.<br>CT3 Application of mathematical models to find generally accurate solutions.<br>CT4 Generally appropriate and effective use of electronic technology to find mostly accurate solutions to routine problems. | RC1 Generally appropriate interpretation and evaluation of mathematical results, with some understanding of their reasonableness and possible limitations.<br>RC2 Generally appropriate use of mathematical notation, representations, and terminology with some inaccuracies.<br>RC3 Generally effective communication of mathematical ideas and reasoning to develop some logical arguments.<br>RC4 Formation of an appropriate conjecture and some attempt to test it using mathematical evidence. |
| **D** | CT1 Basic knowledge and some understanding of concepts and relationships.<br>CT2 Some effective selection and use of mathematical techniques and algorithms to find some accurate solutions to routine problems in some contexts.<br>CT3 Some application of mathematical models to find some accurate solutions.<br>CT4 Some appropriate use of electronic technology to find some accurate or partially accurate solutions to routine problems. | RC1 Some interpretation and attempted evaluation of mathematical results, with some awareness of their reasonableness and possible limitations.<br>RC2 Some appropriate use of mathematical notation, representations, and terminology, with some accuracy.<br>RC3 Some communication of mathematical ideas with attempted reasoning and/or arguments.<br>RC4 Attempted formation of a conjecture with limited attempt to test it using mathematical evidence. |
| **E** | CT1 Limited knowledge or understanding of concepts and relationships.<br>CT2 Attempted selection and limited use of mathematical techniques or algorithms, with limited accuracy in solving routine problems.<br>CT3 Attempted application of mathematical models, with limited accuracy.<br>CT4 Attempted use of electronic technology, with limited accuracy in solving routine problems. | RC1 Limited interpretation or evaluation of mathematical results, with limited awareness of their reasonableness and possible limitations.<br>RC2 Limited use of appropriate mathematical notation, representations, or terminology, with limited accuracy.<br>RC3 Attempted communication of mathematical ideas, with limited reasoning.<br>RC4 Limited attempt to form or test a conjecture. |

### ASSESSMENT INTEGRITY

The SACE Assuring Assessment Integrity Policy outlines the principles and processes that teachers and assessors follow to assure the integrity of student assessments.  This policy is available on the SACE website (www.sace.sa.edu.au) as part of the SACE Policy Framework.

The SACE Board uses a range of quality assurance processes so that the grades awarded for student achievement, in both the school assessment and the external assessment, are applied consistently and fairly against the performance standards for a subject, and are comparable across all schools.

Information and guidelines on quality assurance in assessment at Stage 2 are available on the SACE website (www.sace.sa.gov.au)

## SUPPORT MATERIALS

### SUBJECT-SPECIFIC ADVICE

Online support materials are provided for each subject and updated regularly on the SACE website (www.sace.sa.edu.au). Examples of support materials are sample learning and assessment plans, annotated assessment tasks, annotated student responses, and recommended resource materials.

### ADVICE ON ETHICAL STUDY AND RESEARCH

Advice for students and teachers on ethical study and research practices is available in the guidelines on the ethical conduct of research in the SACE on the SACE website (www.sace.sa.edu.au).