

2015 SAMPLE MATHEMATICAL APPLICATIONS PAPER

The external assessment requirements of this subject are listed on page 36.

**FOR OFFICE
USE ONLY**

SUPERVISOR
CHECK

RE-MARKED

**ATTACH SACE REGISTRATION NUMBER LABEL
TO THIS BOX**

Graphics calculator ☐

Brand _____

Model _____

Computer software ☐

Time: 2 hours

Examination material: one 36-page question booklet
one SACE registration number label

Pages: 36
Questions: 15

Approved dictionaries, notes, calculators, and computer software may be used.

Instructions to Students

- You will have 10 minutes to read the paper. You must not write in your question booklet or use a calculator during this reading time but you may make notes on the scribbling paper provided.
- This paper consists of three topics. ***Tick the boxes by the two topics you have studied for the examination:***
 Topic 2: Investment and Loans (Questions 1 to 5), pages 2 to 11 ☐
 Topic 4: Matrices (Questions 1 to 5), pages 12 to 23 ☐
 Topic 7: Statistics and Working with Data (Questions 1 to 5), pages 24 to 35. ☐
- The total mark for each topic is 45.
- Answer **all** questions on the ***two topics you have studied for the examination.***
- Write your answers in the spaces provided in this question booklet. There is no need to fill all the space provided.
- Show all working in this booklet. (You are strongly advised **not** to use scribbling paper. Work that you consider incorrect should be crossed out with a single line.)
- Each topic has a spare answer page that you can write on if you need more space. Make sure to label each answer clearly.
- Use only black or blue pens for all work other than graphs and diagrams, for which you may use a sharp dark pencil.
- Appropriate steps of logic and correct answers are required.
- Marks may be deducted if you do not clearly show all steps in the solution of problems, if your answers have an inappropriate number of decimal places, or if you use incorrect units.
- Diagrams, where given, are not necessarily drawn to scale.
- Complete the box on the top right-hand side of this page with information about the electronic technology you are using in this examination.
- Attach your SACE registration number label to the box at the top of this page.

TOPIC 2: INVESTMENT AND LOANS (Questions 1 to 5)

(45 marks)

Answer **all** questions on this topic. Page 11 is a spare answer page if you need more space.

1. The local country club borrowed \$300 000 to pay for extensions to its hall. The club arranged an interest-only loan at 8.5% per annum. The \$300 000 loan must be repaid within 12 years. The club also set up a sinking fund that earns interest of 6.75% per annum, compounded monthly.

- (a) Calculate the yearly interest payable on the loan.

(1 mark)

- (b) Calculate the monthly deposit that the club must make to accumulate the \$300 000 in the sinking fund by the end of the 12 years.

AMPL

(2 marks)

- (c) Find the value of the sinking fund after 6 years.

(2 marks)

- (d) Explain why, after 6 years (halfway through the term of the loan), the club has not saved half of the \$300 000.

(2 marks)

- (e) Find the total amount that the extensions to the hall will cost the club.

OLE

(2 marks)

2. Georgina has been paying off her home loan for 15 years and has reduced the debt to \$356 000. She has made regular monthly repayments of \$4357.37 over this time. Her original home loan was for 25 years at an interest rate of 8.2% per annum, compounded monthly.

(a) Show that Georgina originally borrowed approximately \$555 000.

(2 marks)

- (b) (i) Calculate how much of the loan Georgina has paid off after 15 years.

DLV

(1 mark)

- (ii) Calculate the total repayments Georgina has made over 15 years.

CAM

(1 mark)

- (c) (i) Suppose that, after the 15 years, Georgina adds an extra \$120 a month to the repayments.

Show that this will reduce the term of the loan by approximately half a year.

(3 marks)

- (ii) Alternatively, suppose that, after the 15 years, Georgina transfers the balance of the loan to a bank that has a lower interest rate.

If Georgina is able to repay the balance of the loan in 9 years, what interest rate has the bank been charging? Assume that her monthly repayment remains \$4357.37.

CAMK

(2 marks)

- (d) Discuss the reasonableness of the two interest minimisation strategies you have investigated in part (c) above.

(2 marks)

3. Jun deposited \$2750 in an account every quarter for 3 years. The account returned interest of 6.95% per annum, compounded quarterly.

(a) Show that Jun deposited \$33 000 in the account over the 3 years.

(1 mark)

(b) Find the value of Jun's investment after 3 years.

(2 marks)

(c) State *one* assumption that you made when calculating the value of Jun's investment.

CAN

(1 mark)

(d) Calculate the interest that Jun's account earned over 3 years.

(1 mark)

- (e) Jun's marginal tax rate is 30%.

Calculate the total amount he will have in his account after he has paid tax.

(2 marks)

- (f) Assume that inflation averaged 3.4% per annum over the 3 years.

Calculate the value of \$33 000 indexed for inflation over 3 years.

APLE

(2 marks)

- (g) Using the answers you have calculated in parts (d), (e), and (f), discuss the effectiveness of the return on Jun's investment.

ST

(2 marks)

4. Tom put \$1200000 in an annuity that had an interest rate of 3.5% per annum, compounded weekly.

- (a) Calculate the weekly withdrawal that Tom could make if the money was to last for 20 years.


(2 marks)

- (b) Calculate how much of Tom's annuity was left after 10 years. Assume that the annuity conditions and his withdrawals remained the same.

MAPLE

(2 marks)

- (c) Why, after 10 years (halfway through the expected term of the annuity), was more than half of the original annuity left?



(1 mark)

- (d) After 10 years Tom realised that, because of changes in the pension rules, the remaining money in the annuity would need to last for at least 15 years.

Using calculations, determine whether or not the money would last for 15 years if Tom reduced his weekly withdrawal to \$1200.

(3 marks)

SAMPLE

Write the comparison interest rate for each loan in the boxes below.

APLE

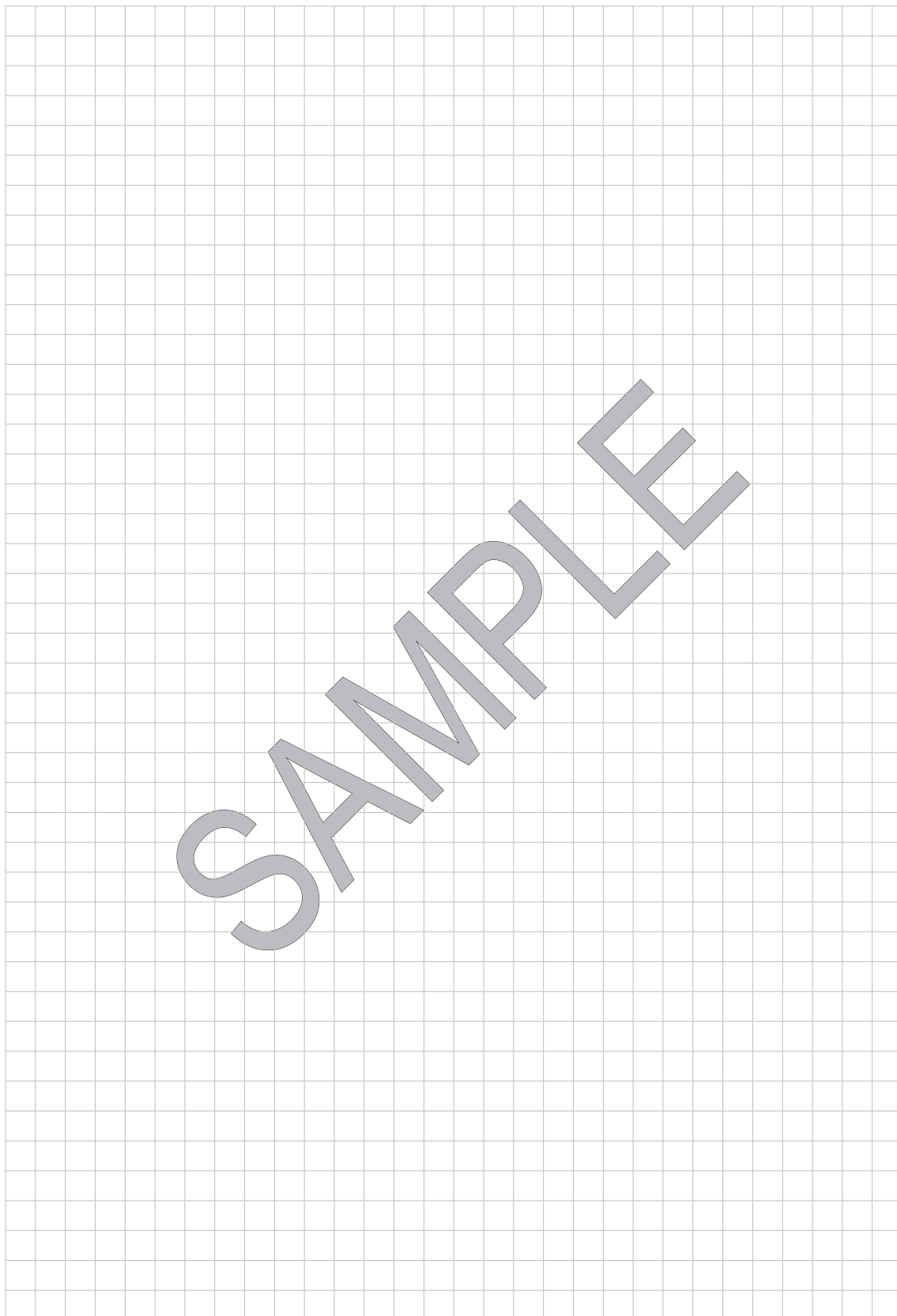
Loan B comparison interest rate:	%
----------------------------------	---

Loan	
------	--

(c) State *one* reason why Hannah may decide not to choose the loan with the better comparison interest rate.

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You may write on this page if you need more space to finish your answers to Topic 2.
Make sure to label each answer carefully (e.g. 'Question 2(c)(i) continued').



End of Topic 2

This sample Mathematical Applications paper shows the format of the examination for 2015, based on questions from past examination papers.

TOPIC 4: MATRICES (Questions 1 to 5)
(45 marks)

Answer **all** questions on this topic. Page 23 is a spare answer page if you need more space.

1. A mechanic (M), a salesperson (S), and an accounts clerk (C) work in a local car yard. The hours that they work each day in 1 week are shown in matrix H below:

$$H = \begin{bmatrix} & \text{M} & \text{S} & \text{C} \\ \text{M} & 7.0 & 9.0 & 8.0 \\ \text{T} & 7.5 & 8.0 & 9.0 \\ \text{W} & 8.0 & 8.5 & 8.0 \\ \text{T} & 8.0 & 8.0 & 7.5 \\ \text{F} & 5.0 & 0.0 & 8.5 \end{bmatrix}$$

- (a) The gross (before tax) hourly wage is \$18.50 for the mechanic, \$32 for the salesperson, and \$26.50 for the accounts clerk.

- (i) Construct a column matrix, G , for these gross hourly wages.

APR 2015

(1 mark)

- (ii) Using matrix methods, calculate the total amount of gross wages paid each day.

SA

(2 marks)

- (iii) Show how matrix methods are used to calculate the total amount of gross wages paid for the week and find this value.

(2 marks)

- (b) Let $A = [1 \ 1 \ 1 \ 1 \ 1]$.

What does the calculation $A \times H$ show in the context of this question?

(1 mark)

- (c) Describe *one* limitation of using matrix methods to solve the problems in this question.

NAME _____

(1 mark)

$$M = \begin{matrix} & \begin{matrix} A & B & C & D \end{matrix} \\ \begin{matrix} A \\ B \\ C \\ D \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

-

(b) (i) (1) Find $P = M + M^2$.

SAMPLE

(2) Why are the main diagonal elements ($P_{1,1}$, $P_{2,2}$, $P_{3,3}$, and $P_{4,4}$) not useful in this context?

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- (ii) Using your result from part (b)(i)(1) or otherwise, explain which town or towns would be most at risk in a bushfire situation.

(2 marks)

SAMPLE

- (c) Matrix C^3 has been calculated and the result is shown below:

$$C^3 = \begin{matrix} & \begin{matrix} T & M & V & B & G \end{matrix} \\ \begin{matrix} T \\ M \\ V \\ B \\ G \end{matrix} & \begin{bmatrix} 2 & 3 & 3 & 2 & 3 \\ 3 & 4 & 3 & 6 & 5 \\ 1 & 3 & 2 & 4 & 4 \\ 4 & 5 & 3 & 6 & 7 \\ 4 & 6 & 2 & 4 & 6 \end{bmatrix} \end{matrix}$$

Explain the meaning of the value 2 in the fifth row of matrix C^3 . Support your answer with one path from the network diagram.

(2 marks)

- (d) Gianni decides to fly to Bologna, spend 7 days using a bus pass to travel around the region, and then fly out from Genoa.
- (i) Using a matrix calculation, find out how many ways Gianni has of travelling during the 7 days. (There is no need to write down the full matrix answer but you must indicate the calculation used.)

(2 marks)

- (ii) State *one* limitation of using matrices to solve problems such as the one in part (d)(i).

(1 mark)

4. Frances and Christine are shopping for a number of items of clothing and plan to check prices at two different stores. Frances needs four shirts, three pairs of jeans, and a hoody. Christine needs six shirts, one pair of jeans, and three hoodies.

(a) Write the clothing requirements of the two girls as a 2×3 matrix, R .

(2 marks)

- (b) At Store X, shirts cost \$35, jeans cost \$62, and hoodies cost \$51.

At Store Y, shirts cost \$28, jeans cost \$75, and hoodies cost \$49.

Write the costs of the items of clothing as a 3×2 matrix, C .

MPLE

(1 mark)

(c) Calculate matrix RC .

(2 marks)

(d) Interpret the values in the first column of matrix RC .

(2 marks)

(e) Using a matrix calculation or otherwise, explain which store has cheaper prices overall for the clothing that Frances and Christine need.

DOLE

(2 marks)

		Size tomorrow				
		XL	L	S	N	
$T =$	Size today	XL	0.01	0.07	0.12	0.80
		L	0.01	0.55	0.39	0.05
		S	0.02	0.47	0.45	0.06
		N	0.01	0.33	0.65	0.01

Complete this matrix.

	XL	L	S	N
XL	0.0112	_____	_____	_____
L	0.0139	_____	_____	_____
S	0.0145	0.4912	0.4272	0.0671
N	0.0165	0.4910	0.4289	0.0636

(ii) If a hen lays an extra large egg today, what is the probability that she will lay an extra large egg in 2 days' time?

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- (iii) Today two of the fifty hens laid an extra large egg, twenty-one laid a large egg, and twenty-four laid a standard egg.

Express this as an appropriately labelled row matrix, N .

(2 marks)

- (iv) Using matrix methods, calculate how many of the hens that laid a standard egg today could be expected to lay a standard egg in 2 days' time.

APLE

(2 marks)

- (c) (i) Using matrix methods, calculate the percentage of eggs in each category that will be laid per day in the long term.

SK

(2 marks)

- (ii) Calculate the number of large eggs that will be laid per day in the long term.

(1 mark)

- (d) After a certain time the hens' laying pattern changes slightly. When the hens lay a large egg, the probability that they will lay a large egg the next day decreases from 55% to 45%, and the probability that they will lay a standard egg increases correspondingly. The probability that the hens will lay a large egg or a standard egg the day after laying a standard egg also changes, as shown in transition matrix T_{new} below:

		Size tomorrow				
		XL	L	S	N	
$T_{\text{new}} =$	Size today	XL	0.01	0.07	0.12	0.80
		L	0.01	————	————	0.05
		S	0.02	0.41	0.51	0.06
		N	0.01	0.33	0.65	0.01

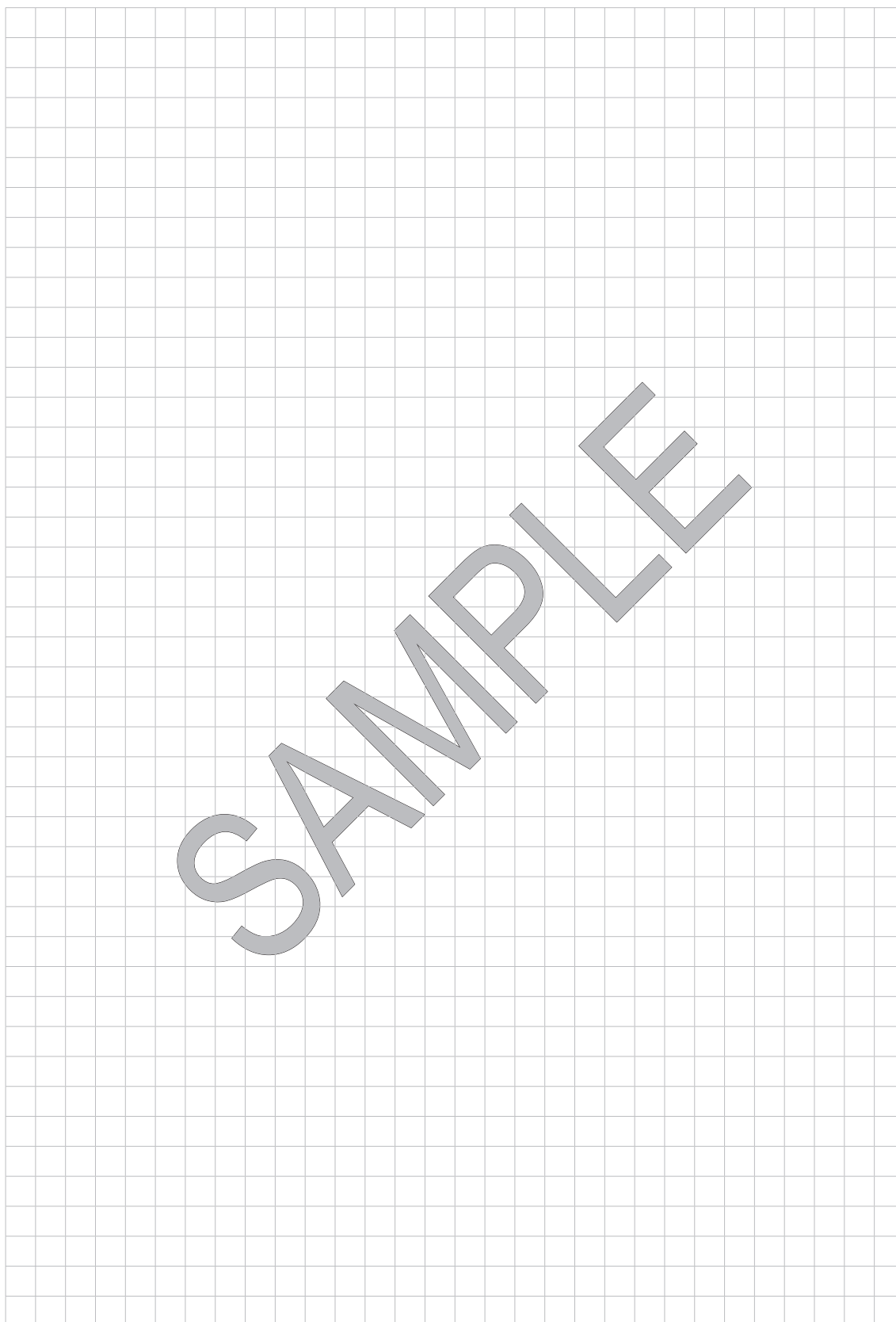
- (i) Complete transition matrix T_{new} above. (2 marks)
- (ii) The new steady state for the categories of eggs laid is now 1.5% extra large, 41.6% large, 50.5% standard, and 6.4% none.

Comment on the reasonableness of assuming that the same percentage of eggs in each category will be laid per day in the long term.

APL

(1 mark)

You may write on this page if you need more space to finish your answers to Topic 4.
Make sure to label each answer carefully (e.g. 'Question 5(b)(iii) continued').



End of Topic 4

This sample Mathematical Applications paper shows the format of the examination for 2015, based on questions from past examination papers.

TOPIC 7: STATISTICS AND WORKING WITH DATA (Questions 1 to 5)
(45 marks)


Answer **all** questions on this topic. Page 35 is a spare answer page if you need more space.

1. The call centre for a marketing company employs people to enter information into a database. A new employee is trained and then monitored at random intervals, using a standard test. The test is checked to see how many mistakes the employee has made.

The following scatter plot shows the test results over the first 6 weeks of employment of one particular employee:



- (a) Name the dependent variable in this scenario.



(1 mark)

- (b) Tick the appropriate box to indicate which one of the following values for Pearson's correlation coefficient (r) is the *most* appropriate for the scatter plot on page 24. Give a reason for your answer.

$r = 0.51$

$r = 0.84$

$r = -0.84$

(1 mark)

- (c) With reference to the variables, describe the relationship shown in the scatter plot on page 24.

(1 mark)

- (d) (i) On the scatter plot on page 24, circle the data point that is most likely to be considered an outlier. (1 mark)

- (ii) The coefficient of determination for the original data, including the outlier, is $r^2 = 0.71$.

Tick the appropriate box to indicate which one of the following values is most likely to be the new r^2 value, once the outlier has been removed from the data. Give a reason for your answer.

$r^2 = 0.71$

$r^2 = 0.82$

$r^2 = -0.65$

\$1

(1 mark)

SA

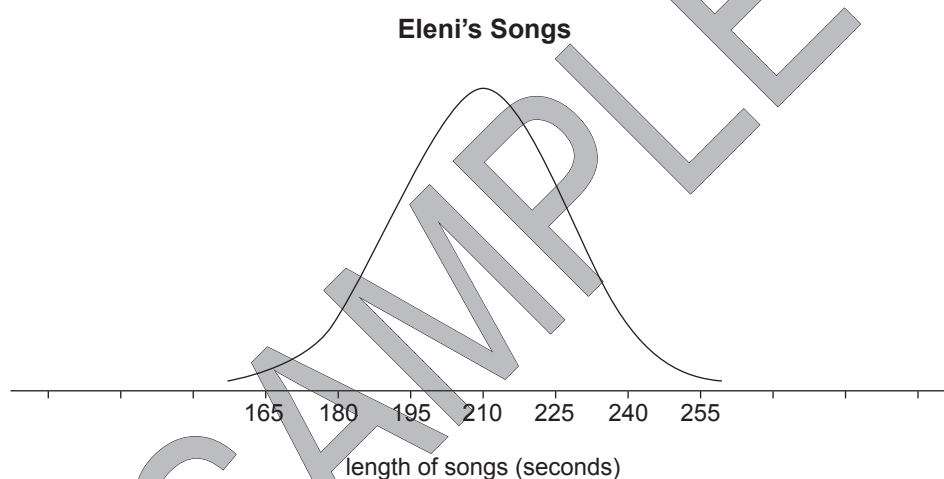
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- (d) Michaela decides to delete 12% of the songs. She does this by removing the shortest songs.

What will be the length (to the nearest whole second) of the shortest song left on her audio player when she has finished? Show your working.

(3 marks)

Michaela's friend Eleni also keeps her favourite songs on a digital audio player. The following graph shows the distribution of lengths of songs on Eleni's audio player:



- (e) (i) Estimate the mean and the standard deviation of the distribution shown in the graph above.

(2 marks)

- (ii) Using the mean and the standard deviation, compare the lengths of Eleni's songs and Michaela's songs.

(2 marks)

- (f) What percentage of songs on Eleni's audio player is likely to be less than 195 seconds in length?

(2 marks)

3. All Year 12 students in South Australia were surveyed about their Internet use.

The students were asked the following survey question:

'How many hours do you spend on the Internet each day?'

The population mean for the students was found to be 3.5 hours.

- (a) A teacher selected a group of eighteen male and eighteen female Year 12 students from her school to answer the same survey question.

The sample mean was found to be 5.0 hours.

Suggest *one* reason why the sample mean did not accurately reflect the population mean.

(1 mark)

- (b) Describe a sampling process that the teacher could use to minimise bias in selecting the sample of eighteen male and eighteen female Year 12 students.

AMPL

(2 marks)

Plant	1	2	3	4	5	6	7	8	9
spray concentration (%)	10	25	36	50	52	56	64	72	80
number of dead bugs after 2 weeks	2	39	19	38	39	38	53	51	66

A Pesticide Study

number of dead bugs

spray concentration (%)

spray concentration (%)	number of dead bugs
10	2
25	39
36	19
50	38
52	39
56	38
64	53
72	51
80	66

- (a) Calculate the coefficient of determination (r^2).

\$K^2\$

(1 mark)

- (b) (i) On the scatter plot above, circle a possible outlier in the data. (1 mark)

- (ii) If the outlier is due to an error, suggest *one* possible reason for this error.

(1 mark)

- (c) (i) Remove the outlier and recalculate the coefficient of determination (r^2).

(1 mark)

- (ii) Interpret the strength of the association between the two variables with the outlier removed.

(1 mark)

- (d) Determine the least squares regression line (line of best fit) with the outlier removed.



(1 mark)

- (e) (i) Using the least squares regression line (line of best fit) that you determined in part (d), predict the number of dead bugs after 2 weeks if the spray concentration was 49%.

SK

(2 marks)

- (ii) Using the term 'interpolation' or 'extrapolation', discuss the accuracy of the prediction you made in part (e)(i).

(2 marks)

5. A health agency wants to launch an advertising campaign to encourage people to eat more healthily. The agency considers the average daily intake of calories to be 1650 calories for women and 1850 calories for men. The health agency conducts an online survey to gather information. The first fifteen women and fifteen men to respond are asked to calculate their average daily intake of calories (to the nearest 10).

The following table shows the data for the fifteen women and the fifteen men:

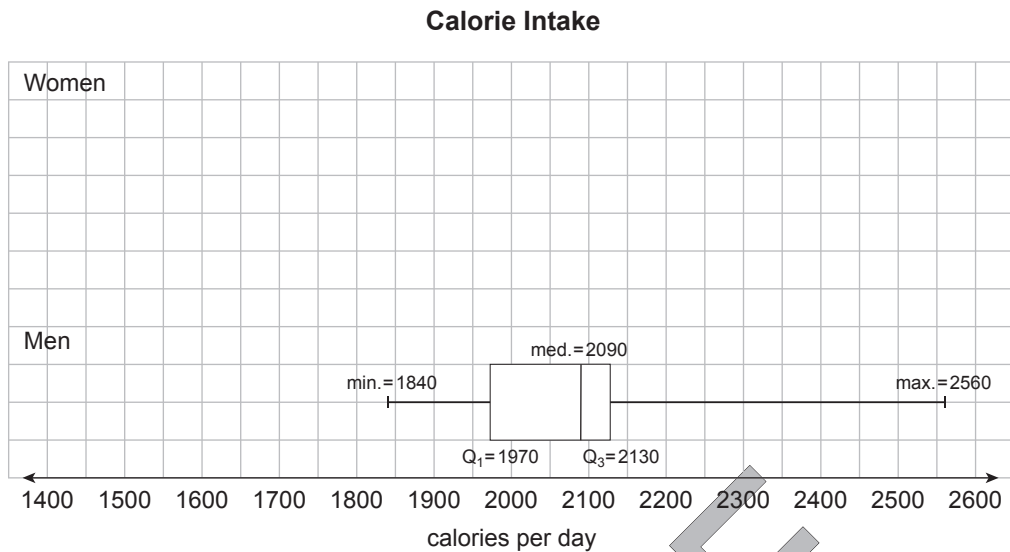
Women	Men
1470	1840
1480	1860
1550	1950
1560	1970
1560	1980
1630	2060
1630	2070
1640	2090
1670	2090
1780	2120
1790	2130
1800	2130
2030	2160
2120	2240
2310	2560

- (a) Complete the following table.

Statistical Measure	Women	Men
mean	1734.7	
median		2090.0
standard deviation	245.1	
interquartile range		160.0

(3 marks)

- (b) Draw and label a box-and-whisker diagram for the women's data, using the axis provided.



(2 marks)

- (c) Fill in the blanks in the following sentences with 'men' or 'women':

'The standard deviation suggests that _____ demonstrate a higher level of variability in the number of calories they consume on an average day.'

'On average, _____ consume fewer calories each day.'

(2 marks)

- (d) There was an error in the recording of the data. The men's value of 2560 calories should have been 1860.

Replace 2560 with 1860 and recalculate the mean, median, standard deviation, and interquartile range for the men's data.

Statistical Measure	Men
mean	
median	
standard deviation	
interquartile range	

(2 marks)

Question 5 continues on page 34.

- (e) Tick the pair of statistical measures that, in general, are most affected by the presence of outliers.

mean and standard deviation	<input type="checkbox"/>
median and interquartile range	<input type="checkbox"/>
mean and median	<input type="checkbox"/>

(1 mark)

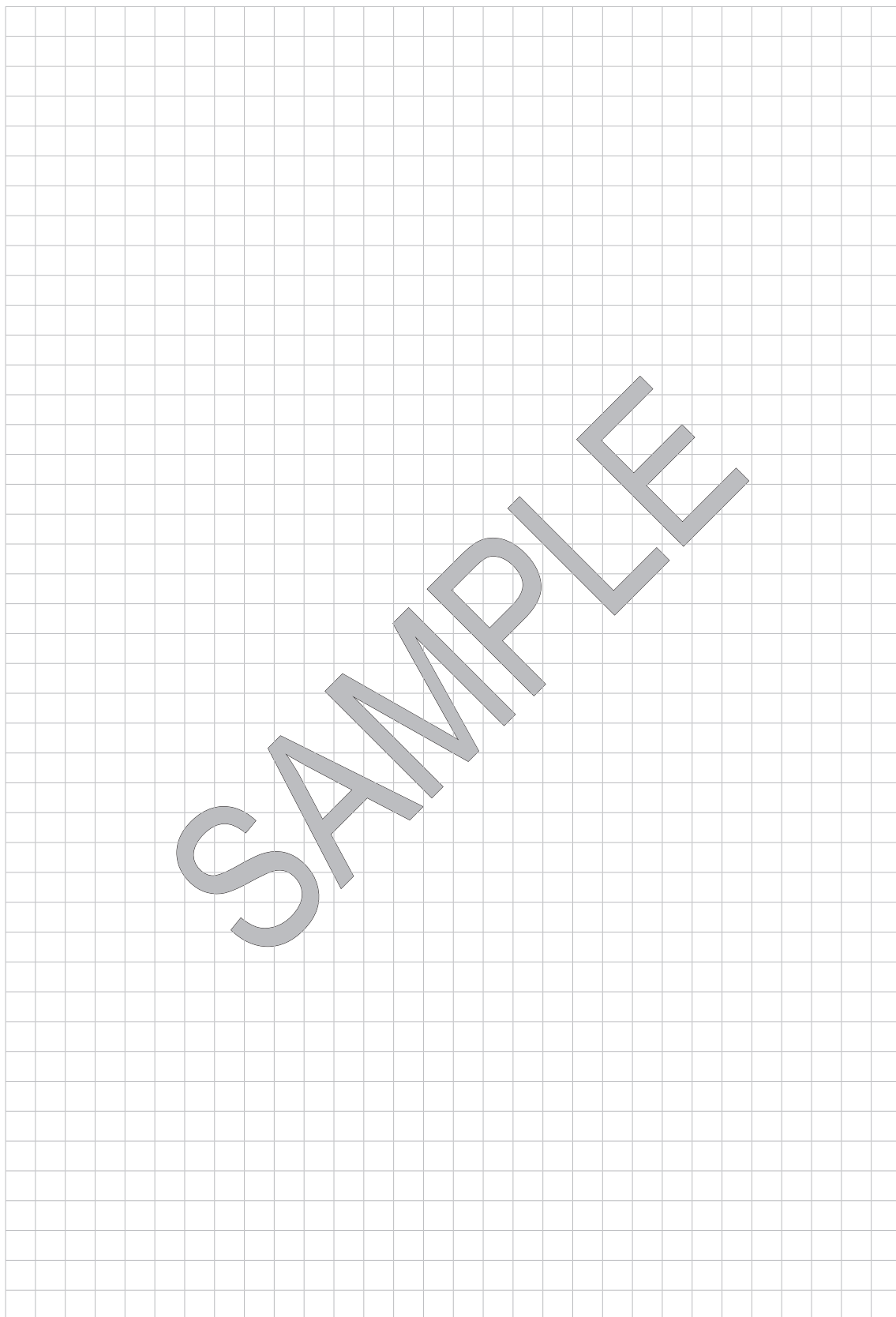
- (f) The agency's promotional material includes the following claim:

'The average daily intake of calories is exceeded by a far greater amount by women than by men.'

Using the information given in this question, and the calculations you have made, explain whether or not you think this claim is accurate. (Assume that the outlier has been removed from the men's data.)

(2 marks)

You may write on this page if you need more space to finish your answers to Topic 7.
Make sure to label each answer carefully (e.g. 'Question 2(d) continued').



End of Topic 7

This sample Mathematical Applications paper shows the format of the examination for 2015, based on questions from past examination papers.

2015 SAMPLE MATHEMATICAL APPLICATIONS PAPER

The purpose of this sample paper is to show the structure of the Mathematical Applications examination and the style of questions that may be used. The following extract is from the Stage 2 subject outline for Mathematical Applications:

EXTERNAL ASSESSMENT

Assessment Type 3: Examination (30%)

Students undertake a 2-hour examination in which they answer questions on two of the three examined topics. The three examined topics are:

- Topic 2: Investment and Loans
- Topic 4: Matrices
- Topic 7: Statistics and Working with Data.

The external examination is based on the subtopics, key questions, and key ideas outlined in the examined topics. The considerations for developing teaching and learning strategies are provided as a guide only, although applications described under this heading may provide useful contexts for examination questions. The examination is set by the SACE Board and conducted under supervision in schools on a day and time to be nominated by the Board.

The examination consists of a range of questions, some focusing on knowledge, routine skills, and applications, and others focusing on analysis and interpretation. Students are required to provide explanations and arguments, and use notation, terminology, and representation correctly throughout the examination.

Students must have access to approved electronic technology during the external examination. However, students need to be discerning in their use of electronic technology to solve questions in the examination.

For this assessment type, students provide evidence of their learning in relation to the following assessment design criteria:

- mathematical knowledge and skills and their application
- mathematical modelling and problem-solving
- communication of mathematical information.

Source: Mathematical Applications 2015 Subject Outline Stage 2, pp. 43–4, on SACE website, www.sace.sa.edu.au