**STAGE 1 MATHEMATICS**

**PROGRAM 3 – SEMESTER 2**

This program is for a cohort of students intending to continue to Mathematical Methods at Stage 2. The following program describes the second semester of learning.

**SEMESTER TWO – 17 WEEKS INCLUDING EXAM WEEK AND STUDENT DEVELOPMENT/ACTIVITY WEEK**

* Topic 3 – Trigonometry Subtopics 3.2 and 3.3 (3 weeks)
* Topic 4 – Counting and Statistics Subtopic 4.1 (3 weeks)
* Topic 12 – Real and Complex Numbers Subtopic 12.3 (2 weeks)
* Topic 2 – Polynomials Subtopic 2.2 (2 weeks)
* Topic 1 – Functions and Graphs Subtopic 1.2 (1 week)
* Topic 5 – Growth and Decay Subtopics 5.1, 5.2 and 5.3 (4 weeks)

**Topic 3 – Trigonometry Subtopics 3.2 and 3.3 (3 weeks)**

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| Termweek | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| WEEK 9 TERM 2 BEGINNING OF SEMESTER TWO - STUDENT DEVELOPMENT WEEK - CAREERS, POSITIVE EDUCATION |
| 2-10 | 3.2Circular Measure and Radian Measure | Introduction to the unit circle and its propertiesHow the unit circle is linked to graphs of $cosθ$ and $sinθ$* The link between the unit circle and $cosθ$, $sinθ$ and $tanθ$ in degrees
* The unit circle definition of $cosθ$, $sinθ$ and $tanθ$ and periodicity using degrees

Definition of radian measure* Conversion between radian and degree measure

Calculation of the lengths of arcs and areas of sectors of circle |  |
| 3-1 | 3.3Trigonometric Functions | Connection between unit circle and $\cos(θ)$, $\sin(θ)$ and $\tan(θ)$ in radians Determine the exact value of cosine and sine from multiples of $\frac{π}{6}$ and $\frac{π}{4}$ using unit circle or graphsMaking the connection that the functions $y=\cos(θ)$ and $y=\sin(θ)$ best describe the horizontal and vertical positions around a circleExplore the features of $y=\sin(θ)$ and$ y=\cos(θ)$* Amplitude $y=A\sin(x)$ and $y=A\cos(x)$
* Period $y=\sin(Bx)$ and $y=\cos(Bx)$
* Phase $y=\sin((x+C))$ and $y=\cos((x+C))$

Solve practical problems in a range of different contexts |  |
| 3-2 | Solve trigonometric equations both algebraically and graphically* Only consider cases such as $cosx=\frac{1}{2}$ and $\sin(\left(2x\right))=\frac{1}{2}$

Special relationships observed of sine and cosine functions * $\sin(\left(-x\right))=-sinx$
* $cos \left(-x\right)=cosx$
* $sin \left(x+\frac{π}{2}\right)=cosx$
* $cos \left(x-\frac{π}{2}\right)=sinx$

Tangent function* Consider the relationship between the angle of inclination and the gradient of a line
* The relationship $tan⁡(x)=\frac{sin⁡(x)}{cos⁡(x)}$
* Graphs of the functions
	+ $y=tanx$
	+ $y=tanBx$
	+ $y=tan⁡(x+C)$
 |  |
| 3-3 |  | **Revision and SAT 1** | **SAT 1**Subtopics3.2 and 3.3Calculator permitted |

**Topic 4 – Counting and Statistics Subtopic 4.1 (3 weeks)**

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| **Term Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-4 | 4.1Counting | The Multiplication Principle* The idea that if there are **a** ways of doing something and **b** ways of doing another thing, then there are **a** × **b** ways of performing both actions. Examples of tree diagrams, tables etc

Factorials and Factorial Notation* The factorial of a [non-negative integer](http://en.wikipedia.org/wiki/Non-negative_integer) n, denoted by n!, is the [product](http://en.wikipedia.org/wiki/Product_%28mathematics%29) of all positive integers less than or equal to *n*. For example, 4!=4×3×2×1=24

Permutations* Counting of all possible arrangements of a collection of things (discrete), where the order is important

 $P\_{r}^{n}=\frac{n!}{(n-r)!}$* Using only discrete variables, students explore various examples. Initially algebraically, then using technology.
 |  |
| 3-5 | Combinations* The number of ways to select different groups in which the order does not matter
* The number of combinations of $r$ objects taken from a set of $n$ distinct objects is

$$C\_{r}^{n}=\frac{P\_{r}^{n}}{r!}=\frac{n!}{(n-r)!r!}$$* Using only discrete variables, students explore various examples. Initially algebraically, then using technology.

Use of the notation $(\_{r}^{n})=\frac{n!}{(n-r)!r!}$* The coefficients of the expansion of $(x+y)^{n}$
	+ Expand $(x+y)^{n}$ for integers $n=1,2,3,4$
	+ Recognise the numbers $(\_{r}^{n})$ as binomial coefficients
	+ The pattern connecting the values of $(\_{r}^{n})$ leading to Pascal’s triangle
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| 3-6 |  | **Revision and consolidation** |  |

**Topic 12 – Real and Complex Numbers Subtopic 12.3 (2 weeks)**

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| **Term- Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-7 | 12.3Complex Numbers | Introduction to the imaginary number $i$, and its definition as $i=\sqrt{-1}$. Exemplify its use in solutions to equations such as $x^{2}+1=0$Introduction to complex numbers: $a+bi$ and defining the real and imaginary componentsOperations with complex numbers, including the use of$ i^{2}=-1$* Addition, subtraction, multiplication and division
* Complex conjugates
* Readdress the quadratic formula in the context of complex number solutions
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**Topic 2 – Polynomials Subtopic 2.2 (2 weeks)**

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| **Term- Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-8 | 2.2Cubic and quartic polynomials | Definition of a cubic Terminology, degree and forms* General $y=ax^{3}+bx^{2}+cx+d, a\ne 0$
* Point of Inflection $y=a(x-b)^{3}+c, a\ne 0$
* Factored $y=a\left(x-α\right)\left(x-β\right)\left(x-γ\right), a\ne 0$

Features* Shape reference to leading coefficient

$$a>0, increasing shape$$$$a<0, decreasing shape $$* Behaviour as $x\rightarrow \pm \infty and y\rightarrow \pm \infty $
* Nature and number of zeros of the graph of a cubic

Explore features of cubics written as a product of:* A linear factor and a quadratic (both real and non-real zeros)
* Three linear factors

Determining cubic functions from given zeros and one other piece of dataDefinition of a quarticTerminology, degree and forms as an extension of the work on cubicsCubic and quartic modelling (using technology)Determining unknown variablesOptimisation such as dimensions for maximum volume | **INVESTIGATION**Features of Polynomials |
| 3-9 |  | **Revision and SAT 2** **Investigation Due**  | **SAT 2** Subtopics 4.1, 12.3, 2,2Calculator permitted |

**Topic 1 – Functions and Graphs Subtopic 1.2 (1 week)**

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| **Term- Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 3-10 | 1.2Inverse Relationships | Exploring the mathematical relationship where one variable decreases as the other increasesConsider the graph of the basic hyperbola $y=\frac{1}{x}$* Define asymptote, both horizontal and vertical

Consider translations of the basic hyperbola i.e. $y=\frac{a}{x-c}$The use of technology is incorporated in the graphs above |  |

**Topic 5 – Growth and Decay Subtopics 5.1, 5.2 and 5.3 (4 weeks)**

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| **Term- Week** | **Subtopic** | **Concepts and Content**Technology is incorporated into all aspects of this topic as appropriate | **Assessment Task** |
| 4-1 | 5.1 Indices and Index Laws | Indices* Review indices and index laws including negatives and fractional
* Algebraic application to all laws including simplification using positive, negative and fractional indices
* Conversions from radical to fractional indices

Surds* Definition of rational and irrational numbers
* Operations with surds and fractional indices (rational indices)
* Discussion on the real number system and its inclusion of irrationals
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| 4-2 | 5.2Exponential Functions | Exponentials* Exponential functions - their algebraic properties and uses
* Behaviour of exponential functions
* Technology will be used to explore the qualitative features of the graph of $y=a^{x}$, its translations $y=a^{x}+b$ and $y=a^{x+c},$ and dilation $y=ka^{x}$
* Discussion on characteristics such as asymptotes, intercepts and behaviour as $x\rightarrow \pm \infty $
* \*Use of real life situations to determine variables in the contexts such as bacteria growth, radioactive decay, half-life, population models and compounding interest. Technology is used to support interpretation of situations.
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|  4-3 | 5.3Logarithmic Functions | Definition of a logarithm, initially base 10* Rules, initially base 10

This could be extended to logarithms with bases other than 10 (e.g. base *e*)* Application of rules with other bases

Solving of logarithmic equations (base 10)Solving exponential equations using logarithms (base 10) threaded back to exponentials dot point 5\* (from subtopic 5.2) |  |
| 4-4 |  | **Revision and SAT 3** | **SAT 3**Subtopics 1.2, 5.1, 5.2 and 5.3Calculator permitted |
| 4-5 | **EXAMINATION REVISION** |
| 4-6 | **YEAR 11 EXAMS** |