NAME: ………………………………….

PART A: NO CALCULATOR

QUESTION 1 **5 marks**

Solve algebraically to find the *exact* value for x:

(a) $3^{2x-1} = \sqrt{3}$

(2 marks)

(b) $25^{x} = \left(\frac{1}{5}\right)^{1+2x}$

(3 marks)

QUESTION 2 **9 marks**

(a) Expand and simplify (where possible):

(i) $(a^x - 7)(2a^x + 5)$

(2 marks)

(ii) $2^{-x}(2^{3x} - 2^{-2x})$

(2 marks)

(b) Fully factorise:

(i) $49^x - 9$

(2 marks)

(ii) $25^x - 4(5^x) + 3$

(3 marks)

QUESTION 3 **11 marks**

(a) Algebraically determine in simplest form:

(i) $\log\left(\dfrac{10000}{10^{3x}}\right)$

(2 marks)

(ii) $\log_3 (27\sqrt{3})$

(2 marks)

(b) Write the following as a single logarithm or integer:

(i) $\log 50 + \log 2$

(2 marks)

(ii) $\frac{1}{3}\log 8 - \log 4$

(2 marks)

(iii) $4\log 2 + \log 5 - 1$

(3 marks)

QUESTION 4 **10 marks**

(a) Show that $\dfrac{2\log 27}{\log 9} = 3$.

(3 marks)

(b) (i) Simplify $\sqrt{98} - 3\sqrt{8}$

(2 marks)

(ii) Simplify $(a\sqrt{b} - c)(a\sqrt{b} + c)$

(2 marks)

(iii) Explain with algebraic working why $\dfrac{\sqrt{6}}{3 - \sqrt{6}} = 2 + \sqrt{6}$

(3 marks)

PART B: CALCULATOR ALLOWED

NAME: ……………………………….

QUESTION 5 **6 marks**

Solve algebraically (show all working) for the unknown in the following, giving your answers correct to 2 decimal places:

(a) $5^x = 73$

(2 marks)

(b) $2^{x-1} = 17$

(2 marks)

(c) $\log_2(x + 1) = 3$

(2 marks)

QUESTION 6 **5 marks**

The weight, W_t, grams of radioactive material remaining

after t years is given by the formula $W_t = 40 \times 2^{\frac{-1}{5}t}$.

Find:

(a) the initial weight present;

(1 mark)

(b) (i) the amount present after 15 years.

(1 mark)

(ii) the amount present after 30 years.

(1 mark)

(c) How long it would take to decay to a 'safe level' of 5% of its original value?

(2 marks)

NAME: ……………………………….

PART A: NO CALCULATOR

QUESTION 1 **5 marks**

Solve algebraically to find the *exact* value for x:

(a) $3^{2x-1} = \sqrt{3}$

$$3^{2x-1} = 3^{\frac{1}{2}}$$

$$\therefore 2x - 1 = \frac{1}{2} \quad ✓$$

$$\therefore \quad 2x = \frac{3}{2}$$

$$\therefore \quad x = \frac{3}{4} \quad ✓$$

(2 marks)

(b) $25^x = \left(\frac{1}{5}\right)^{1+2x}$

$$5^{2x} = 5^{-1-2x} \quad ✓ \quad ✓$$

$$\therefore 2x = -1 - 2x$$

$$\therefore 4x = -1$$

$$\therefore \quad x = -\frac{1}{4} \quad ✓$$

(3 marks)

QUESTION 2 **9 marks**

(a) Expand and simplify (where possible):

(i) $(a^x - 7)(2a^x + 5) = 2a^{2x} + 5a^x - 14a^x - 35$ ✓ $= 2a^{2x} - 9a^x - 35$ ✓

(2 marks)

(ii) $2^{-x}(2^{3x} - 2^{-2x}) = 2^{2x} - 2^{-3x}$ ✓ ✓

(2 marks)

(b) Fully factorise:

(i) $49^x - 9 = (7^x + 3)(7^x - 3)$ ✓ ✓

(2 marks)

(ii) $25^x - 4(5^x) + 3$

$= (5^x)^2 - 4(5^x) + 3$ ✓

$= (5^x - 3)(5^x - 1)$

✓ ✓

(3 marks)

QUESTION 3 **11 marks**

(a) Algebraically determine in simplest form:

(i) $\log\left(\frac{10000}{10^{3x}}\right) = \log\left(\frac{10^4}{10^{3x}}\right)$ ✓ $= 4 - 3x$ ✓

(2 marks)

(ii) $\log_3(27\sqrt{3}) = \log_3\left(3^3 \times 3^{\frac{1}{2}}\right)$ ✓ $= \log_3\left(3^{\frac{7}{2}}\right) = \frac{7}{2}$ ✓

(2 marks)

(b) Write the following as a single logarithm or integer:

(i) $\log 50 + \log 2 = \log(50 \times 2)$ ✓ $= \log 100 = 2$ ✓

(2 marks)

(ii) $\frac{1}{3}\log 8 - \log 4 = \log\left(\frac{8^{\frac{1}{3}}}{4}\right)$ ✓ $= \log\left(\frac{2}{4}\right) = \log\left(\frac{1}{2}\right)$ ✓ or $= \log_1 - \log_2 = -\log 2$

(2 marks)

(iii) $4\log 2 + \log 5 - 1 = \log(16 \times 5)$ ✓ $- \log 10$ ✓

$= \log\left(\frac{80}{10}\right)$ ✓

$= \log 8$ ✓

(3 marks)

QUESTION 4 **10 marks**

(a) Show that $\frac{2\log 27}{\log 9} = 3$.

$$\frac{2\log 27}{\log 9} = \frac{2\log 3^3}{\log 3^2} \checkmark$$

$$= \frac{6\cancel{\log 3}}{2\cancel{\log 3}} \checkmark \checkmark$$

$$= 3$$

(3 marks)

(b) (i) Simplify $\sqrt{98} - 3\sqrt{8}$

$\sqrt{49 \times 2} - 3\sqrt{4 \times 2} = 7\sqrt{2} - 6\sqrt{2}$ ✓ $= \sqrt{2}$ ✓

(2 marks)

(ii) Simplify $(a\sqrt{b} - c)(a\sqrt{b} + c)$

$(a\sqrt{b} - c)(a\sqrt{b} + c) = a^2b - c^2$ ✓ ✓

(2 marks)

(iii) Explain with algebraic working why $\frac{\sqrt{6}}{3 - \sqrt{6}} = 2 + \sqrt{6}$

$$\frac{\sqrt{6}}{3-\sqrt{6}} \times \frac{3+\sqrt{6}}{3+\sqrt{6}} \checkmark \quad = \frac{3\sqrt{6} + 6}{9 - 6} \checkmark$$

$$= \frac{3\sqrt{6}}{3} + \frac{6}{3} \checkmark$$

$$= \sqrt{6} + 2 \quad \text{or } 2 + \sqrt{6}$$

(3 marks)

PART B: CALCULATOR ALLOWED

NAME: ....

QUESTION 5 **6 marks**

Solve algebraically (show all working) for the unknown in the following, giving your answers correct to 2 decimal places:

(a) $5^x = 73$

$x \log 5 = \log 73$ ✓

$\therefore \quad x = \dfrac{\log 73}{\log 5} \approx 2.67$ ✓

(2 marks)

(b) $2^{x-1} = 17$

$(x-1)\log 2 = \log 17$ ✓

$\therefore \quad x - 1 = \dfrac{\log 17}{\log 2}$

$\therefore \quad x = \dfrac{\log 17}{\log 2} + 1 \approx 5.09$ ✓

(2 marks)

(c) $\log_2(x+1) = 3$

$x + 1 = 2^3 = 8$ ✓

$\therefore \quad x = 8 - 1 = 7$ ✓

(2 marks)

QUESTION 6 **5 marks**

The weight, W_t, grams of radioactive material remaining

after t years is given by the formula $W_t = 40 \times 2^{\frac{-1}{5}t}$.

Find:

(a) the initial weight present;

$W_0 = 40$ grams ✓

(1 mark)

(b) (i) the amount present after 15 years.

$W_{15} = 5$ grams ✓

(1 mark)

(ii) the amount present after 30 years.

$W_{30} = 0.625$ grams ✓

(1 mark)

(c) How long it would take to decay to a 'safe level' of 5% of its original value?

5% of 40 $= 0.05 \times 40 = 2$ grams ✓

Using gcalc in graph mode $t \approx 21.6$ years ✓

(2 marks)