

## Self-directed Clarifying Activity – Assessment Type 1: Skills and Applications Tasks – Quadratic and other Polynomials

1. Use the annotated performance standards and student work sample to compare your interpretation of the performance standards and recalibrate your assessment decision (if necessary).

# Assessment Decision = B

Please see annotations below

|          | Mathematical Knowledge and Skills and Their Application                                                                                                                                                                                                                                                                                                                                | Mathematical Modelling and Problem-solving                                                                                                                                                                                                                                                                                                                                                                                                    | Communication of Mathematical Information                                                                                                                                                                       |
|----------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>B</b> | <p>Some depth of knowledge of content and understanding of concepts and relationships.</p> <p>Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find some correct solutions to complex questions.</p> <p>Accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.</p>                   | <p>Attempted development and appropriate application of mathematical models.</p> <p>Mostly accurate and complete solutions to mathematical problems set in applied and theoretical contexts.</p> <p>Complete interpretation of the mathematical results in the context of the problem.</p> <p>Some depth of understanding of the reasonableness and possible limitations of the interpreted results, and recognition of assumptions made.</p> | <p>Effective communication of mathematical ideas and reasoning to develop mostly logical arguments.</p> <p>Mostly accurate use of appropriate notation, representations, and terminology.</p>                   |
| <b>C</b> | <p>Generally competent knowledge of content and understanding of concepts and relationships.</p> <p>Use of mathematical algorithms and techniques (implemented electronically where appropriate) to find mostly correct solutions to routine questions.</p> <p>Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts.</p> | <p>Appropriate application of mathematical models.</p> <p>Some accurate and generally complete solutions to mathematical problems set in applied and theoretical contexts.</p> <p>Generally appropriate interpretation of the mathematical results in the context of the problem.</p> <p>Some understanding of the reasonableness and possible limitations of the interpreted results, and some recognition of assumptions made.</p>          | <p>Appropriate communication of mathematical ideas and reasoning to develop some logical arguments.</p> <p>Use of generally appropriate notation, representations, and terminology, with some inaccuracies.</p> |

# STAGE 1 MATHEMATICS

## SKILLS AND APPLICATIONS TASK – Quadratic and Other Polynomials

1. Use factorisation to solve

(a)  $x^2 + 3x - 28 = 0$  [2 marks]

$$\begin{aligned} 7-4 &= 3 & 7 \times -4 &= -28 \\ (x-4)(x+7) &= 0 \\ x &= 4 \text{ or } -7 \end{aligned}$$

✓✓

(b)  $6x^2 - 7x - 3 = 0$  [2 marks]

$$\begin{aligned} 6x^2 + 2x - 9x - 3 \\ 2x(3x+1) - 3(3x+1) \\ (3x+1)(2x-3) &= 0 \\ x &= -\frac{1}{3} \text{ or } \frac{3}{2} \end{aligned}$$

-, +

2. Determine the equation of the axis of symmetry of  $y = 5x^2 - 2x + 9$  [1 mark]

$$-\frac{b}{2a} = -\frac{-2}{2 \times 5} = \frac{1}{5}$$

3. Use "Completing the Square" to solve the equation  $x^2 + 6x - 1 = 0$  [2 marks]

$$\begin{aligned} x^2 + 6x - 1 &= 0 \\ x^2 + 6x + 9 &= 10 \\ (x+3)^2 &= 10 \\ x+3 &= \pm\sqrt{10} \\ x &= -3 \pm \sqrt{10} \end{aligned}$$

✓✓

4. A quadratic function has a vertex at  $(-1, 3)$  and a y-intercept at  $(0, 8)$ . Write the equation of this function in the form  $y = a(x-d)^2 + e$ . [2 marks]

$$\begin{aligned} y &= a(x-1)^2 + 3 \\ 8 &= a(0-1)^2 + 3 \\ 5 &= a \\ y &= 5(x-1)^2 + 3 \end{aligned}$$

5. Find all quadratic equations with roots of -5 and 7 [1 mark]

$$y = a(x+5)(x-7) \quad a \neq 0$$

✓

**Mathematical Knowledge and Skills and Their Application**  
Evidence of mostly correct solutions to routine questions (Q1-5).

6. Expand and Simplify.

(a)  $(5x + 2\sqrt{3})(5x - 2\sqrt{3})$

[1 mark]

$$25x^2 - 12 \quad \checkmark$$

(b)  $(7x - \sqrt{5})^2$

[1 mark]

$$\begin{aligned} & (7x - \sqrt{5})(7x - \sqrt{5}) \\ & = 49x^2 - 7x\sqrt{5} - 7x\sqrt{5} + 5 \\ & = 49x^2 - 14x\sqrt{5} + 5 \end{aligned} \quad \checkmark$$

(c)  $(3 - 2i)(3i + 1)$  where  $i = \sqrt{-1}$

[2 marks]

$$\begin{aligned} & = 9i + 3 - 6i^2 - 2i \\ & = -6i + 9i - 2i + 3 \\ & \quad \quad \quad \downarrow \\ & = 7i + 6 + 3 \\ & = 7i + 9 \quad \checkmark \end{aligned}$$

$$\begin{aligned} i^2 &= -1 \\ -6 \times -1 &= 6 \end{aligned}$$

7. Solve the equation  $-7x^2 + 11x + 3 = 0$  using the quadratic formula.

[2 marks]

$$\begin{aligned} x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-11 \pm \sqrt{11^2 - 4 \times (-7) \times 3}}{2 \times (-7)} \end{aligned}$$

$$\begin{aligned} x &= \frac{-11 \pm \sqrt{121 - 12} \rightarrow 109}{2} \\ &= \frac{-11 \pm \sqrt{109}}{2} \end{aligned}$$

8. Solve  $x = 3 - \frac{2}{x}$

[3 marks]

$$\begin{aligned} x &= 3 - \frac{2}{x} \\ x^2 &= 3x - 2 \\ x^2 - 3x + 2 &= 0 \quad \checkmark \end{aligned}$$

$$\begin{aligned} -2 \times -1 &= 2 \\ -2 + -1 &= -3 \\ (x-1)(x-2) &= 0 \\ x &= 1 \text{ or } 2 \quad \checkmark \end{aligned}$$

9. Write  $\sqrt{\frac{-49}{81}}$  in terms of  $i$ .

[1 mark]

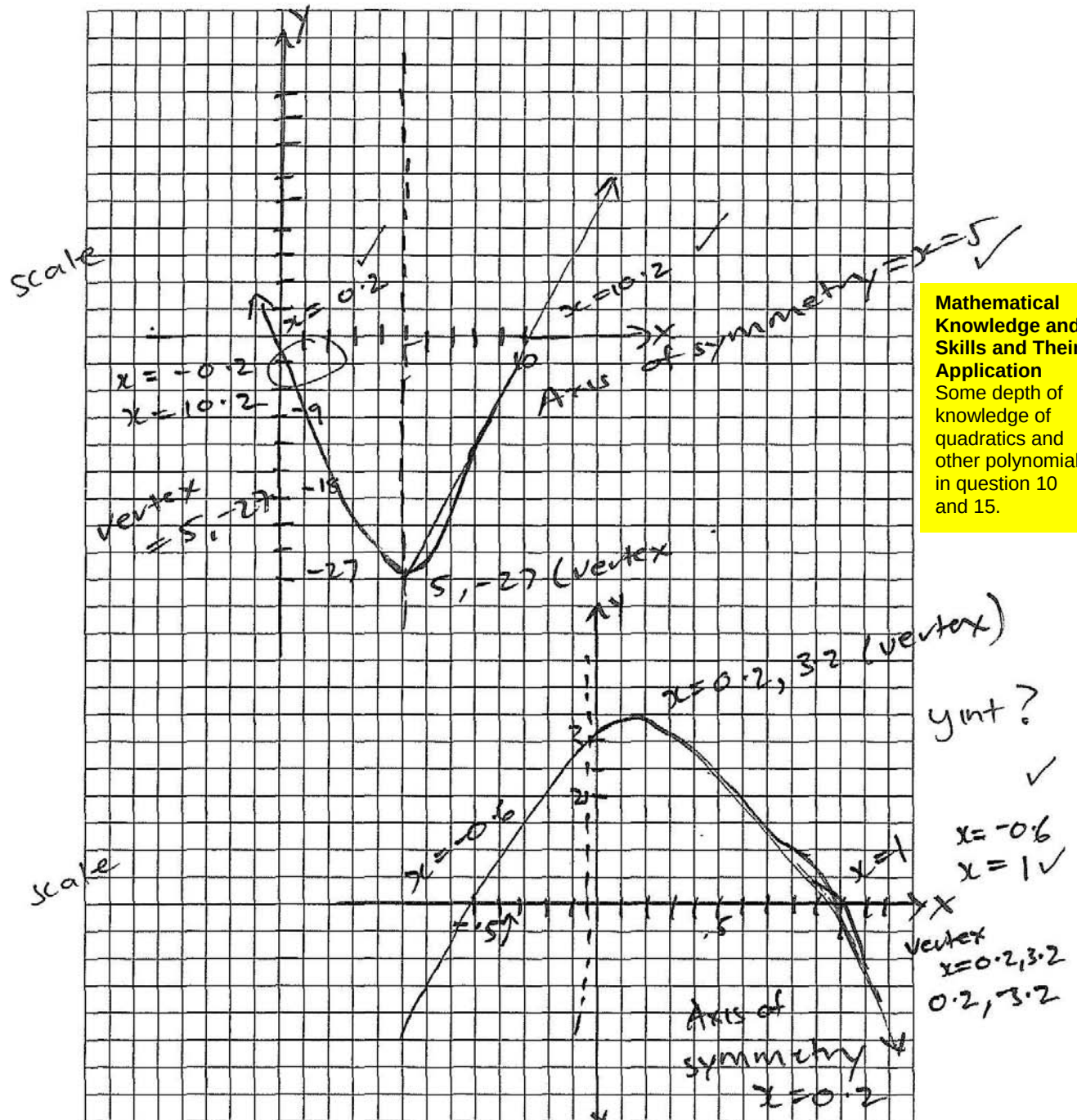
$$\begin{aligned} \frac{\sqrt{-49}}{\sqrt{81}} &= \frac{\sqrt{-1} \times \sqrt{-49}}{9} \\ &= \frac{i\sqrt{49}}{9} \\ &= \frac{7i}{9} \quad \checkmark \end{aligned}$$

10. Sketch (on separate axes) the graphs of

(a)  $y = x^2 - 10x - 2$

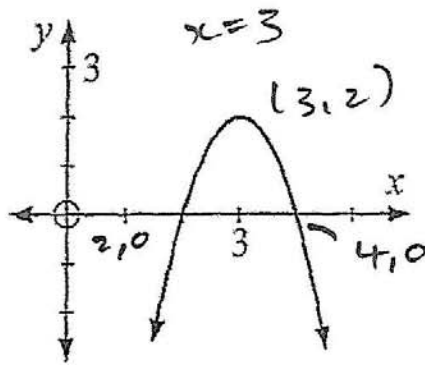
(b)  $y = -5x^2 + 2x + 3$

showing clearly any axis intercepts, axes of symmetry and vertices. [6 marks]



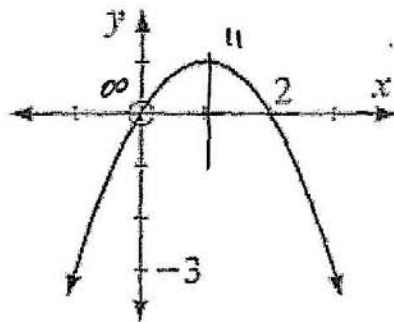
11. Find the equation of each of the graphs below.

[6 marks]



$$y = -x^2 + 5x - 2$$

↓ this correct!



$$y = -\frac{1}{4}(x-2)^2 + 1$$

$$y = a(x-2)^2 + 1$$

$$0 = a(x-2)^2 + 1$$

$$0 = 4a + 1$$

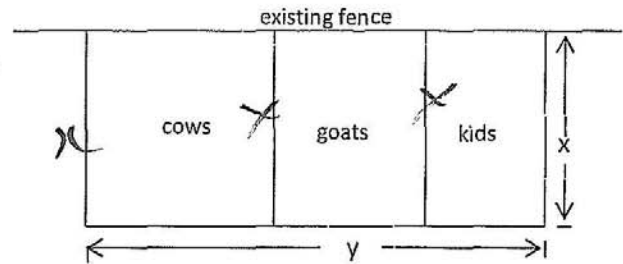
$$-1 = 4a$$

$$4a = -1$$

$$a = -\frac{1}{4}$$

### Question 12

Farmer Eccles needs to construct 3 rectangular paddocks for cows, goats and kids as shown in the diagram. He has 4000m of fencing available and will use an existing fence as one side of his pens.



- (a) Show that  $y = 4000 - 4x$ . (2 marks)

$$4x + y = 4000$$

$$y = 4000 - 4x \quad \checkmark \checkmark$$

- (b) Show that the total area  $A \text{ m}^2$ , is given by  $A = 4000x - 4x^2$ . (2 marks)

$$A = x \times y$$

$$= x(4000 - 4x) \quad \checkmark \checkmark$$

$$= 4000x - 4x^2$$

- (c) Find the dimensions which will give the paddock a maximum area. (2 marks)

$$= -4x^2 + 4000x$$

$$-\frac{b}{2a} = \frac{-4000}{-8}$$

$$x = 500 \text{ m}$$

$$y = 4000 - 4x$$

$$= 4000 - 2000$$

$$= 2000 \text{ m}$$

Dimensions are  
 $x = 500 \text{ m}$   
 $y = 2000 \text{ m} \quad \checkmark \checkmark$

- (d) What is the maximum area? (1 mark)

$$1\,000\,000 \text{ m}^2$$

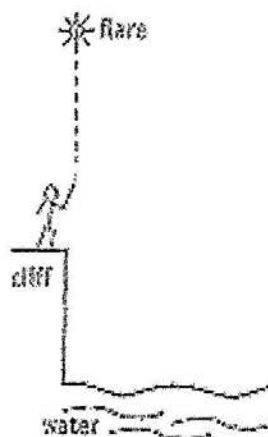
**Mathematical Knowledge and Skills and Their Application**

Use of mathematical algorithms to find some correct solutions to complex questions (Questions 12 and 13)



### Question 13

When a group of bushwalkers became lost, they decided to launch a distress flare vertically into the air from the top of a cliff. At time  $t$  seconds after firing, the height  $h$  metres of the flare above the water was  $h = -5t^2 + 30t + 35$



- (a) What was the height of the flare above the water at  $t = 0$  seconds? (1 mark)

$$h = (-5 \times 0)^2 + (30 \times 0) + 35$$

$$= 35 \text{ m} \quad \checkmark$$

- (b) What does this distance represent? (1 mark)

How high the flare was when it got fired.  $\checkmark$

yes and...?

- (c) How many seconds after firing did the flare reach its maximum height? (2 marks)

$$\frac{-b}{2a} = \frac{-30}{-10} = 3 \text{ s} \quad \checkmark$$

- (d) For how many seconds is the flare in the air? (3 marks)

6 seconds

x

- (e) What was the maximum height the flare reached above the water and at what time after firing, did that occur? (2 marks)

$$h = (-5 \times 3)^2 + (30 \times 3) + 35$$

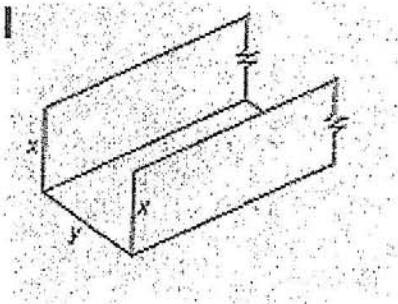
$$= 350 \text{ m} \quad \times$$

The flare reached 350 m above the water level 3 seconds after it was fired.

#### Mathematical Knowledge and Skills and Their Application

Generally accurate application of knowledge and skills to answer questions set in applied and theoretical contexts. For example, questions 12 and 13.

Question 14



A home guttering company makes metal gutters from material which is 36 cm wide. The gutter is open at the top and has a rectangular cross section.

- (a) Find an expression for  $y$  in terms of  $x$ . (2 marks)

$$2x + y = 36$$

$$y = 36 - 2x$$

$$y = 18 - 2x$$

- (b) Find an expression for the area of X-section in terms of  $x$ . (2 marks)

- (c) Find the value of  $x$  for which the area is a maximum. What is that area? (3 marks)

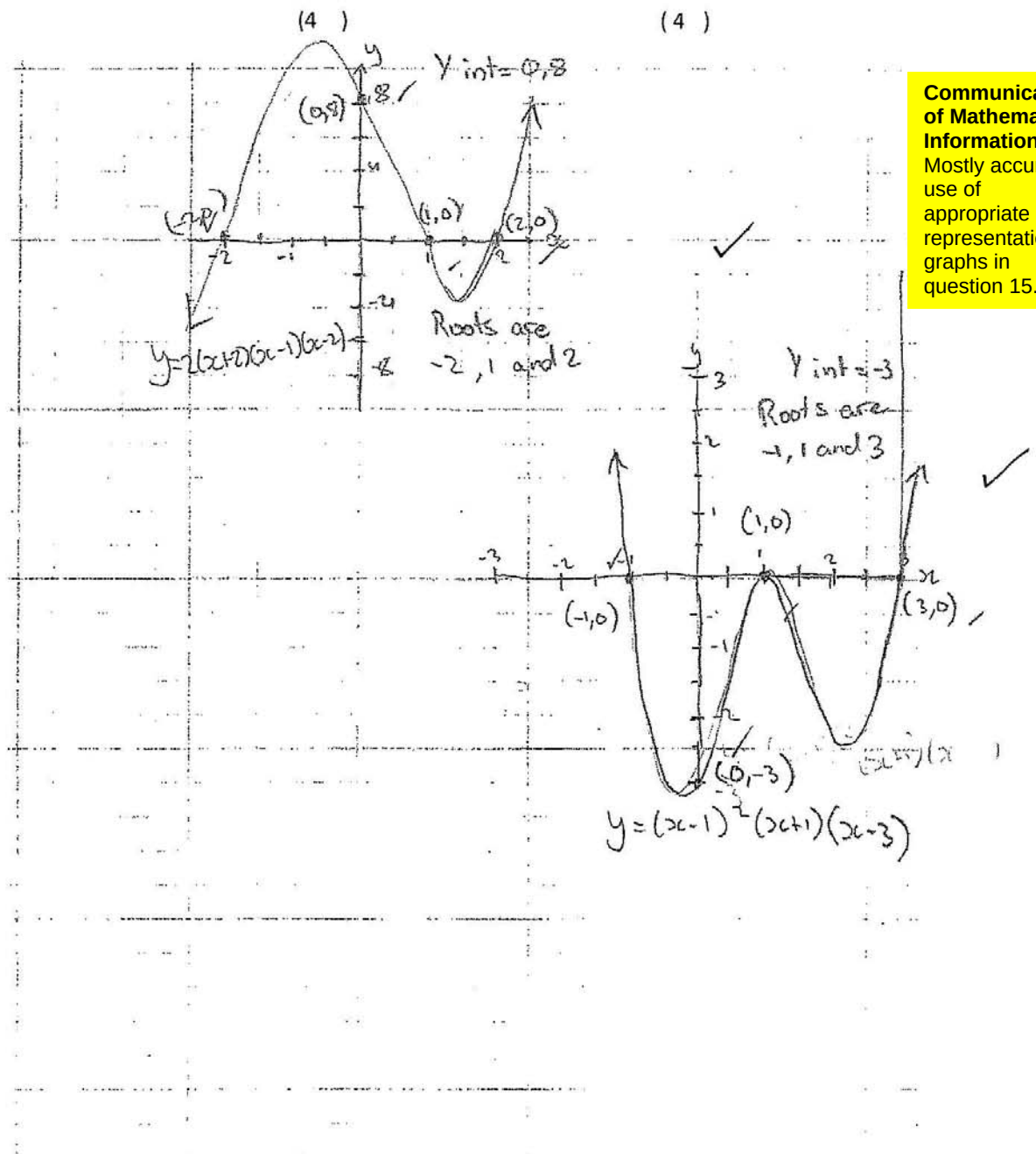


Question 15

In the space provided, sketch the graphs of the given polynomials, showing their x and y intercepts.

(a)  $y = 2(x+2)(x-1)(x-2)$

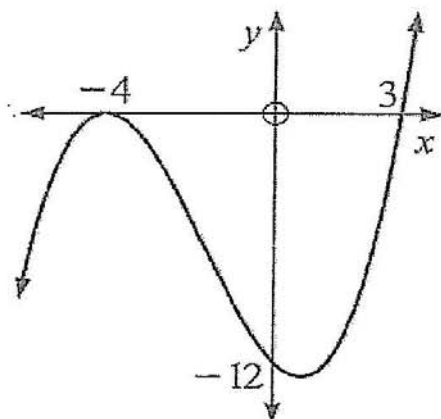
(b)  $y = (x-1)^2(x+1)(x-3)$



**Communication of Mathematical Information**  
Mostly accurate use of appropriate representation of graphs in question 15.

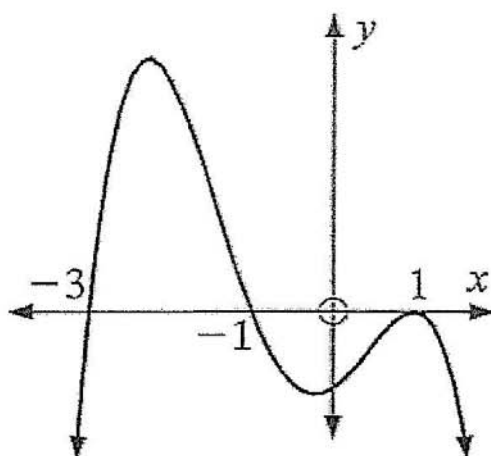
Question 16

For each of the graphs shown, state its equation and explain how you obtained it.



$y = a(x - 4)(x - 3)$  ?  
 $a = a \cdot (10 + 4)(15 + 3)$   
 $a = 1$

(4 marks)



(4 marks)