South Australian Certificate of Education

1

Specialist Mathematics
2025

Question booklet 1

Questions 1 to 7 (55 marks)
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on pages 17 and 18 if you need more space
- Allow approximately 65 minutes
- Approved calculators may be used — complete the box below

Examination information

Materials
- Question booklet 1
- Question booklet 2
- Formula sheet
- SACE registration number label

Instructions
- Show appropriate working and steps of logic in the question booklets
- State all answers correct to three significant figures, unless otherwise instructed
- Use black or blue pen
- You may use a sharp dark pencil for diagrams

Total time: 130 minutes
Total marks: 100

The SACE Board of South Australia acknowledges that this examination was created on Kaurna Land. We acknowledge First Nations Elders, parents, families, and communities as the first educators of their children, and we recognise and value the cultures and strengths that First Nations students bring to the classroom. We respect the unique connection and relationship that First Nations peoples have to Country, and their ever-enduring cultural heritage.

Attach your SACE registration number label here

Graphics calculator

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Question 1 (6 marks)

Liquid is poured into a chemical flask at a rate of $2\,\text{cm}^3$ per second. The volume, V, of the liquid in the flask is given by $V = \frac{11\pi}{2}h^2 - \frac{\pi}{3}h^3$, where h is the height of the liquid in the flask measured in centimetres and $0 \le h \le 5$.

(a) Find h when the volume is $\frac{200\pi}{3}\,\text{cm}^3$.

(1 mark)

(b) Show that $\frac{dh}{dt} = \frac{1}{\pi h(11-h)}\frac{dV}{dt}$.

(2 marks)

(c) **Hence**, find the rate of change of the height of the liquid in the flask at the instant when the volume of the liquid is $\frac{200\pi}{3}\,\text{cm}^3$. State the answer correct to three significant figures.

(3 marks)

Question 2 (6 marks)

It is known that $\int xe^{2x}dx = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + c$, where c is a constant.

This result may be assumed.

(a) Using the information given above and integration by parts, show that

$$\int x^2e^{2x}dx = \frac{1}{2}e^{2x}\left(x^2 - x + \frac{1}{2}\right) + k, \text{ where } k \text{ is a constant.}$$

(3 marks)

Figure 1 displays the graph of $y = xe^x$.

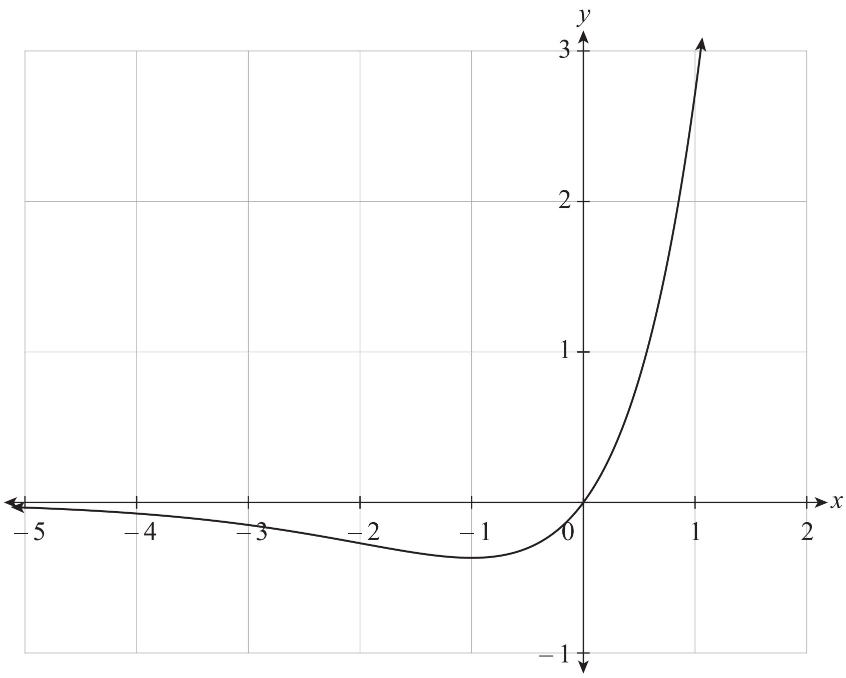

Figure 1

(b) The graph of $y = xe^x$ in **Figure 1** is rotated 2π radians about the x-axis from $x = -1$ to $x = 0$.

Find the **exact** value for the volume of the solid formed.

(3 marks)

Question 3 (9 marks)

(a) (i) Show that $2cis\frac{3\pi}{5}$ is a solution of $z^5 = -32$.

(1 mark)

(ii) Use De Moivre's theorem to show that all the solutions to $z^5 = -32$ are $-2, 2cis\left(\frac{\pi}{5}\right), 2cis\left(\frac{3\pi}{5}\right), 2cis\left(-\frac{3\pi}{5}\right)$, and $2cis\left(-\frac{\pi}{5}\right)$.

(2 marks)

(b) (i) Show that $\frac{z^5+32}{z+2} = z^4 - 2z^3 + 4z^2 - 8z + 16,\ z \neq -2$.

(1 mark)

(ii) **Hence**, state in **exact** $rcis\theta$ form the solutions to $z^4 - 2z^3 + 4z^2 - 8z + 16 = 0$.

(1 mark)

(c) The four solutions found in part (b)(ii) are shown in **Figure 2**.

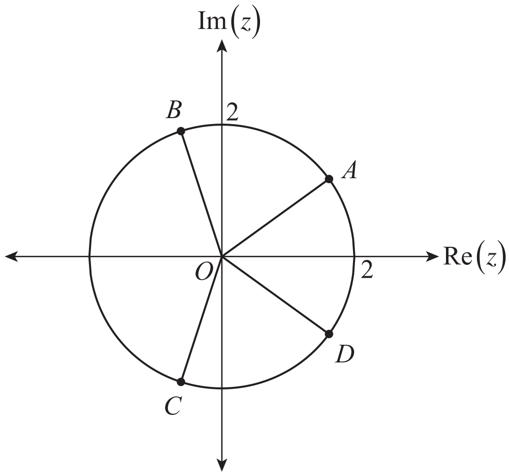

Figure 2

(i) Show that the area of triangle $AOD = 2\sin\frac{2\pi}{5}$.

(2 marks)

(ii) The area of quadrilateral $ABCD = a\sin\frac{2\pi}{5} + b\sin\frac{4\pi}{5}$, where a and b are real constants.

Find the values of a and b.

(2 marks)

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Question 4 (7 marks)

Consider the function $y=(2x+1)e^{x}$ and its derivatives, where

$\frac{dy}{dx}$ is the first derivative,

$\frac{d^2y}{dx^2}$ is the second derivative,

$\frac{d^3y}{dx^3}$ is the third derivative, and so on.

In general, the n^{th} derivative of y is $\frac{d^ny}{dx^n}$.

(a) Show that $\frac{dy}{dx}=(2x+3)e^{x}$.

(2 marks)

(b) For the function $y=(2x+1)e^x$, use mathematical induction to prove that for all positive integers n

$$\frac{d^n y}{dx^n}=\left(2x+(2n+1)\right)e^x.$$

(5 marks)

Question 5 (8 marks)

(a) Show that $(2\boldsymbol{a}+3\boldsymbol{b})\times(\boldsymbol{a}+5\boldsymbol{b})=7\boldsymbol{a}\times\boldsymbol{b}$.

(2 marks)

(b) **Figure 3** shows the vectors $\overrightarrow{OC}=\boldsymbol{a}$, $\overrightarrow{CD}=\boldsymbol{a}$, $\overrightarrow{DE}=3\boldsymbol{b}$, and $\overrightarrow{CF}=5\boldsymbol{b}$.

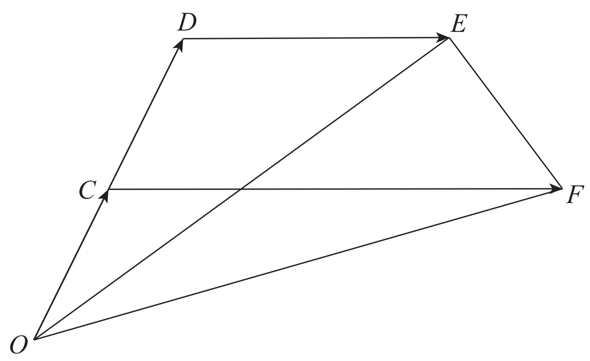

Figure 3

(i) Find $\overrightarrow{OE}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

(1 mark)

(ii) Show that the area of triangle $OEF=\frac{7}{2}|\boldsymbol{a}\times\boldsymbol{b}|$.

(2 marks)

(iii) Show that the area of quadrilateral $ODEF = \frac{13}{2}|\boldsymbol{a} \times \boldsymbol{b}|$.

(1 mark)

(c) If $\boldsymbol{a} = [1, 2, 4]$ and $\boldsymbol{b} = [3, 1, -5]$, find the area of quadrilateral $ODEF$.

(2 marks)

Question 6 (9 marks)

The circle shown in the Argand diagram in **Figure 4** has centre at the positive real number k, and radius k units.

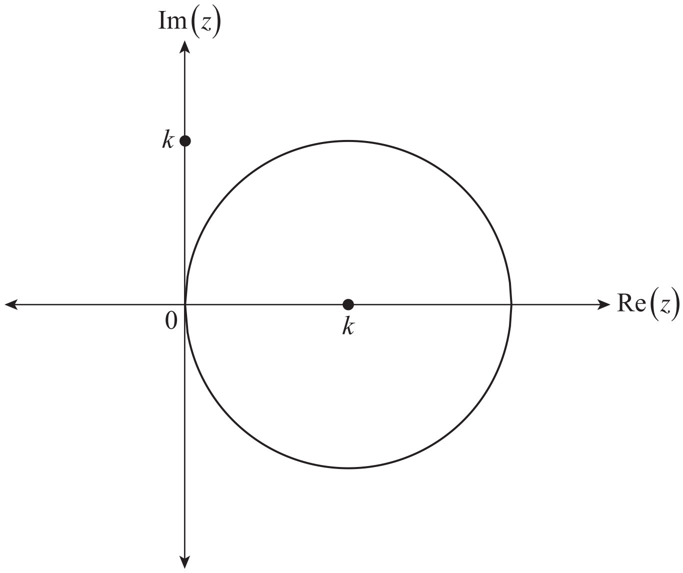

Figure 4

(a) Draw the ray defined by $\arg z = \frac{\pi}{4}$ on **Figure 4**. (1 mark)

(b) The ray intersects the circle at point P in the first quadrant.

(i) Find in terms of k the complex number at P in Cartesian form.

(1 mark)

(ii) Find in terms of k the complex number at P in polar form.

(1 mark)

The Argand diagram in **Figure 5** shows circles C_0, C_1, and C_2. With the sequence continuing, consider the centre of C_n is at the real number 2^n and the radius of C_n is 2^n units.

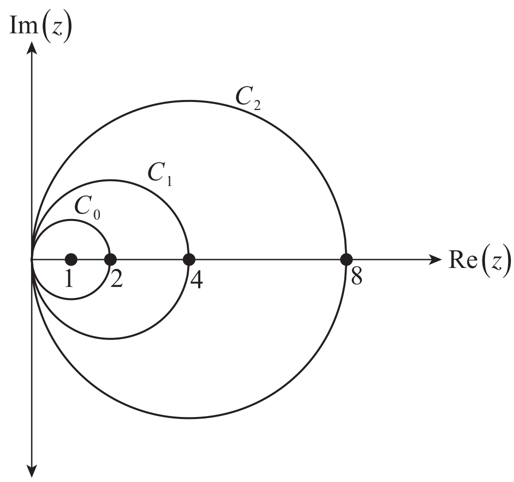

Figure 5

(c) Write the equation of circle C_n in terms of z.

(2 marks)

(d) Consider the complex number $v = 5 + 3i$.

Show that v lies inside the circle C_2.

(2 marks)

(e) Consider the complex number $w = 10cis\frac{\pi}{4}$.

Find the smallest positive integer value of n such that w lies inside the circle C_n. Support your answer using appropriate mathematical reasoning.

(2 marks)

Question 7 (10 marks)

(a) (i) Show that $\sin^3 x = \sin x - \sin x \cos^2 x$.

(1 mark)

(ii) Hence, show that $\int_0^{\frac{\pi}{3}} \sin^3 x \, dx = \frac{5}{24}$.

(3 marks)

Figure 6 shows the graph of $f(x) = \sin^3 x, -\frac{\pi}{3} \le x \le \frac{\pi}{3}$.

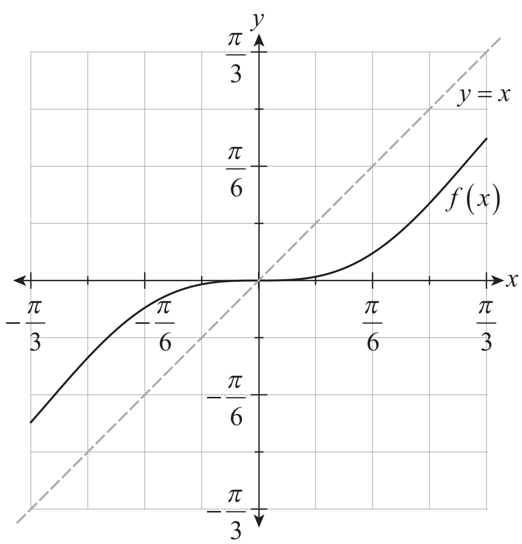

Figure 6

(b) Sketch and label the graph of $f^{-1}(x)$ on the axes in **Figure 6**, above. (2 marks)

Figure 7 shows the graph of a function $g(x)$.

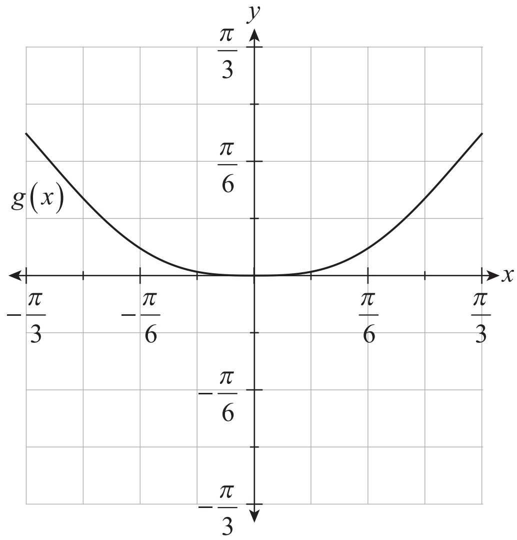

Figure 7

(c) Given that the function $g(x)$ may be written in terms of $f(x)$, circle the correct answer from the following options.

$g(x)=f(\|x\|)$	$g(x)=f(-x)$	$g(x)=f^{-1}(x)$	$g(x)=f^{-1}(\|x\|)$

(1 mark)

Figure 8 shows the graph of functions $f(x)$ and $f^{-1}(x)$ reflected around each of the axes.

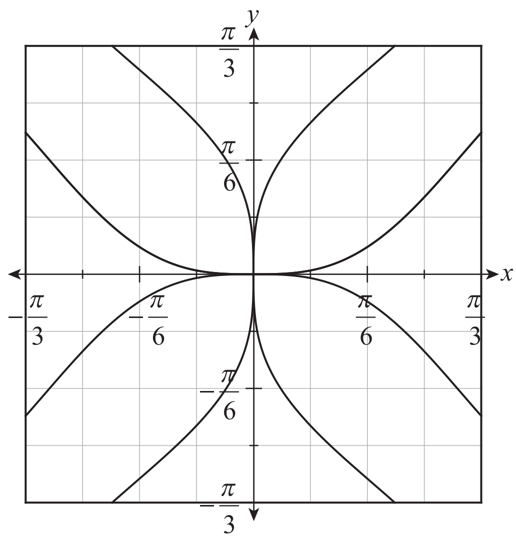

Figure 8

Question 7 continues on page 16.

In **Figure 9** below, the graphs from **Figure 8** are shown contained within the square defined by $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$ and $-\frac{\pi}{3} \le y \le \frac{\pi}{3}$.

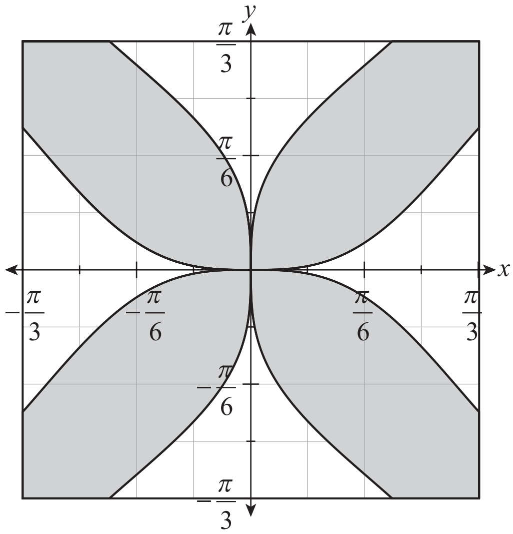

Figure 9

(d) Find the **exact** area of the shaded portions of **Figure 9**.

(3 marks)

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each answer carefully (e.g. 4(b) continued).

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2025

Question booklet 2

Questions 8 to 10 (45 marks)
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on pages 7 and 12 if you need more space
- Allow approximately 65 minutes
- Approved calculators may be used — complete the box below

2

Copy the information from your SACE label here

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Question 8 (15 marks)

(a) (i) Show that the point $\left(\frac{d_1}{a}, 0, 0\right)$ is on the plane $ax + by + cz = d_1$, where $a \neq 0$.

(1 mark)

(ii) Show that the distance between the planes $ax + by + cz = d_1$ and $ax + by + cz = d_2$ is $\frac{|d_1 - d_2|}{\sqrt{a^2 + b^2 + c^2}}$.

(1 mark)

Consider the planes $P_1 : 2x - y + z = 20$ and $P_2 : 2x - y + z = -10$.

(b) (i) Find the distance between the planes.

(2 marks)

(ii) Find the equation of the normal to P_1 through $A(10, 2, 2)$.

(2 marks)

(iii) Show that this normal meets P_2 at $B(0, 7, -3)$.

(3 marks)

Figure 10 shows plane P_3 is parallel to P_1 and P_2, with P_1 between P_2 and P_3.

The distance between P_2 and P_3 is three times the distance between P_1 and P_3.

(c) Find the equation of P_3.

(2 marks)

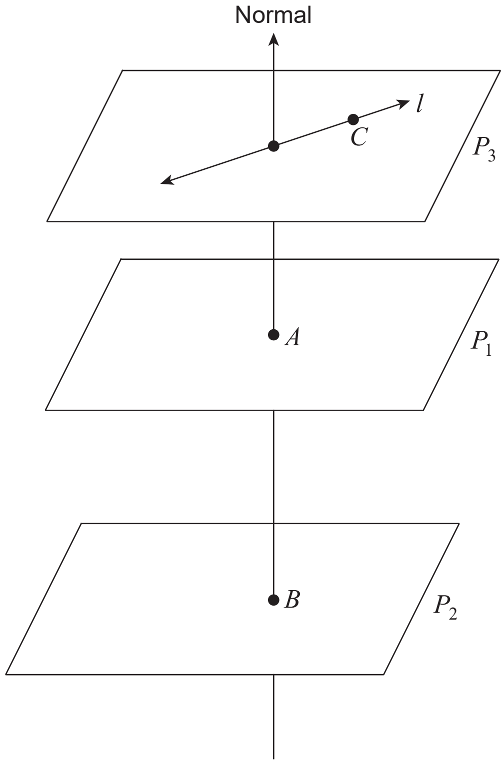


Figure 10

(d) The line l is on P_3 and passes through the normal found in part (b)(ii).

Shown in **Figure 10** is C, which is any point on l.

(i) Explain why $|BC| \leq |AC| + |AB|$.

(1 mark)

(ii) Find the coordinates of C if $|BC| = |AC| + |AB|$.

(3 marks)

Question 9 (14 marks)

(a) Show that $\int \sqrt{1+\cos 2x}\, dx = \sqrt{2}\sin x + c$, where $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ and c is a constant.

(2 marks)

Figure 11 shows a pendulum connected by string to point A, allowing it to swing from side to side. As the pendulum swings, it partially wraps against the solid structure shaded in **Figure 11**.

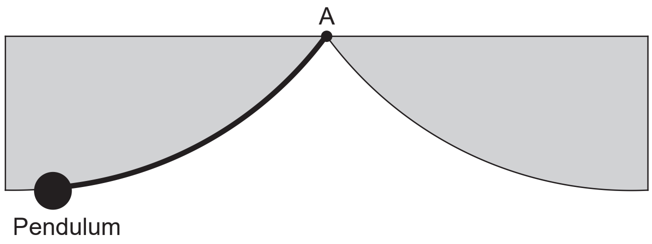


Figure 11

(b) The parametric equations for the path of the pendulum shown in **Figure 11** for a string length of 4 units are

$$\begin{cases} x(t) = 2t + \sin 2t \\ y(t) = -2\cos^2 t \end{cases} \quad \text{for } -\frac{\pi}{2} \le t \le \frac{\pi}{2}, \text{ where } t \text{ is a real parameter.}$$

Sketch on **Figure 12** the path of the pendulum.

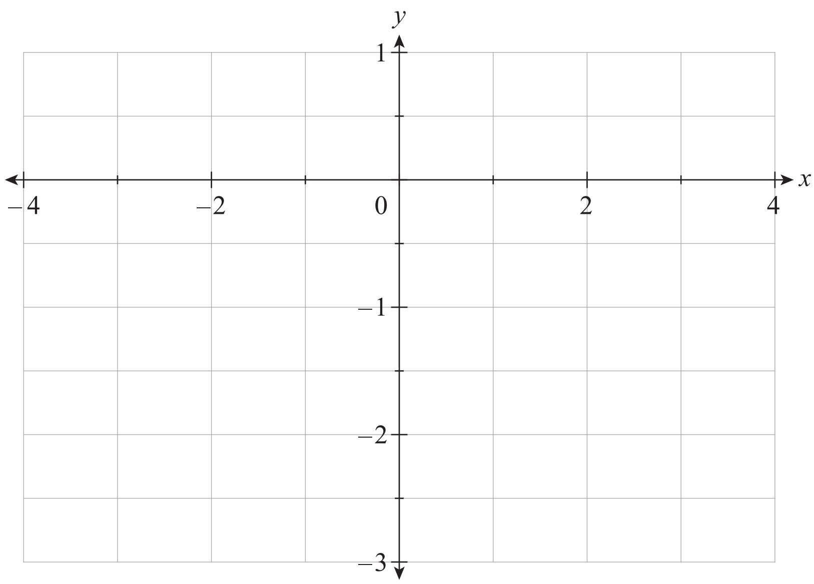


Figure 12

(3 marks)

In general, the parametric equations for the path of a pendulum with string length $4a$ units are given by

$$\begin{cases} x(t) = a(2t + \sin 2t) \\ y(t) = -2a\cos^2 t \end{cases} \text{ for } -\frac{\pi}{2} \le t \le \frac{\pi}{2}.$$

(c) (i) Show that

$$\begin{cases} x'(t) = 2a + 2a\cos 2t \\ y'(t) = 2a\sin 2t \end{cases}.$$

(2 marks)

(ii) **Hence**, show that the path of the pendulum has an arc length

$$l = 2a\sqrt{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sqrt{1+\cos 2t}\, dt.$$

(3 marks)

Question 9 continues on page 6.

(d) (i) Given that the arc length for the path of the pendulum is $l = k \times (\text{string length})$.

Use parts (a) and (c)(ii) to find the value of k, where k is an integer.

(3 marks)

(ii) **Hence or otherwise**, state the arc length for the path of the pendulum when $a = 1$.

(1 mark)

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each answer carefully (e.g. 9(c)(ii) continued).

Question 10 (16 marks)

Two chemicals react to form a new compound.

The unreacted mass of one chemical is $A\,\text{kg}$.

The unreacted mass of the other chemical is $B\,\text{kg}$.

The mass of the new compound is $X\,\text{kg}$.

The rate of change of unreacted mass A is given by $\frac{dA}{dt} = -\frac{1}{4}\frac{dX}{dt}$ where t is measured in minutes.

(a) Use integration to show that the solution to the differential equation $\frac{dA}{dt} = -\frac{1}{4}\frac{dX}{dt}$ is $A = -\frac{1}{4}X + 2$, given that $A = 2$ when $X = 0$.

(2 marks)

The rate of change of unreacted mass B is given by $\frac{dB}{dt} = -\frac{3}{4}\frac{dX}{dt}$, where t is measured in minutes.

Given that $B = 3$ when $X = 0$, it is known that $B = -\frac{3}{4}X + 3$.

The differential equation for the rate of increase in mass X is given by $\frac{dX}{dt} = \frac{1}{3}AB$.

(b) Show that $\frac{dX}{dt} = \frac{1}{16}(X - 8)(X - 4)$.

(2 marks)

(c) Show that $\frac{1}{(X-8)(X-4)} = \frac{1}{4}\left(\frac{1}{X-8} - \frac{1}{X-4}\right)$.

(2 marks)

Question 10 continues on page 10.

(d) (i) **Hence**, solve the differential equation $\frac{dX}{dt} = \frac{1}{16}(X-8)(X-4)$ using integration techniques and the condition that $t = 1$ when $X = 1$ to show that

$$X = \frac{24 - 28e^{\frac{1}{4}(t-1)}}{3 - 7e^{\frac{1}{4}(t-1)}}.$$

(6 marks)

(ii) **Hence**, show that $X = 4 + \dfrac{12}{3 - 7e^{\frac{1}{4}(t-1)}}$.

(1 mark)

(iii) Using part (d)(ii), state the limiting value of X as t increases.

(1 mark)

(e) **Figure 13** shows the slope field for the differential equation $\dfrac{dX}{dt} = \dfrac{1}{3}AB$.

Draw the solution curve for $t \geq 1$, which **begins** at the given point $(1,1)$.

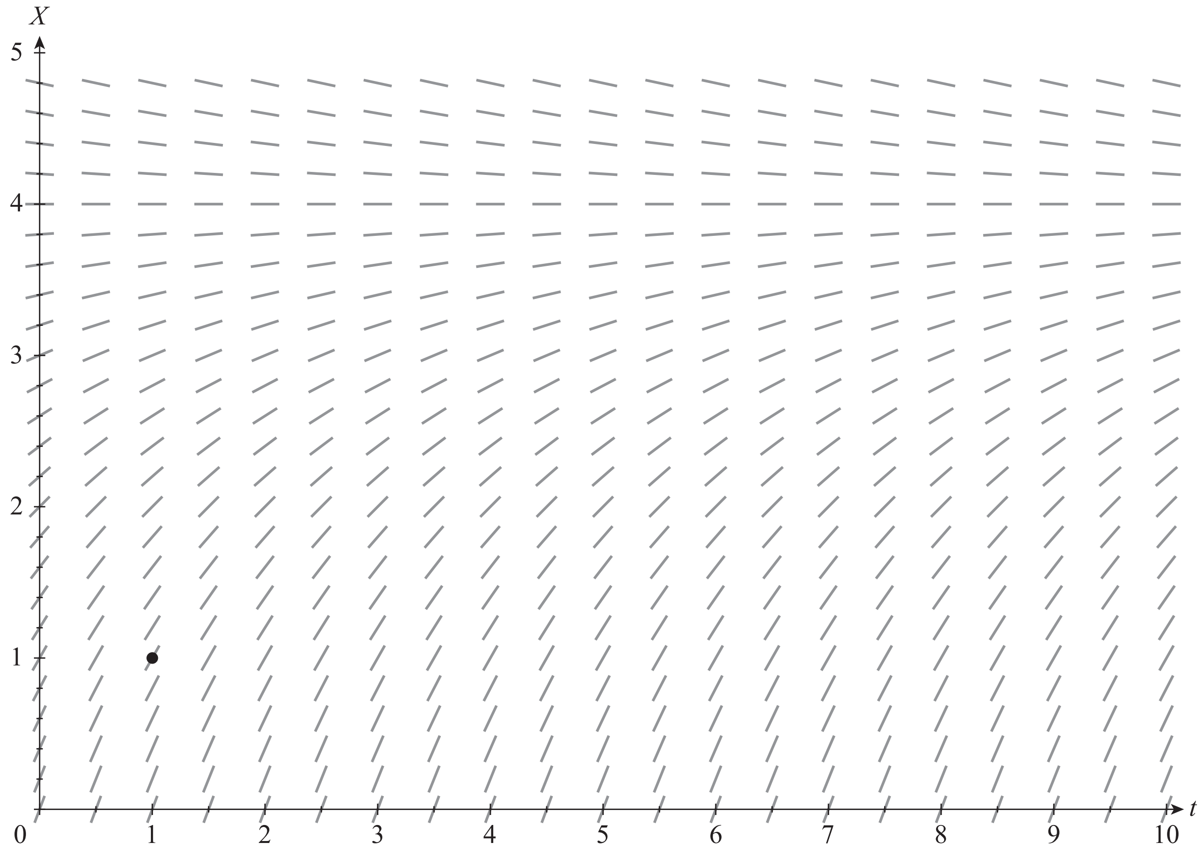


Figure 13

(2 marks)

You may write on this page if you need more space to finish your answers to any of the questions in this question booklet. Make sure to label each answer carefully (e.g. 10(d)(i) continued).

SPECIALIST MATHEMATICS FORMULA SHEET

Circular functions

$\sin^2 A + \cos^2 A = 1$

$\tan^2 A + 1 = \sec^2 A$

$1 + \cot^2 A = \operatorname{cosec}^2 A$

$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$

$\sin 2A = 2 \sin A \cos A$

$$\begin{aligned}\cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A\end{aligned}$$

$\tan 2A = \dfrac{2 \tan A}{1 - \tan^2 A}$

$2 \sin A \cos B = \sin(A + B) + \sin(A - B)$

$2 \cos A \cos B = \cos(A + B) + \cos(A - B)$

$2 \sin A \sin B = \cos(A - B) - \cos(A + B)$

$\sin A \pm \sin B = 2 \sin \frac{1}{2}(A \pm B) \cos \frac{1}{2}(A \mp B)$

$\cos A + \cos B = 2 \cos \frac{1}{2}(A + B) \cos \frac{1}{2}(A - B)$

$\cos A - \cos B = -2 \sin \frac{1}{2}(A + B) \sin \frac{1}{2}(A - B)$

Matrices and determinants

If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $\det A = |A| = ad - bc$ and

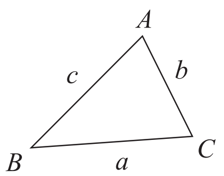
$A^{-1} = \dfrac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Measurement

Area of sector, $A = \frac{1}{2} r^2 \theta$, where θ is in radians.

Arc length, $l = r\theta$, where θ is in radians.

In any triangle ABC:



Area of triangle $= \frac{1}{2} ab \sin C$

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Quadratic equations

If $ax^2 + bx + c = 0$ then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Distance from a point to a plane

The distance from (x_1, y_1, z_1) to $Ax + By + Cz + D = 0$ is given by

$\dfrac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$.

Derivatives

$f(x) = y$	$f'(x) = \dfrac{dy}{dx}$
$\arcsin x$	$\dfrac{1}{\sqrt{1 - x^2}}$
$\arccos x$	$\dfrac{-1}{\sqrt{1 - x^2}}$
$\arctan x$	$\dfrac{1}{1 + x^2}$

Properties of derivatives

$\dfrac{d}{dx}\big(f(x)\,g(x)\big) = f'(x)g(x) + f(x)g'(x)$

$\dfrac{d}{dx}\left(\dfrac{f(x)}{g(x)}\right) = \dfrac{f'(x)g(x) - f(x)g'(x)}{\big(g(x)\big)^2}$

$\dfrac{d}{dx} f\big(g(x)\big) = f'\big(g(x)\big)g'(x)$

Arc length along a parametric curve

$l = \displaystyle\int_a^b \sqrt{\boldsymbol{v} \bullet \boldsymbol{v}}\, dt$, where $a \le t \le b$.

Integration by parts

$\displaystyle\int f'(x)g(x)\,dx = f(x)g(x) - \int f(x)g'(x)\,dx$

Volumes of revolution

About x axis, $V = \displaystyle\int_a^b \pi y^2\, dx$, where y is a function of x.

About y axis, $V = \displaystyle\int_c^d \pi x^2\, dy$, where y is a one-to-one function of x.