## Essential Mathematics

## Question booklet

Topic 2: Measurement (Questions 1 to 3) 30 marks
Topic 4: Statistics (Questions 4 to 6) 30 marks
Topic 5: Investments and loans (Questions 7 to 9) 30 marks

- Answer all questions
- Write your answers in this question booklet
- You may write on pages 12 and 24 if you need more space
- Allow approximately 40 minutes for each topic


## Examination information

## Materials

- Question booklet
- SACE registration number label


## Instructions

- Show appropriate working and steps of logic in this question booklet
- Use black or blue pen
- You may use a sharp dark pencil for diagrams and graphical representations
- Approved calculators may be used - complete the box below

Total time: 130 minutes
Total marks: 90
Attach your SACE registration number label here

| Graphics calculator |
| :---: |
| 1. Brand |
| Model |
| 2. Brand |
| Model |

## Question 1 (9 marks)

Eight identical rhombus shapes are used to make a star pattern on a square panel of material.
The star pattern and the dimensions of the square panel of material are shown below in Diagram 1.
The dimensions of one rhombus are shown in Diagram 2.

(a) Calculate the perimeter of the star pattern.

(b) Calculate the area of the star pattern.

(c) (i) Show that the area of the square panel of material is approximately $420 \mathrm{~cm}^{2}$.

(ii) Calculate the area of the square panel of material not covered by the star pattern (the shaded area in Diagram 1).

(1 mark)

The square panels of material are joined together to make square quilts.
(d) Nine square panels of material are used to make a square quilt, as shown in Diagram 3

Calculate the area of this square quilt, in square metres $\left(\mathrm{m}^{2}\right)$.


Diagram 3

(2 marks)
(e) Calculate the minimum number of square panels of material that would be needed to make another square quilt that has an area of at least $4 \mathrm{~m}^{2}$.

(2 marks)

## Question 2 (12 marks)

On an island, patrol dogs are used to protect penguins from foxes. Park rangers want to calculate the area of the island that the dogs will patrol.

A diagram representing the outline of the island with some measurements is shown below.


Diagram 4
(a) (i) Show that the length of $D F$ is 0.6 km .

(ii) Calculate the area of triangle $D E F$.

(1 mark)

The following measurements are also known for Diagram 4.
Angle $G H F=44^{\circ}$
Angle $G F H=30^{\circ}$
(b) (i) On Diagram 4, write the values for angle GHF and angle GFH.
(ii) Show that the length of $G H$ is approximately 0.73 km .

(3 marks)
(iii) Show that the area of triangle $F G H$ is approximately $0.35 \mathrm{~km}^{2}$.

## Note:

Area of triangle $A B C$, Area $=\frac{1}{2} a b \sin C$

(1 mark)

Each region of the island is identified by the numbers $1,2,3$, and 4 , as shown below in Diagram 5 . On the map, Region 2 has an area of $0.43 \mathrm{~km}^{2}$ and Region 4 has an area of $0.32 \mathrm{~km}^{2}$.


Diagram 5
(c) Calculate the total area of the island that the dogs need to patrol.

(1 mark)

When the tide goes out, a greater area of land is uncovered around the island, as shown below in Diagram 6. Note: The original land area is shaded.

(d) (i) Using Simpson's rule, calculate the approximate area of the land when the tide is out.

(ii) Calculate how much more land area the dogs will need to patrol when the tide is out (the non-shaded area in Diagram 6).


## Question 3 (9 marks)

A council plans to install a new fountain in a local park. The fountain is made from a central cube section with a quarter sphere joined to each side of the cube at the top.


Source: adapted from © Katerynabibro | Dreamstime.com

## Top view

Cross-section view
Image of a quarter sphere
[The diagrams are not drawn to scale.]
(a) (i) Calculate the volume of water held in the cube section of the fountain.

(1 mark)
(ii) State the radius of a quarter sphere, given that the cube has sides of length 3 metres.

(iii) Show that the total volume of water held in the four quarters is approximately $14 \mathrm{~m}^{3}$.

Note: Volume of a sphere, $V=\frac{4}{3} \pi r^{3}$

(2 marks)
(iv) Hence, calculate the total volume of water that the fountain can hold.



Source: adapted from © Nicolae Cirmu | Shutterstock.com
The fountain (shown above) has a motor that pumps water through one spout.
Every minute, 85.5 litres of water is pumped through the spout.
(b) Calculate how many litres of water will be pumped through the spout in 1 hour.

(c) (i) State the number of litres of water in the fountain if it contains $38.5 \mathrm{~m}^{3}$ of water.


The spout malfunctioned and water began pumping out of the fountain onto the ground.
(ii) Calculate how many hours it would take for all the water to be pumped out of the fountain.

(1 mark)
(iii) State one assumption that may impact the time calculated in part (ii).

(1 mark)

## Question 4 (7 marks)

The owner of a travel agency in a town with a population of 17000 residents wanted to find out the likelihood of the residents in the town travelling over the next 12 months.

The owner sent an email to all 250 clients who had booked travel through the agency. The survey asked the clients if they intended to travel in the next 12 months.

The owner received responses from 125 clients. All the respondents indicated that they were intending to travel in the next 12 months.

The owner made the following statement to the staff:
Fifty per cent of the population in the town were going to be booking travel in the next 12 months.
(a) Discuss the reasonableness of the statement made by the owner of the travel agency, referring to the survey method.

(2 marks)
(b) The owner decided to use a different survey method by getting a staff member to deliver a survey form to every second house in the town.
(i) State one advantage of this method.

(ii) State one disadvantage of this method.

(1 mark)
(c) State what type of survey is being used if the staff were to collect responses from every resident.

(1 mark)

One of the staff at the travel agency recommended using a stratified sampling method based on age groups. They intend to survey 500 people from the town's population of 17000 .
(d) (i) There are 680 people in the 20-25 age group in the town.

Calculate the number of people in this age group that should be included in the stratified survey.

(ii) There are 40 people in the 50-55 age group that will be surveyed using the stratified sampling method.
Calculate the total number of people in this age group in the town.

(1 mark)

You may write on this page if you need more space to finish your answers to any questions. Make sure to label each answer carefully (e.g. 3(c)(ii) continued).
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## Question 5 (15 marks)

A research group investigated the average amount of money (in dollars) spent on single takeaway food purchases for different age groups.
Table 1 shows the data for the amount of money (in dollars) spent on a single purchase by 12 individuals from the Gen $Z$ age group who were aged 18 to 25 years.

Table 1

| 70 | 86 | 65 | 50 | 73 | 62 | 56 | 74 | 49 | 68 | 81 | 90 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

(a) Using the data in Table 1, complete the ordered stem plot below for the Gen Z age group.

Amount of money spent on a single takeaway food purchase (\$)


Key: stem 3 leaf $\mathbf{1}=31$
(b) The three statements below compare the data for the Boomers to the data for Gen Z. Circle the appropriate highlighted word to make each statement correct.
(i) The data for the Boomers are less / more symmetrical than the data for Gen Z .
(ii) The data for the Boomers have a larger / smaller range than the data for Gen Z.
(iii) One of the Boomers / Gen Z has spent the lowest amount on a single takeaway food purchase.
(c) The amount spent on a single takeaway food purchase by one Boomer appears to be an outlier.
(i) State the outlier.

(ii) Explain which statistical value would be a better measure of how much Boomers generally spend on a single takeaway food purchase.

(d) Complete Table 2 below (correct to one decimal place).

Table 2: Money spent on a single takeaway food purchase (\$)

| Statistical measure | Boomers | Gen Z |
| :--- | :---: | :---: |
| minimum |  | 49.0 |
| mean | 53.9 |  |
| Q1 | 41.5 | 59.0 |
| median | 69.5 |  |
| Q3 |  | 77.5 |
| maximum |  | 90.0 |

The research group collected an additional data set (Data set 3). It is shown on the box-and-whisker diagram below.
(e) Complete and label the box-and-whisker diagrams below for the Boomers and Gen Z.

Money spent on a single takeaway food purchase (\$)

(3 marks)
(f) Tick the appropriate box below for the age group that you believe Data set 3 best represents the spending on a single takeaway food purchase.individuals in the age group 42 to 49 yearsindividuals in the age group 10 to 17 years

Justify your answer using information from the box-and-whisker plot.

(2 marks)

## Question 6 (8 marks)

A class of students was given data showing changes in global temperatures over time. The data recorded the annual mean temperature (in ${ }^{\circ} \mathrm{C}$ ) for a selection of years from 1900 that are provided in the table below.

Table 3: Changes in global temperatures

| Calendar year | 1900 | 1910 | 1920 | 1930 | 1950 | 1960 | 1970 | 1980 |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Number of years <br> since $1900(N)$ | 00 | 10 | 20 | 30 | 50 | 60 | 70 | 80 |
| Annual mean <br> temperature in ${ }^{\circ} \mathrm{C}(T)$ | 13.74 | 13.72 | 13.83 | 13.96 | 13.98 | 13.99 | 14.00 | 14.18 |

(a) State the dependent variable.

(b) Circle the appropriate highlighted word in the statement below to correctly describe the relationship between the two variables.

As the number of years since $1900(N)$ increases, the annual mean temperature ( $T$ ) increases / decreases.
(c) Calculate the coefficient of determination $\left(r^{2}\right)$ and state the strength of the relationship between the number of years since 1900 and the annual mean temperature.

(2 marks)

The students found that the equation of the least squares regression line (line of best fit) for this data was:

$$
T=0.0049 N+13.73
$$

where $\quad T=$ annual mean temperature
$N=$ number of years since 1900
(d) Using the least squares equation $T=0.0049 N+13.73$, calculate the predicted annual mean temperature for 2025 , which is 125 years after 1900.


The teacher then gave the students two extra data points. These two points are provided in the table below:

| Calendar year | 1840 | 1990 |
| :--- | :---: | :---: |
| Number of years since $1900(N)$ | -60 | 90 |
| Annual mean temperature in ${ }^{\circ} \mathrm{C}(T)$ | 13.50 | 14.31 |

(e) (i) Calculate the new coefficient of determination $\left(r^{2}\right)$ for the data with the two extra data points included.

(ii) Discuss whether the new equation of the least squares regression line, including the two extra data points, would be expected to give a better prediction for the annual mean temperature in 2025.

(2 marks)

## Question 7 (8 marks)

Jai has $\$ 57000$ to live on while attending university for 4 years. Jai deposits the money in an account that pays $3.25 \%$ per annum, compounded monthly.
(a) Show that Jai will be able to make regular monthly withdrawals of approximately $\$ 1270$ from the account.

(b) Calculate how much Jai will have in the account after 1 year of making the monthly withdrawals.


After 1 year, Jai withdraws a lump sum of $\$ 3500$ from the account to purchase textbooks and equipment.
(c) (i) Show that Jai will have approximately $\$ 39900$ in his account after he withdraws the lump sum of $\$ 3500$.

(1 mark)
(ii) Show that Jai will only have approximately 33 months of regular withdrawals after using the lump sum of $\$ 3500$ to purchase textbooks and equipment. Assume his regular withdrawals and the account conditions remain the same.

(1 mark)

Jai needs the remaining money to last for 3 years while he completes university.
(d) Calculate how much less Jai would need to withdraw every month for the money to last for 3 years. Assume the account conditions remain the same.

(2 marks)

| Planning for retirement |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Age now | Status | Expected | uring r | tirement ${ }^{\text {A }}$ | Amount needed in superannuation by age 65 |
| 57 years | Single |  | htly \$) | (annually \$) | (\$) |
|  |  | Low | 1269 | 33000 | 89000 |
|  |  | Average | 1654 | 43000 | 313000 |
|  |  | High | 2077 | 54000 | 742000 |
|  | Couple | Low | 1846 | 48000 | 115000 |
|  |  | Average | 2385 | 62000 | 409000 |
|  |  | High | 3077 | 80000 | 1034000 |
| (A)Spending levels are in today's dollars. Retirement is defined as age 65 to 90 and spending in retirement rises with price inflation (stays the same in real terms). |  |  |  |  |  |

Source: data from Super Consumers Australia 2022, 'Consultative Report: retirement spending levels and savings target March 2022', viewed 29 August 2022, www.superconsumers.com.au, p 4
(a) Referring to the table above, state the amount a couple aged 57 years with average spending during retirement would need to save by the age of 65 years.


The table suggests that a single person aged 57 years with average spending during retirement would need to save $\$ 313000$ by the age of 65 years.
Sam, aged 57 years and single, has $\$ 150000$ in a superannuation fund.
(b) (i) Show that Sam has 8 years to reach the goal of $\$ 313000$ before retirement.

(ii) Show that Sam would need to make a quarterly contribution of approximately $\$ 1650$ to reach the goal of $\$ 313000$ in 8 years if their fund earns $6.2 \%$ per annum, compounded quarterly.

(iii) State one assumption made in the calculation to find this quarterly contribution.


The notes under the table state that these estimates are in today's dollars.
(c) (i) Show that the estimated retirement amount that Sam would need to reach is approximately $\$ 370000$ if inflation is predicted to be at an average of $2.1 \%$ per annum over the 8 years.

(ii) Calculate how much longer it would take Sam to save the estimated retirement amount with inflation factored in. Assume that the superannuation fund conditions and payments remain the same.

(iii) Sam received a promotion at work and earned a higher salary.

State one effect this may have on Sam's retirement plan.

(1 mark)

## Question 9 (11 marks)

Sansa was spending an average of $\$ 35$ per week on fast food. Sansa's parents suggested that she eat at home and save the money in a savings account.
Sansa finds a savings account that pays $1.37 \%$ per annum, compounded weekly.
(a) Calculate how much Sansa saves in 6 months if she invested the $\$ 35$ per week in the savings account.

(b) Calculate the interest Sansa earns in the 6 months.

(1 mark)

After 6 months of this saving, Sansa withdrew all the funds and her parents gave her additional money so that she had a total of $\$ 1000$.

Sansa invested the $\$ 1000$ in a term-deposit investment for 6 months with the interest paid at the end of the term.
(c) (i) Calculate the simple interest rate required so that Sansa's term-deposit investment would have $\$ 1020$ in total at the end of the 6-month term.


Sansa also continued paying $\$ 35$ per week into the savings account earning $1.37 \%$ per annum, compounded weekly, for a second 6-month period.
(ii) Calculate the total amount of money that Sansa would have after the second 6-month period.


Sansa was able to invest $\$ 1900$ every year into an account that had interest compounded annually for 9 years.
(d) (i) Calculate the interest rate required so that Sansa's account would reach \$20000 in 9 years.

(ii) State one assumption made in part (d)(i).

(1 mark)
(iii) Explain the impact on the length of time it would take Sansa to have $\$ 20000$ in savings if the assumption in part d(ii) changed.

(2 marks)

You may write on this page if you need more space to finish your answers to any questions. Make sure to label each answer carefully (e.g. 9(d)(iii) continued).
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