South Australian Certificate of Education

The purpose of this sample paper is to show the structure of the 130-minute Specialist Mathematics examination and the style of questions that might be used. The examination will consist of questions that assess a *selection* of the key questions and key concepts from across the six topics.

1

Specialist Mathematics
November 2020 sample paper

Question booklet 1

- Questions 1 to 7 (51 marks)
- Answer ***all*** questions
- Write your answers in this question booklet
- You may write on page 17 if you need more space
- Allow approximately 65 minutes
- Approved calculators may be used — complete the box below

Examination information

Materials

- Question booklet 1
- Question booklet 2
- Formula sheet
- SACE registration number label

Instructions

- Show appropriate working and steps of logic in the question booklets
- State all answers correct to three significant figures, unless otherwise instructed
- Use black or blue pen
- You may use a sharp dark pencil for diagrams

Total time: 130 minutes
Total marks: 97

Attach your SACE registration number label here

Graphics calculator

1. Brand ____________

Model ____________

2. Brand ____________

Model ____________

Government of South Australia

Question 1 (6 marks)

Figure 1 shows rhombus $OPQR$ with $\overrightarrow{OP} = \boldsymbol{p}$ and $\overrightarrow{OR} = \boldsymbol{r}$.

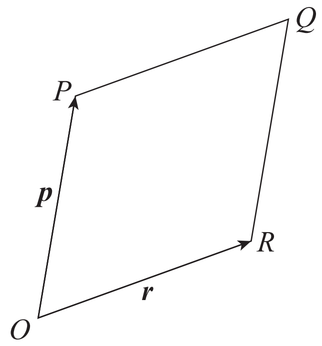

Figure 1

(a) (i) Find $\overrightarrow{OQ}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{r}$.

(1 mark)

(ii) Find $\overrightarrow{PR}$ in terms of $\boldsymbol{p}$ and $\boldsymbol{r}$.

(1 mark)

(b) (i) Show that $\overrightarrow{OQ} \bullet \overrightarrow{PR} = |\boldsymbol{r}|^2 - |\boldsymbol{p}|^2$.

(2 marks)

(ii) Hence prove that the diagonals of the rhombus $OPQR$ are perpendicular, giving reasons.

(2 marks)

Question 2 (7 marks)

A curve has the following parametric equations:

$$\begin{cases} x(t) = \sqrt{\cos t} \\ y(t) = \sin t \end{cases} \quad \text{where } 0 \le t \le \frac{\pi}{2}.$$

(a) Draw a graph of this curve on the axes in Figure 2.

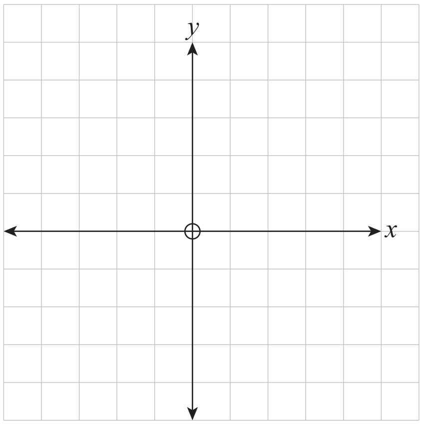

Figure 2

(3 marks)

(b) Show that all points (x, y) on the curve that you drew in Figure 2 satisfy the equation

$$x^4 + y^2 = 1.$$

(1 mark)

(c) Use implicit differentiation to show that $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-2x^3}{y}$, where $y \neq 0$.

(2 marks)

(d) Find the slope of the tangent to the curve at $t=\frac{\pi}{6}$.

(1 mark)

Question 3 (6 marks)

(a) (i) Write $\sqrt{2}+i\sqrt{2}$ in polar form.

(1 mark)

(ii) Write $\sqrt{2}-i\sqrt{2}$ in polar form.

(1 mark)

(b) (i) Write $z=\dfrac{\left(\sqrt{2}+i\sqrt{2}\right)^m}{\left(\sqrt{2}-i\sqrt{2}\right)^n}$ in simplest polar form, where m and n are integers.

(2 marks)

(ii) State a positive integer value for m and a positive integer value for n such that z is real.

(1 mark)

(iii) State a positive integer value for m and a positive integer value for n such that z is purely imaginary.

(1 mark)

Question 4 (7 marks)

(a) Use mathematical induction to prove that $2^{4n} - 3^n$ is divisible by 13 for all positive integers n.

(5 marks)

(b) Explain why $2^{2020} - 3^{505}$ is divisible by 13.

(2 marks)

Question 5 (7 marks)

A cone is formed as grain is poured into a large cylindrical container of fixed radius.

The volume of the cone of grain (in cubic metres) is given by

$$V = \frac{1}{3}\pi h^3 \tan^2\theta$$

where h is the height of the cone in metres and θ (in radians) is the angle shown in Figure 3.

Note that V, h, and θ are all functions of time.

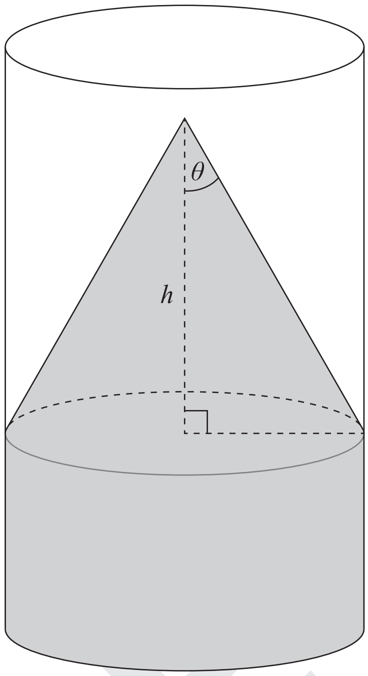

Figure 3

(a) Show that $\dfrac{dV}{dt} = \dfrac{1}{3}\pi h^2\left(2h\tan\theta\sec^2\theta\dfrac{d\theta}{dt} + 3\dfrac{dh}{dt}\tan^2\theta\right)$.

(3 marks)

(b) Consider the instant when $\theta = \frac{\pi}{6}$ radians and $V = 3\pi$ cubic metres.

(i) Find h at this instant.

(1 mark)

(ii) At this instant, $\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{\pi}{12}$ radians per second and the height of the cone is decreasing at approximately 1.81 metres per second.

Find $\frac{\mathrm{d}V}{\mathrm{d}t}$.

(3 marks)

Question 6 (10 marks)

(a) Show that $\frac{1}{x-2} - \frac{1}{x+3} = \frac{5}{(x-2)(x+3)}$.

(1 mark)

Let $f(x) = \frac{5}{(x-2)(x+3)}$.

(b) (i) Draw the graph of $y = f(x)$ on the axes in Figure 4.

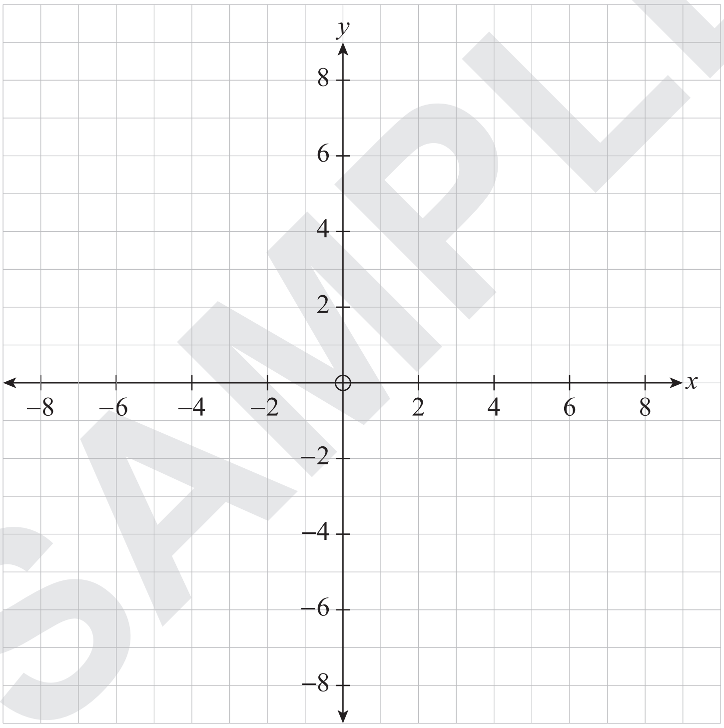

Figure 4

(3 marks)

(ii) Draw the graph of $y=\left|f\left(x\right)\right|$ on the axes in Figure 5.

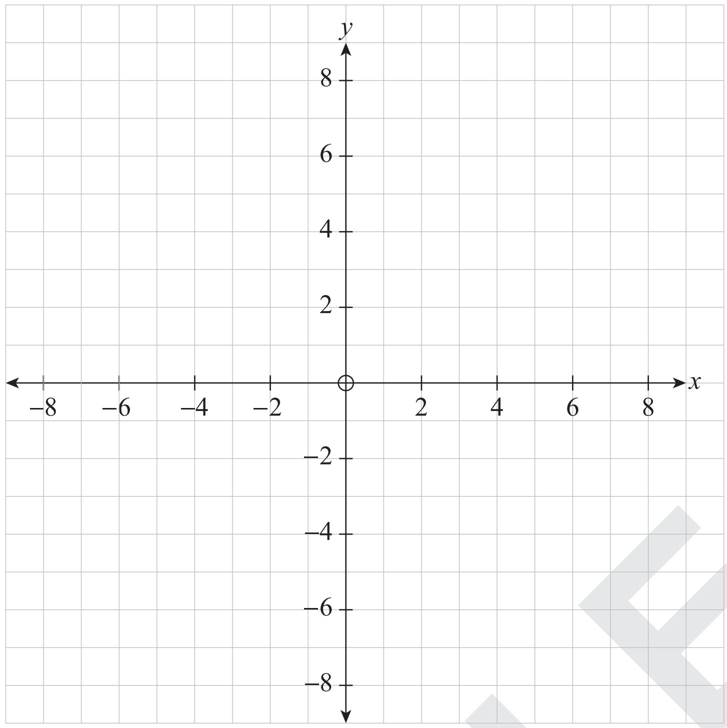

Figure 5 (1 mark)

(iii) Draw the graph of $y=\left|f\left(x\right)\right|-f\left(x\right)$ on the axes in Figure 6.

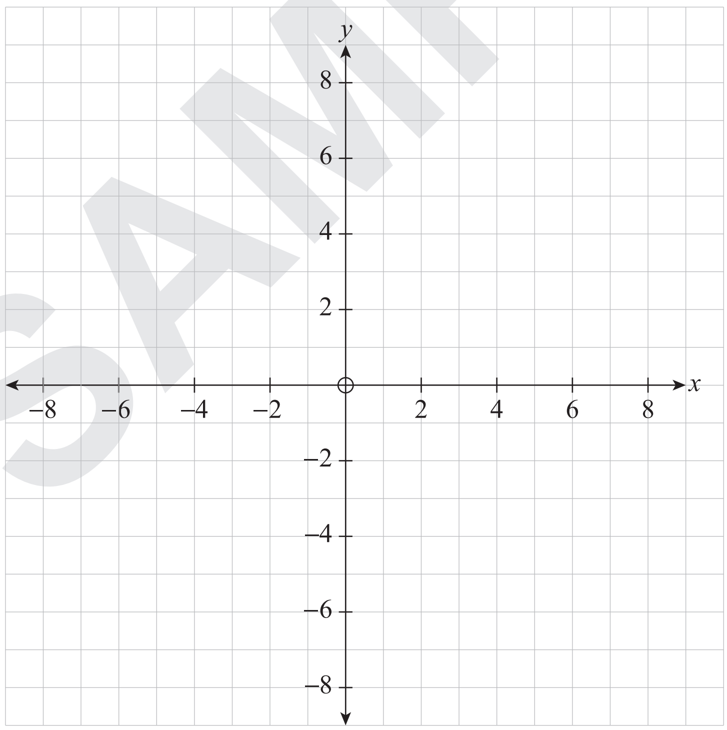

Figure 6 (2 marks)

(c) Find the exact area between the graph of $y = \left| f(x) \right| - f(x)$, the x-axis, and the lines $x = -2$ and $x = 1$.

(3 marks)

Question 7 (8 marks)

(a) (i) Use integration by parts to find $\int xe^{2x}\mathrm{d}x$.

(3 marks)

(ii) Use integration by parts to show that

$$\int x^2e^{2x}\mathrm{d}x = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + c\text{, where } c \text{ is a constant.}$$

(2 marks)

(b) Let $f(x) = xe^x$.

The graph of $y = f(x)$ for $x \geq 0$ is shown in Figure 7.

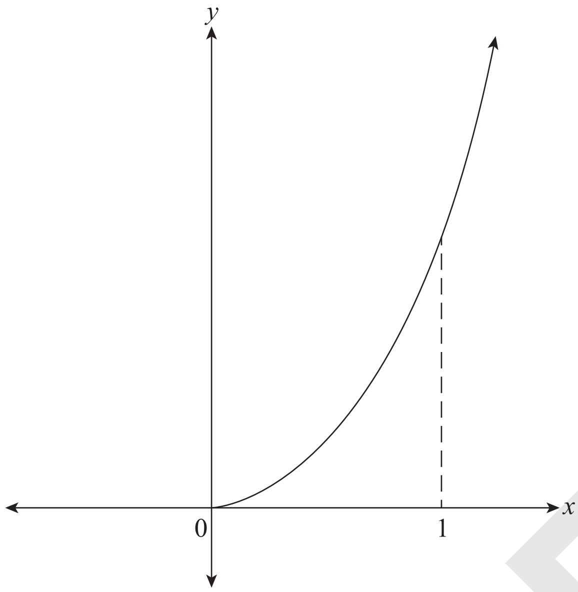

Figure 7

Find the exact volume of the solid obtained when the region bounded by the graph of $f(x)$ on the interval $[0, 1]$ is rotated about the x-axis.

(3 marks)

You may write on this page if you need more space to finish your answers to any of the questions in Question booklet 1. Make sure to label each answer carefully (e.g. 7(a)(i) continued).

This sample Specialist Mathematics paper shows the format of the examination from November 2020.